Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(6):1339-1346

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

ENO morphological wavelet and its application in signal processing

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ABSTRACT

The one dimension ENO (essentially non-oscillatory) MW (morphological wavelet) algorithm is proposed, using the characteristics of ENO interpolation basically without oscillation, and improved reconstruction strategy. The original signal is decomposed by MW algorithm, and reconstructed according to ENO interpolation method. The algorithm is applied to the signal processing, results show the mentioned ENO MW algorithm has better profermance than the existing MW and has good application value.

Key words: Morphological wavelet(MW); essentially non-oscillatory (ENO) interpolation; ENO morphological wavelet; mathematical morphology

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INTRODUCTION

In 2000, Goustias [1] proposed the concept of morphological wavelet (MW), unified the most linear and nonlinear wavelet successfully and formed a united framework for multi-resolution analysis. Morphological wavelet as a branch of wavelet theory nonlinear extension studies, and is the description methods that can be completely reconstructed and non-redundant, which is based on the nonlinear characteristics of mathematical morphology.

Morphological wavelet transform is used to the time-frequency characteristics of signal by Linearity of wavelet transform and non-linearity of morphological operator combining, and has an extremely important significance in theory and in application. Morphological Wavelet analysis is a nonlinear analysis; it is closer to the nonlinear characteristics of the signal itself, the sub-band signals is better reflected the original signal after decomposed. At present, there are two branches in the morphological wavelet researching, one is the morphological wavelet that proposed based on [1], which is a non-dual wavelet essentially; the other is a dual wavelet [2] that based on various mathematical morphology operators, which is a multi-level morphological filters essentially that meeting pyramid reconstruction conditions, which is the generalized morphological wavelet. The morphological wavelet notable feature is non-linear and can well reflect the characteristics of the original signal, which is the approximate signal got by decomposition, can well describe the details of the original signal.

However, morphological wavelet reconstruction strategy is interpolation in every two points for approximation signal, the newly inserted point value is the same as the previous point, and then the detail signal is joined in the odd and even bits using different methods, which naturally ensures the signal perfect reconstructed, so when the detail signal is intercepted through the threshold value, the modified detail signal bits are mostly zero, resulting the next data is identical with the previous data in the reconstructed signal, that is a stepped waveform appearing in the reconstruction. This phenomenon is extremely unfavorable in one-dimensional or two signal processing. ENO (Essentially Non-Oscillatory) interpolation technique [3] provides a good reference for the non-linear reconstruction of data-dependent; the advantage of this method has no data concussion basically after interpolated, but also avoiding the Kyrgyzstan Booth effects of discontinuity.Multi-resolution analysis is proposed based on ENO interpolation in [4], however, it is not a wavelet technique essentially, and the frequency invariant characteristics are not guaranteed when signal analyzing[5-6].

Therefore, ENO morphological wavelet algorithm is proposed, and it had two steps: morphological wavelet decomposition and ENO interpolation reconstruction. The signal is expanded into a one-dimensional ENO morphological wavelet algorithm, and its characteristics and effects were analyzed.

ONE-DIMENSIONAL ENO MORPHOLOGICAL WAVELET

Mathematical Morphological basic transformation

Mathematical morphology is mathematical methods that developed on the basis of set theory and is different from the time, frequency [8-10].the target signal is describe by set, a structure element, called "probe", is designed in signal processing, the useful information can be extracted to characterize and describe through which the probe is constantly moving in the signal. The common operations are erosion, dilation, opening, closing, etc., are defined as follows.

Definition 1 assume A is a set, B is the structuring element, then the corrosion transform A to B is defined as:

$$A \Theta B = \{x : B + x \supset A\} \tag{1}$$

Definition 2 assume A is a set, B is the structuring element, then the expansion transform A to B is defined as:

$$A \oplus B = \cup\{x : A + b : b \in B\} \tag{2}$$

Definition 3 assume A is a set, B is the structuring element, then the opening transform A to B is defined as:

$$A \circ B = (A \Theta \hat{B}) \oplus B \tag{3}$$

Definition 4 assume A is a set, B is the structuring element, then the closing transform A to B is defined as:

$$A \bullet B = (A \oplus \hat{B}) \Theta B \tag{4}$$

Four operations as above is the morphological basic operation. The realization of morphological transformation generally only contains Boolean operations, addition and subtraction from the above definitions, calculation is simple and computing speed is fast.

One-dimensional morphological wavelet

Morphological wavelet as a branch of wavelet theory nonlinear extension studies, and is the description methods that can be completely reconstructed and non-redundant, which is based on the nonlinear characteristics of mathematical morphology. Multi-resolution pyramidal decomposition, including linear and non-linear pyramidal decomposition, which is the platform the morphological wavelet generated. Morphological pyramidal decomposition is a nonlinear pyramidal decomposition, the morphological operators such as erosion and expansion et al are introduced successfully into multi-resolution technology, and a series of non-linear morphological pyramid is obtained. It has a sound theoretical structures and algorithms framework.

First, the wavelet is transformed into the morphological wavelet; the main difference is that the wavelet using a linear signal analysis filter and the morphological wavelet filters using corrosion or swelling filter in signal analyzing.

Analysis and synthesis operators in linear wavelet can be expressed as convolution form.

$$\Psi^{\uparrow}(x)(n) = \sum_{k=-\infty}^{\infty} \tilde{h}(2n-k)x(k) \tag{5}$$

$$\Psi^{\downarrow}(x)(n) = \sum_{k=-\infty}^{\infty} h(2n-k)x(k) \tag{6}$$

Take the kernel function

$$\begin{cases} \tilde{h}(-1) = \tilde{h}(0) = \dfrac{1}{2}; \tilde{h}(n) = 0, n \neq -1, 0 \\ h(0) = h(1) = 1; h(n) = 0, n \neq 0, 1 \end{cases} \tag{7}$$

The corresponding operators of synthesis and decomposition are:

(8) $\Psi^{\uparrow}(x)(n) = 0.5 \times x(2n) + x(2n+1)$

(9) $\Psi^{\downarrow}(x)(2n) = \Psi^{\downarrow}(x)(2n+1) = x(n)$

$\Psi^{\uparrow}\Psi^{\downarrow} = \sum_{k=-\infty}^{\infty} \tilde{h}(2n-k)h(k) = \delta(n)$ can be proved, that is analysis and synthesis operators meet bi-orthogonal condition. In fact, the analysis and synthesis operators are consistent with the low pass filter of the Haar wavelet, that the above formula is the Haar wavelet transform.

Morphological filtering operator such as erosion and expansion replaced linear analysis and synthesis operators in above equation, there have

(10) $$\Psi^{\uparrow}(x)(n) = (x\Theta A)(2n) = \wedge_{k\in A} e_{k-2n}(x(k))$$

(11) $$\Psi^{\downarrow}(x)(k) = (x \oplus A)(2n) = \vee_{n\in A} d_{k-2n}(x(n))$$

"∧", "∨"are operators that take the minimum and maximum.

If A= {0, 1} was taken, there has

(12) $\Psi^{\uparrow}(x)(n) = x(2n) \wedge x(2n+1)$

(13) $\omega^{\uparrow}(x) = x(2n) - x(2n+1)$

(14) $\Psi^{\downarrow}(x)(2n) = \Psi^{\downarrow}(x)(2n+1) = x(n)$

(15) $\omega^{\downarrow}(y)\ (2n) = y(n) \vee 0,\ \omega^{\downarrow}(y)\ (2n) =: (y(n) \wedge 0)$

Formula (12) - (15) constitute a one-dimensional morphological Haar wavelet. The morphological Haar wavelet is a non-dual wavelet that can be proved.

The difference between the morphological wavelet and the wavelet defined by formula (5), (6) is the minimum operation is used in the morphological wavelet signal analysis operator and the averaging operation is used in the wavelet signal analysis operator from the definition of one-dimensional morphological wavelet. Only simple maximum, minimum and addition and subtraction are involved in one-dimensional morphological wavelet, the operation process is simple and the operation speed is fast. The morphological wavelet is the morphological filtering operator instead of the linear analysis and synthesis operators of the wavelet, so the morphological wavelet is a nonlinear wavelet.

ENO interpolation

ENO interpolation method is used to calculate shock capturing in the hydrodynamic. The method is that the piecewise polynomial is defined adaptively to approach a given signal function according to this signal function smoothness.

Definition 4 assume $H_m\ (x_j;w)$ is the continuous m order interpolation function at the point $\{x_i\}$ and w piecewise, that is

(16) $H_m(x_j;w) = w(x_j)$

(17) $H_m(x;w) = q_{m,j+1/2}(x;w) \quad x_j \leqslant x \leqslant x_{j+1}$

$q_{m,j+1/2}(x;w)$ is the m order interpolation polynomial that obtained by interpolated to w (x) on m +1 consecutive nodes $\{x_i\}$, $i_m(j)$ is grid cell number of the leftmost in interpolation base frame.

(18) $$q_{m,j+1/2}(x_i;w) = w(x_i),\quad \begin{matrix} i_m(j) \leqslant i \leqslant i_m(j)+m \\ 1-m \leqslant i_m(j)-j \leqslant 0 \end{matrix}$$

There have different reconstruction polynomial correspond to different $i_m(j)$, so there are m polynomials of order m. the most smooth base frame is selected as the interpolation base frame from m base frames including x_j, x_{j+1}, so $w(x)$ is the most smooth distribution to gradual sense on $(x_{im(j)}, x_{im(j)+m})$.

The most crucial idea of ENO method is interpolation base frame selected by using adaptive technology, thus ensuring the method has ENO properties. The smoothness of $w(x)$ distribution is reflected by the difference quotient of w. the difference quotient table as follows is established by recursive method.

$$w[x_j]=w(x_j)$$

(19) $$w[x_j, \cdots, x_{j+k}]=(w[x_j, \cdots, x_{j+k}]-w[x_j, \cdots, x_{j+k-1}])/(x_{j+k}-x_j)$$

If w is C_∞ on $[x_j, x_{j+k}]$, then there has

(20) $$w[x_i,...,x_{i+k}]=\frac{1}{k!}\frac{d^k}{dx^k}w(\xi_{i,k}) \quad , \quad x_i \le \xi_{i,k} \le x_{i+k}$$

However, when the p-order $(0 \leqslant p \leqslant k)$ derivative is discontinuity on the interval,

(21) $$w[x_i,...,x_{i+k}]=o(h^{-k+p}[w^{(p)}])$$

$[w^{(p)}]$ represents the p-order derivative is in intermittent.

Formula (20) and (21) show that $|w[x_i, \cdots, x_{i+k}]|$ is smooth distribution criterion of w to gradual sense on (x_i, x_{i+k}), that is w is smooth distribution on (x_{i_1}, x_{i_2+k}),and there has intermittent on (x_{i_2}, x_{i_2+k}).There will always have $\left|w[x_{i_1},...,x_{i_1+k}]\right| < \left|w[x_{i_2},...,x_{i_2}+k]\right|$ when h is small enough, thus the interpolated base frame is determined, $i_m(j)$ can be determined by recursive method in programmer. Basic steps as follows:

1) A first-order reconstruction polynomial is $q_{1,j+1/2}$ on (x_j, x_{j+1}),there has $i_1(j)=j$;

2) A k-order reconstruction polynomial is $q_{k,j+1/2}$ on $x_{i_k(j)},...,x_{i_k(j)+k}$, the base frame start point is $i_k(j)$.

3) A k +1 order reconstruction polynomial $q_{k+1,j+1/2}$ is constructed by joining a cell on the left or right of $x_{i_k(j)},...,x_{i_k(j)+k}$,that is $i_{k+1}(j)= i_k(j)-1$or $i_{k+1}(j)= i_k(j)$.

The interpolation base frame is determined by selecting the minimum absolute difference quotient of w on these two base frames.

(22) $$i_{k+1}(j)=\begin{cases} i_k(j)-1 & \left|w[x_{i_k(j)-1},...,x_{i_k(j)+k}]\right| < \\ & \left|w[x_{i_k(j)},...,x_{i_k(j)+k+1}]\right| \\ i_k(j) & \text{others} \end{cases}$$

The reconstruction polynomial $H_m(x;w)$ that obtained on the base frame above mentioned has no oscillation substantially.

One-dimensional ENO morphological wavelet algorithm

The approximate signal is obtained by taking bigger on adjacent elements when in signal decomposing in morphological wavelet algorithm, and thus after multi-layer decomposition and effectively filtering, the original signal periodical features can extracted.

At the same time, the original signal boundary feature is retained, and avoided losing the edge information because of too smooth compared the approximate signal after decomposed to the original signal. However, the morphological wavelet reconstruction strategy, the first is interpolation in every two points for approximation signal, the newly inserted point value is the same as the previous point, and then the detail signal is joined in the odd and even bits using different methods, which naturally ensures the signal perfect reconstructed, so when the detail signal is intercepted through the threshold value, the modified detail signal bits are mostly zero, resulting the next data is identical with the previous data in the reconstructed signal, that is a stepped waveform appearing in the reconstruction. Therefore, it is necessary to improve the morphological wavelet reconstruction strategy. The focus to be improved is that make the signal reconstruction smooth, and without affecting the signal frequency distribution.

ENO morphological wavelet algorithm is proposed in this paper; basically no oscillation interpolation is achieved to the approximate signal using ENO algorithm, both the boundary information and the original signal period are used which is reflected accurately by the approximate signal obtained by the morphological wavelet decomposition, and basically no oscillation interpolation is used too.

One-dimensional ENO morphological wavelet algorithm consists of two main steps:
1) The approximate signal ($x_{2j}(j=1, \cdots,N/2)$) and the detail signal ($y_{2j}(j=1, \cdots,N/2)$) are obtained by decomposition the one-dimensional morphological wavelet which the length of the original signal x_{1j} ($j=1, \cdots,N$) is N according to formula (12), (13).
2) Take the approximate signal and expand, $x_{2j}{}' =[x_{2j}\ x_{2N/2}]$ ($j=1,\cdots,N/2$) will be got, every point interpolation is achieved as above mention according to ENO interpolation algorithm to signal $x_{2j}{}' =[x_{2j}\ x_{2N/2}]$, and $x_{1j}{}''(j=1,\cdots,N+1)$ will be got, the first N points are truncated, and $x_{1j}{}'$ ($j=1,\cdots,N$) will be got, $x_{1j}{}'$ is the ENO reconstructed signal of the approximation signal obtained by the one-dimensional morphological wavelet decomposition.

The above is an algorithm of the one-layer ENO morphological wavelet decomposition and reconstruction; multi-layers decomposition should be expanded based on one-layer algorithm. The ENO morphological wavelet can not be reconstructed completely the original signal, but will be got better results comparing a morphological wavelet threshold reconstruction or single reconstruction.

ONE-DIMENSIONAL ENO MORPHOLOGICAL WAVELET ALGORITHMS IN SIGNAL PROCESSING
In order to discuss the results that one-dimensional ENO morphological wavelet in signal processing, vibration signals of X that a power plant steam turbine bearing generated are analyzed, the turbine speed is 3030/min, take 1024 points for analyzing, and shown in Figure 1, It can be seen that the signal is in periodic and with noise.

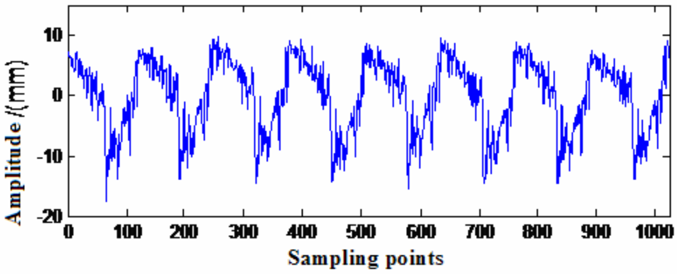


Fig.1 Original signal

Three-layer morphological wavelet decomposition is carried out to signal shown in Figure 1, as shown in Figure 2, Ca1, Ca2, Ca3 are the approximation signals of the first layer, the second layer, the third layer decomposition respectively, Cd1, Cd2, Cd3 are the detail signals of the first layer, the second layer, the third layer decomposition respectively, morphological wavelet decomposition is filtering decomposition, approximation signal is the low frequency filtered of original signal, the detail signal is the high frequency filtered of original signal. As can be seen, Ca1 is able to respond to the original signal cycle regularity, but still with noise, further decomposition Ca2 is smoother than Ca1 and Ca3 is the better than Ca2. As can be seen, Ca3 well extracted the low frequency portion of the original signal.

Threshold filtering reconstruction are carried to the approximate signal Ca3 and detail signal Cd1, Cd2, Cd3, the results shown in Figure 3 (a). From figure 3(a) there appeared many steps in reconstructed waveform and the signal is not smooth enough.Ca3 can be well expressed the waveform characteristics of the original signal in Figure 3 (b), but the length of the signal is shorter than the original signal's, the length is of the signal is 1/8 of the original signal's.

The waveform in Figure 3 (c) is generated according to the one-dimensional ENO morphological wavelet algorithm. Figure 3 (c) is the results of the Figure 3 (b) basis interpolation without oscillation and well extracted the characteristics of the original signal. For further observation and comparison, Figure 3 are local amplified and obtained Figure 4, the reconstructed effect of the one-dimensional ENO morphological wavelet is the best from Figure 4, with no step waveform, and is more smooth than the approximation signal (Figure 3(b)) obtained by the morphological wavelet decomposition, the step waveform is evident in morphological wavelet threshold reconstruction, and the reconstructed signal is distortion.

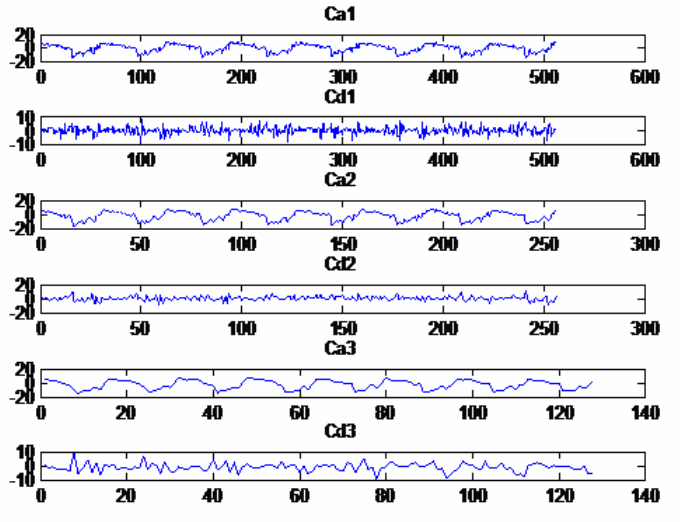
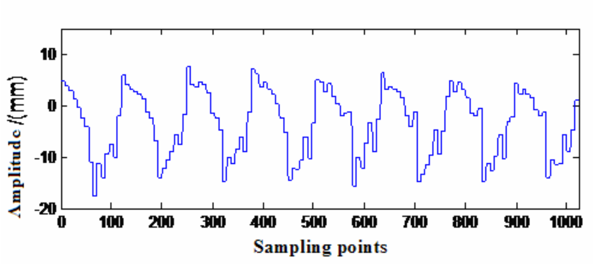
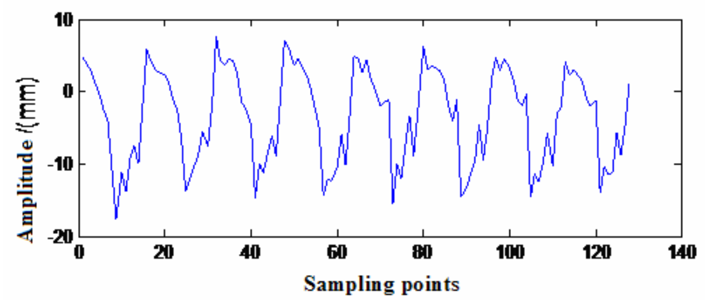


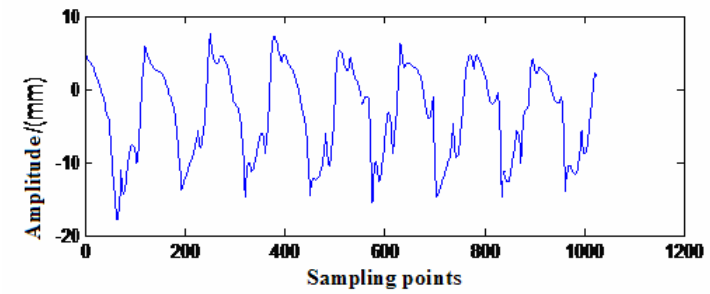
Fig.2 Signals after 3-layer MW decomposition



(a) MW threshold reconstructed signal

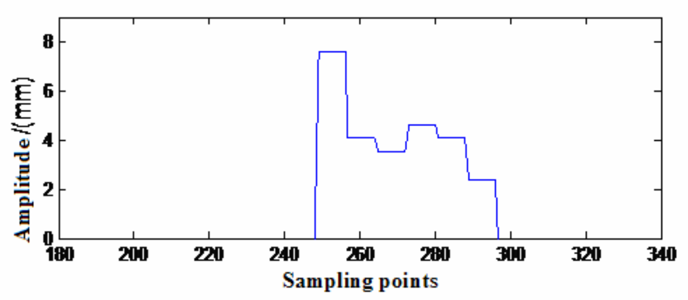


(b) MW decomposition signal Ca3

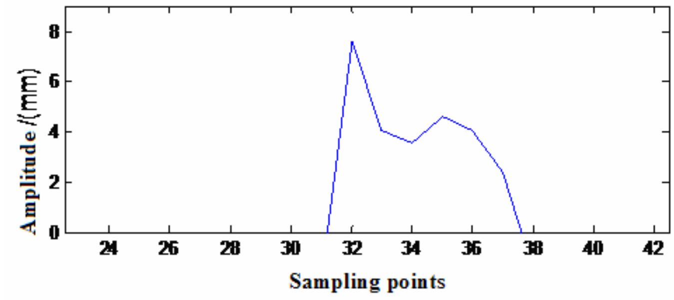


(c) ENO MW reconstructed signal

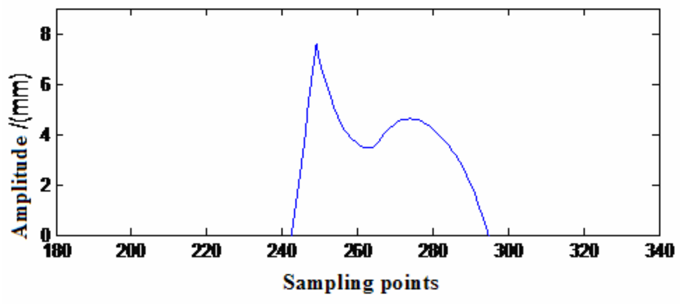
Fig.3 Reconstructed signals with various methods



(a) MW threshold reconstructed signal



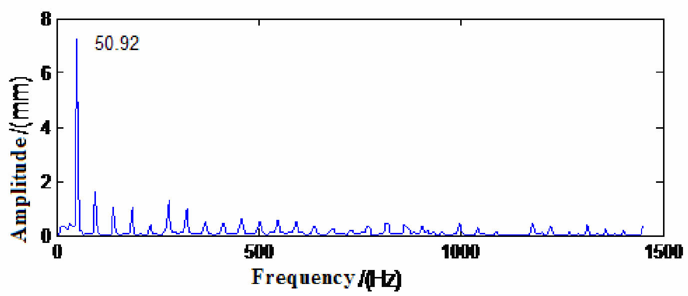
(b) MW decomposition signal Ca3



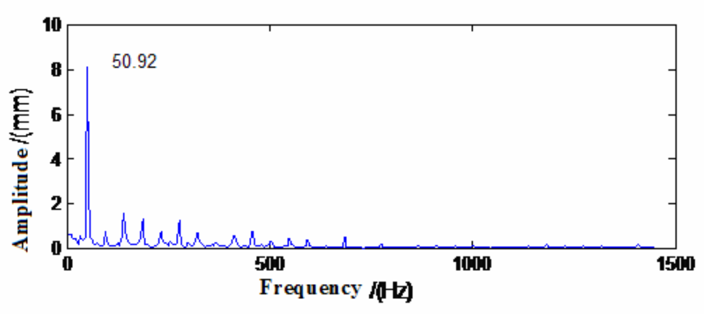
(c) ENO MW reconstructed signal

Fig.4 Local amplification of Fig.3

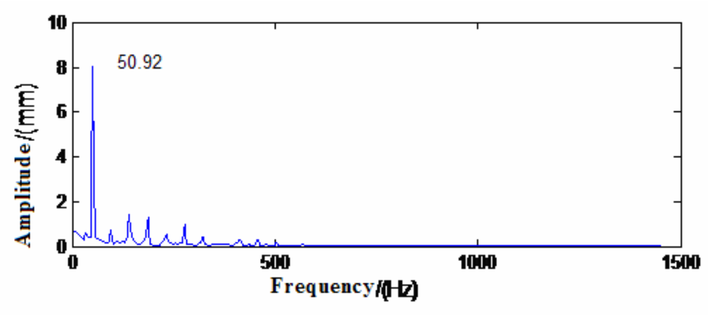
Signal decomposition and reconstruction algorithm must meet the signal frequency unchanged to the original signal. Figure 5(a),(b),(c) are the spectrum of the original signal, Ca1 and the signal analyzed by one-dimensional ENO morphological wavelet for Ca1 signal respectively. 50.92Hz is prominent, with noise signal in part of high-frequency in Figure 5 (a). 50.92Hz is prominent, while the high frequency (higher than 500Hz) is attenuated in Figure 5 (b), the approximate signal decomposed by morphological wavelet is the low frequency filtered to the original signal. 50.92Hz is still prominent in Figure 5 (c), which indicating ENO morphological wavelet algorithm does not change the signal frequency distribution, and compared to Fig 5 (b), the high frequency (500Hz up and down) have a section of attenuation, which is consistent with signal in Figure 4 (c) is more smooth than signal in Figure 4 (b).ENO morphological wavelet algorithm can ensure that the signal frequency unchanged which is validated, while there is a certain filtering effect, and suitable for applying to the signal processing.



(a) Spectrum of original signal



(b) Spectrum of MW the approximate signal Ca3



(c) Spectrum of 1-dimension ENO MW reconstructed signal
Fig.5 Spectrum of original signal and reconstructed signals with various methods

CONCLUSION

Morphological wavelet as a nonlinear wavelet, and is better to extract detail characteristics of the signal, and ENO interpolation method can realize interpolation no oscillation on the whole, one-dimensional ENO morphological wavelet algorithm is proposed combining the both advantages in this paper and applied to the one-dimensional signal processing, the results show that the algorithm is better than morphological wavelet algorithm and has better results.

Acknowledgments
This work was supported in part by a grant from the Education Department Zhejiang Province (the number is Y201018585),under which the present work was possible, to express my gratitude.

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