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Edge collapse improved model-based three dimensional surface model data compression algorithm

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ABSTRACT

With computer technology and three-dimensional imaging software and hardware technology rapidly development, three-dimensional digital technology is more and more concerned and applied by people. But limit graphics card handling ability and network bandwidth seriously affect such media transmission, simply rely on increasing hardware equipment input is not enough to solve problems, it should adopt corresponding algorithms to compress three-dimensional surface model data so that can fundamentally solve the drawback. Though some common used compression algorithms and tools can be used to reduce three-dimensional geometric data sizes, efficiency is not ideal or it is improper for some specific applications, three-dimensional model and previous voice frequency(one-dimensional), image(two-dimensional), video(two-dimensional+time) the three kinds of medias types are different, three-dimensional network model also reflects as random crook, complex and lacking of continuous natural parameterization features, and each kind of attributes that are defined in model surface is non-regular sampled, which let classic orthogonal analysis tools cannot be directly used to handle 3D geometric signal. Therefore, look for new algorithms to effective express three-dimensional model is imperative. The paper based on Garland proposed surface model compression algorithm, it puts forward a kind of edge collapse-based surface data compression algorithm, the algorithm calculates edge collapse cost and defines each edge collapse order according to quadric error measure criterion, judges shrinkage legality according to half-space test method, by the algorithm, it implements surface model boundary zone and interior zone synchronization compression, it not only can ensure original model geometric features on the condition of high compression ratio, but also can effective ensure compression model quality.

Key words: Edge collapse, compression algorithm, three-dimensional surface, quadric error, half-space test

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INTRODUCTION

Three-dimensional laser scanning system, is also called three-dimensional laser imaging system, is mainly composed of three-dimensional laser scanner and system software, the purpose is fast, convenient, correct gaining object space three-dimensional coordinate, establishing three-dimensional space model so that can further analyze, make data processing and apply on model [1-3]. Three-dimensional model as the fourth kind of multimedia data type plays increasingly important roles in industrial manufacturing, architecture, medical science, analog simulation and entertainment as others numerous fields. Internet rapid development also let three-dimensional model application to be more and more widely, network bandwidth restriction seriously hinders the media transmission, though PC machine graphics card data handling ability and network bandwidth have already made a lot of progress, it is still poor by comparing to abundant details and complicated three-dimensional network model amazing data sizes, therefore it should adopt high efficient coding way so that can solve its data storage and network transmission problems [4-8]. In virtual reality, three-dimensional geographic information system, interactive visualization and other fields, generally it takes triangulated irregular network, TIN as descriptive space objects surface features basic elements, and these triangulated network is forming by connection three-dimensional features points that lie in objects surface. By far, in view of research fields, surface modeling technique has expanded from traditional researching on surface expression, surface intersection and surface joining to surface deforming, surface reconstruction, surface simplification, surface transformation and surface potential difference, surface reconstruction needs to collect lots of data, the established model is generally very complicated [9, 10].

Lots of domestic and foreign scholars have made widely and deepen researches on three-dimensional surface model data compression method, from which the representative were: Schroeder and others put forward vertex deletion-based surface model data compression algorithm [1]; Garland proposed edge collapse-based surface model data compression method [2]; Hamann put forward triangle contracted-based surface model data compression method [3]; Turk put forward network redistributed-based surface model data compression method [4]; Hoppe and others proposed overall gridding optimization criterion-based surface model data compression method [5]. Its research result was well applied, which provided well theoretical basis for the technology development. In addition:Xu Min and Zhang Yong-Sheng from School of Geodesy and Geomatics, University of information engineering in “Three-dimensional model data compression technical analysis”, they introduced three-dimensional model data compression algorithms classification, and then combined with different classification status, made comparative analysis of some representative kinds of compression algorithms, and put forward summary and expectation in some aspects on the technique development [6]; Zhejiang University’s CAD& Wang Peng-Jie ,Pan Zhi-Geng as well as CG national key laboratory’s Liu Yong-Kui in “Point-based three-dimensional model compression technique research progress”, they summarized point model compression flow, and provided point model compression algorithm classification principle, and then took single resolution ratio algorithm and progressive algorithm as main line to detailed introduce point model compressed latest research progress, analyzed and compared corresponding key technologies [7]; Southeast University’s geographic information engineering department’s Cai Xian-Hua and Zheng Tian-Dong in “Digital elevation model data compression algorithm research” proposed DEM data compression, DEM terrain compression error concepts, on this basis, suggested a kind of qualification selection data compression method that based on DEM terrain compression error influence, and utilized the method could provide a kind of technical way for DEM digital integration, reduced DEM storage data sizes, and improved DEM subsequent handling speed [8]; Ocean University of China’s College of information engineering, department of computer science’s Ma Ping and University of science and technology of Qing Dao’s Institution of Qing Dao’s Jia Jian-Feng in “ Three-dimensional surface texture image compression”, started from wavelet analysis-based image compression, systematically introduced wavelet analysis and researched on two-dimensional wavelet analysis application in image compression, in the paper, it introduced three-dimensional surface texture theory frame, on the basis of the frame, it carried out wavelet analysis applied research on three-dimensional surface texture compression, expanded wavelet analysis in image compression field, and verified method validness with experiment and data [9]. By far the uppermost of gridding model compression is topology compression method, the paper studies and analyzes edge collapse principle, and applies it into three-dimensional surface model data compression, which provides theoretical basis for the algorithm implementation.

EDGE COLLAPSE ALGORITHM PRINCIPLE

Variable definition and theorem

As Figure 1shows, surface model sampling point set $V_{sub} = \{v_0, v_1, v_1^n, v_2^n, ..., v_8^n\}$, is composed TIN sampling point itself that is a partial triangulation network of TIN composed of V_{sub}. In order to easy to describe, it makes following definitions:

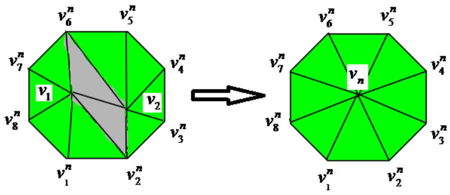

Figure 1: Edge collapse operation schematic diagram

Definition 1 A connected, node interlinked entity surface TIN model is a limited triangle sequence; the sequence meets following five conditions:

In surface model, every edge is shared by two triangles at most:

- Surface model vertex can be shared by any multiple triangles;
- Every triangle at least share a vertex with other triangles;

● If triangle T_i one vertex is simultaneously shared by another triangle T_j, and then the vertex is also the vertex of triangle T_j;

Triangle is complemented and overlapped.

Definition 2 For connected, node interlinked surface model, define edge E_i correlated triangle template P_i as all and edge E_i at least one vertex shared triangle T_i set.

Definition 3 Call triangle P_i vertex set(not including contracted two vertexes v_1, v_2) as boundary vertex set. By boundary vertex set paired order (clockwise or anticlockwise) connection composed polygon is called boundary polygon.

Definition 4 Edge collapse is the process that surface combines requested two vertexes into one vertex and deletes degenerated triangle during surface data compression process.

Figure 1 provided edge collapse operation process: simplify edge (v_1, v_2) into a new vertex v_n, and let each vertex that connects to the edge terminal vertex to connect with new vertex, and meanwhile delete all degenerated edges and surfaces.

There are two strategies for new vertex selection after edge collapsing: one is optimization selection method, that is to calculate space position v_n, let collapsing caused error cost to be minimum, it can look v_n in two points connection line, and can also look for v_n in total space; Another is sub set selection method, that is to select edge vertexes v_1 and v_2, one of them that let collapsing cost to be minimum is new vertex position. The second algorithm not only has advantages in complexity aspect but also is beneficial to form into progressive mesh by comparing to the first algorithm.

Quadric error measurement

Whether one edge can be collapsed or not is decided by its cost size, and cost value is got form edge surrounding information through error measuring. Use quadric error measurement to measures edge collapse cost, allocate every vertex an error measurement matrix, and calculate after edge collapsing generated new vertex error matrix and error value by the matrix, constantly collapse current model's minimum error cost edge to simplify model, till it meets model simplification requests.

Set v is one point in two-dimensional space, coordinate is $[v_x, v_y, v_z, 1]^T$; p represents one plane in space, its equation is as formula (1) shows:

$$ax + by + cz + d = 0 \tag{1}$$

In formula (1), each coefficient meets $a^2 + b^2 + c^2 = 1$, then the plane is recorded as $p = [a, b, c, d]^T$, point v to plane p distance square is as formula (2) shows:

$$d^2(v) = \left(p^T v\right)^2 = v^T \left(pp^T\right) v = v^T K_p v \tag{2}$$

In formula (2), K_p represents 4×4 matrix as formula (3) shows:

$$K_p = pp^T = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix} \tag{3}$$

For a vertex v, its corresponding triangle plane set is $planes(v)$, define the vertex quadric error measure is it to these triangle planes distance squares sum, measure is as formula (4) shows:

$$\Delta'(v) = \sum_{p\in planes(v)} d^2(v) = \sum_{p\in planes(v)} v^T (K_p) v = v^T \left(\sum_{p\in planes(v)} K_p \right) v \tag{4}$$

To make formula (4) expressing easy and simple, let in $Q'(v) = \sum_{p\in planes} K_p$, and use $Q'(v)$ to represent vertex v quadric error measure matrix, it is four orders matrix that is used to measure edge collapsing costs. When edge (v_i, v_j) collapses to new vertex $\bar{v}$, then collapse cost is as formula (5) shows:

$$\Delta'(\bar{v}) = \bar{v}^T \left(Q_i' + Q_j'\right)\bar{v} \tag{5}$$

In formula (5), $Q_i' + Q_j'$ represents new vertex $\bar{v}$ quadric error measure, intuitionally, point $\bar{v}$ and edge (v_i, v_j) correlated triangle plane distance gets further, its cost will also get larger, so is the practice.

New vertex position defining theorem

For assigned edge collapse as Figure 1shows: $(v_1, v_2) \to v_n$, it needs to define v_n position according to certain rules, in order to compress model features, selection of new vertex v_n is crucial.

Due to $\Delta'(v_n)$ is a quadratic equation, looking for $\Delta'(v_n) = \min$ condition becomes solving new vertex's equivalence problem, and because v_n is solution of formula (6) equation, it can utilize the equation as constraint condition to solve $\Delta'(v_n)$ minimum value.

$$\frac{\partial \Delta'(v_n)}{\partial x} = \frac{\partial \Delta'(v_n)}{\partial y} = \frac{\partial \Delta'(v_n)}{\partial z} = 0 \tag{6}$$

Formula (6) applies matrix attribute can be expressed as formula (7) form:

$$\begin{bmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} \\ q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} \\ q_{1,1} & q_{3,2} & q_{3,3} & q_{3,4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

By formula (7), it can get formula (8):

$$\begin{bmatrix} q_{1,1} & q_{1,2} & q_{1,3} \\ q_{2,1} & q_{2,2} & q_{2,3} \\ q_{1,1} & q_{3,2} & q_{3,3} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = 0 \tag{8}$$

Let Q' to represent formula (8) three-order matrix, $v_n = [v_x \quad v_y \quad v_z]^T$, $L = [q_{1,4} \quad q_{2,4} \quad q_{3,4}]$, it can get formula (9) expression by formula (7):

$$Q' v_n + L = 0 \tag{9}$$

If matrix Q' is reversible, then can get new vertex as formula (10) shows:

$$v_n = -Q'^{-1} L \tag{10}$$

If matrix Q' isn't reversible, then can take the second best and select $v_1, v_2, \frac{v_1 + v_2}{2}$ let $\Delta'(v_n)$ to take its minimum value, as formula (11) shows:

$$\Delta'(v_n)_{\min} = \min\left\{\Delta'(v_1), \Delta'(v_2), \Delta'\left(\frac{v_1 + v_2}{2}\right)\right\} \tag{11}$$

In order to effective implement error transmission, it needs to calculate new vertex area error matrix, and further define its quadric error as formula (5)calculation rule, define $\Delta'(v_n)$ as to be collapsed edge $\overrightarrow{v_1 v_2}$ cost of edge collapsing, and according to this, rank all edges that to be collapsed to define contraction order.

Half-space test

In model compression process, in order to effective avoid compression result TIN surface patches' space self intersection, it should adopt proper algorithms to carry out judgment test on edge collapsing operation legality. For specific edge collapsing operation $(v_1, v_2) \to v_n$, adopt half-space test method to do judgment test on legality, its test flow is as following shows:

□ For assigned edge collapse $(v_1, v_2) \to v_n$, according to section 2.3 computational method to work out v_n, and according to vertex (v_1, v_2) surrounding vertexes and new vertex v_n to reconstruct triangle sequence T;

□ According to reconstructed triangle sequence T, solve equation of passing new vertex v_n average plane $\overline{P}$, expression is as formula (12) shows:

$$Ax + By + Cz + D = 0 \tag{12}$$

Individual variable coefficient needs to meet formula (13):

$$A^2 + B^2 + C^2 = 0 \tag{13}$$

Average plane $\overline{P}$ passes through new vertex v_n, and its normal vector $\vec{n}$ can utilize formula (14) to solve:

$$\vec{n} = \frac{\vec{N}}{\left|\vec{N}\right|} \tag{14}$$

In formula (14), $\vec{N}$ is as formula (15) shows:

$$\vec{N} = \frac{\sum \vec{n}_i A_i}{\sum A_i} \tag{15}$$

In formula (15), $\vec{n}_i, A_i$ respectively represents each item diamond triangle patch normal vector and normal area. Average plane normal vector is as Figure 2 shows:

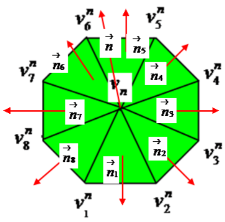

Figure 2: Average plane normal schematic diagram

□ In average plane $\overline{P}$, establish a two-dimensional partial orthogonal coordinate system, the coordinate system's system coordinate origin is v_n, in average plane $\overline{P}$ look for two units' orthogonal vectors $\mathbf{b}_1, \mathbf{b}_2$, and use $\mathbf{b}_1, \mathbf{b}_2$ directions as the coordinate system two coordinate axis directions, then solve normal vector $\mathbf{n}$ orthogonal unit vector $\mathbf{a}$ is as formula (16) shows:

$$\mathbf{a}=\begin{cases}\dfrac{\left(-(\mathbf{n}_y+\mathbf{n}_z),\mathbf{n}_x,\mathbf{n}_x\right)}{\mathbf{n}_x} & ,\mathbf{n}_x\neq 0\\ \dfrac{\left(\mathbf{n}_y,-(\mathbf{n}_x+\mathbf{n}_z),\mathbf{n}_y\right)}{\mathbf{n}_y} & ,\mathbf{n}_y\neq 0\\ \dfrac{\left(\mathbf{n}_z,\mathbf{n}_z,-(\mathbf{n}_x+\mathbf{n}_y)\right)}{\mathbf{n}_z} & ,\mathbf{n}_z\neq 0\end{cases} \quad (16)$$

Unit vector $\mathbf{b_1}$ is expressed as formula (17):

$$\mathbf{b_1}=\frac{\mathbf{a}}{\|\mathbf{a}\|},\|\mathbf{a}\|=\sqrt{\mathbf{a}\bullet\mathbf{a}} \quad (17)$$

Unit vector $\mathbf{b}_2$ is as formula: $\mathbf{b}_2=\mathbf{n}\times\mathbf{b_1}$

□ Calculate (v_1,v_2) corresponding boundary vertex to average plane $\overline{p}$ directed distance $d_j\left(j=0,1,\ldots,n_i\right)$, distance computation method is as formula (18) shows:

$$d_j=dist(y_j,\overline{p})=\frac{Ax_j+By_j+Cz_j+D}{\sqrt{A^2+B^2+C^2}} \quad (18)$$

And due to formula (13), d_j value and formula (18) numerator value are the same, and it changes into formula (19):

$$d_j=dist(y_j,\overline{p})=Ax_j+By_j+Cz_j+D \quad (19)$$

□ Project (v_1,v_2) corresponding boundary vertex to plane $\overline{p}$, and calculate v_1,v_2 coordinates in defined coordinate system, the location distribution is as Figure 3shows.

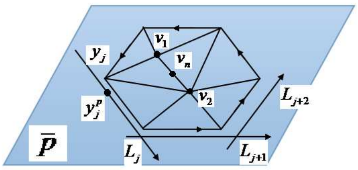

Figure 3: Half-space test

If project all boundary vertexes into average plane $\overline{p}$, then projection coordinate computational method is as formula (20) shows:

$$y_j^p=y_j-d_j\bullet\mathbf{n} \quad (20)$$

In formula (20), y_j^p superscript represents projected, transform y_j^p into partial coordinate system; transformation equation is as formula (21) shows:

$$\left(u_j,v_j\right)=\left(d_j\bullet\mathbf{b_1},d_j\bullet\mathbf{b_2}\right) \quad (21)$$

□ Calculate to be collapsed edge $\overrightarrow{v_1v_2}$ corresponding boundary polygon's each edge linear equation L_j in partial coordinate system, as formula (22) shows:

$$L_j(u,v)=-\Delta' v_j\left(u-u_j\right)+\Delta' u_j\left(v-v_j\right)=0 \quad (22)$$

In formula (22), $\Delta' u_j,\Delta' v_j$ is expressed as formula (23) shows:

$$\begin{cases} \Delta' u_j = u_{(j+1)\mathrm{mod}(n_j+1)} - u_j \\ \Delta' v_j = v_{(j+1)\mathrm{mod}(n_j+1)} - v_j \end{cases} \tag{23}$$

 If v_n meets formula (24) then it shows v_n lies in L_j positive half-space, it can affirm edge collapsing operation legality, otherwise it cannot collapse the edge.

$$L_j(0,0) > 0,\ j = 0,1,...,n_j \tag{24}$$

IMPROVE ALGORITHM

Improved algorithm principle

In computer graphics, three-dimensional grid model most common attributes are color, texture and normal, in order to let model after compressing has good similarity as initial model, it should ensure model geometric information and meanwhile reserve these attributes, due to point to plane distance considers edge collapsing operation effects on vertex surrounding region attributes value changes, it can relative correct describe partial attributes error, and meanwhile is simpler and faster than distance calculation between points and surfaces or surfaces to surfaces. Therefore, adopt distance from point to plane as attribute error measure; apply quadric error measure into attribute error calculation.

Grid model every vertex except for space coordinate, it also has values to describe its attributes, in grid model triangle plane, attributes values are got according to geometric location's interpolation. Therefore, triangle plane attributes values are continuous, and two attributes values distance is measured by Euclidean distance.

Color attribute can use trivector $[r, g, b]^T$ to express, from which $(0 \le r, g, b \le 1)$, all color vectors are composed RGB color space, as Figure 4shows, in RGB color space, point to plane distance square similarly can use quadric error $Q(v)$ to calculate, when new vertex after edge collapsing adopts sub set selection method, no need to recalculate new vertex space position and attribute value, when calculate error, no need to consider space coordinate and attribute value correlations, only need to respectively establish geometry quadric error measure and attribute quadric error measure, and calculate geometry and attribute errors.

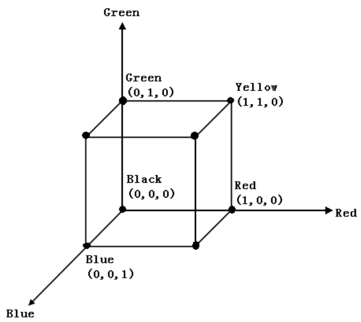

Figure 4: RGB color space

In algorithm description, it adopts triangle grid model with color attribute, grid models with other attributes can similarly deduced. Triangle grid model every vertex v has two vectors $v_g[x, y, z]^T$ and $v_c[r, g, b]^T(0 \le r, g, b \le 1)$ to represent geometry and color information, establish geometry quadric error measure Q_{fg} and color attribute quadric error Q_{fc} each vertex quadric error measure is equal to its adjacent triangle surface quadric error measure sum for every triangle surface, as formula (25) shows:

$$\begin{cases} Q_{vg} = \sum_{(v \in f)} Q_{fg} \\ Q_{vc} = \sum_{(v \in f)} Q_{fc} \end{cases} \tag{25}$$

When collapse edge $(v_i, v_j), j \neq i$ to vertex v, total quadric error measure is as formula (26) shows:

$$\begin{cases} Q_g = Q_{vig} + Q_{vjg} \\ Q_c = Q_{vic} + Q_{vjc} \end{cases} \tag{26}$$

Therefore edge collapsing caused geometric error $E_g = Q_g(v_g)$, color attribute error $E_c = Q_c(v_c)$, then total edge collapsing cost is as formula (27) shows:

$$Cost = E_g + \alpha E_c \tag{27}$$

In formula (26), α represents color attribute error influence coefficient in total cost, it can adjust according to practical demands.

When calculate edge collapsing cost, it needs to focus on two aspects' problems:one is vertex set coordinate value range is $(-\infty, +\infty)$, and color component value range is $[0,1]$, so it should make preprocessing with vertex geometric position data, let the two range to be the same, so as to ensure E_g and E_c effects on $Cost$ are equivalent; the other is it will appear triangle surface three vertexes color vectors are the same, in such case three color vectors cannot compose plane but degenerate into a point, therefore it should calculate and let color vectors quadric error to met formula (28):

$$Q_{fc}(v_c) = (v_c - v_{ic})^2 \tag{28}$$

In formula (27), v_{ic} represents the triangle surface any one color vector.

Algorithm flow

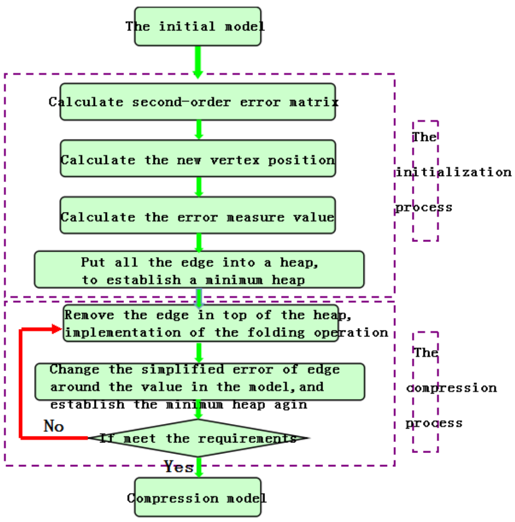

Figure 5: Edge collapsing algorithm flow

Edge collapsing algorithm basic thought is taking edge as deleted basic geometric element in every simplification operation, and increasing new vertex, all points that connect with deleted edges should connect with the new vertex, and let model to still keep as triangle grid, after carrying out multiple times selective edge collapsing, model can be simplified into requested extent. Algorithm flow is as Figure 5 shows:

Algorithm basic steps:

- Step1: Read initial grid model data, and make preprocessing with it, mark boundary edge and feature edge;
- Step2: Calculate initial grid every triangle surface geometric and color attributes quadric error measure values;
- Step3: Calculate every vertex geometry and color attributes quadric error measure values;
- Step4: Calculate every edge collapsing cost and new vertex after collapsing, and carry out legality judgment

test on edge collapsing;

- Step5: All edges that meet legality judgment compose a candidate edge set, rank the set edges according to edge collapsing cost;
- Step6: Select minimum collapsing cost edge from candidate set to execute collapsing operation, and update all relative information;
- Step7: If candidate set is empty or triangles numbers already arrive at users' demands, then end, otherwise transfer to Step6.

Algorithm performance analysis

For storage space perspective, the algorithm with respect to Garland multiple dimensional quadric error measure algorithm, it needs fewer coefficients to save quadric error measure, and no need to save new vertex information after edge collapsing operating every time, storage Q required coefficients are as Table 1 and Table 2 show:

Table 1: Garland multiple-dimensional quadric error measure algorithm

Model type	Garland multiple-dimensional quadric error measure algorithm		
	Vertex expression	Order number	Q coefficient
Only contain geometric information	$[x, y, z]^T$	3×3	$\binom{5}{2}=10$
Geometric information and color	$[x, y, z, r, g, b]^T$	6×6	$\binom{8}{2}=28$
Geometric information and normal	$[x, y, z, a, b, c]^T$	6×6	$\binom{8}{2}=28$

Table 2: Improved edge collapse algorithm

Model type	Improved edge collapse algorithm		
	Vertex expression	Order number	Q coefficient
Only contain geometric information	$[x, y, z]^T$	3×3	$\binom{5}{2}=10$
Geometric information and color	$[x, y, z]^T, [r, g, b]^T$	3×3	$2\times\binom{5}{2}=20$
Geometric information and normal	$[x, y, z]^T, [a, b, c]^T$	3×3	$2\times\binom{5}{2}=20$

By Table 1 and Table 2, it is clear that improved algorithm is superior to Garland multiple-dimensional quadric error measure algorithm in the perspective of storage.

CONCLUSION

It is very necessary to make reasonable, effective compression on three-dimensional surface model, compression algorithm merits has deeply effects on space target surface modeling development; the paper proposes edge collapse-based three-dimensional surface compression algorithm, firstly according to quadric error measure criterion, it defines collapsed edges orders, and then utilizes half-space test algorithm to judge edge collapsing legality so as to implement surface model boundary zone and interior zone synchronization compression; it puts forward researching image compression algorithm, and meanwhile it should understand image attributes, from which it contains geometric attributes, color attributes and normal attributes, then integration compression processing on three-dimensional images can better and faster meet users' demands; by algorithm performance analysis, it is clear that improved algorithm is superior to other algorithms in coefficient storage aspect.

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