



Research Article

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## Dynamical model based parallel movement centripetal force effects on horizontal bar swinging technique

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### ABSTRACT

*With constantly developing of social scientific theory, present correlation theoretical researches are urgently in need of natural science, modern science and technology melting, in the hope of deeply solving relative problems. This paper targeted sports movement horizontal bar event, taking athletes' downward swinging as examples, analyze athletes' status, establish mathematical model, and implement discussion and researching on relative problems. According to body swaying technical features, multi rigid body model, then utilizes dynamics method, corresponding dynamic equation deducing human race swaying. Results show giants turn or grip change motion is more beneficial in  $40^\circ$  -  $50^\circ$  or  $310^\circ$  -  $320^\circ$ .*

**Key words:** Dynamical model, parallel movement, centripetal force, horizontal bar swinging

### INTRODUCTION

The origin of horizontal bar can be derived from human race ancestors primitive men went in for trainings in jungle such as each kind of climbing, ascending, swaying, and swinging and so on. At that time, it was just a kind of practical life skill; subsequently it gradually became a kind of physical training way with social progress. In 1896, horizontal ball was listed into Olympic Games competition events. Horizontal bar technical motions main features, except for knee mounting, kipping and hock arms circle as well as other swaying motions, it has also lots of exerting motions and static postures as well as stepping bar flyaway dismounting [1, 2]. During around half century times since entering into first decade in 20<sup>th</sup> century to the end of 1960s, the main features has eliminated exerting motions and static postures, all is composed of swaying motions that conform to horizontal bar movement features, and it makes progress in dismounting ways, turning, flight with grips releasing and other aspects. Until 1930s, individual technique has been further developed [3-5]. Such as Berlin held 11<sup>th</sup> Olympic Games in 1936, it appeared "el grip circle", "swan astride vault", "flyaway with 2 saltos tucked" and other complicated motions. After that, due to the Second World War, horizontal bar technique affected, the world competition hasn't restored until 1950s. It is clear that modern times horizontal bar has moved forward three big steps that apparatus is changed from wood bar to iron bar; horizontal bar has been listed into world game events; It has made great progress in technique, appearing turning, flight with grip releasing, giants, double somersault and other relative complicated motions. The above 3 steps have built film foundation for modern horizontal bar movement development [6-8].

This research will adopt multi-rigid-body system dynamics Kane method, which applies upper swinging dynamics into human race action Kane's multi-rigid-body system dynamical method, and carries out modeling on horizontal bar human swinging motions. Considering body swinging, especially focus on practical operation techniques movement features, simplifying single pendulum or double pendulum, emphatically carry out segments classification on trunk.

## SWINGING TECHNIQUE DYNAMICS MODEL ESTABLISHMENT

### Horizontal bar swinging technique biomechanical analysis

When athlete rotates in horizontal bar axis, except for human body internal force friction behavior working, human body gravity torque also works. According to rotation law formula (1).

$$\alpha = \frac{M}{J} \quad (1)$$

From which,  $M$  is gravity torque on horizontal bar,  $J$  is human body rotational inertia to bar axis,  $\alpha$  is corresponding angular accelerated speed. When considering movement changes that generated by gravity torque, only take both gravity torque and rotational inertia into consideration, then it can get correct conclusions.

First research on a athlete gravity torque generated angular accelerated speed under two postures. Given an athlete weight 58kg, height 1.73m, body each part proportion can be achieved from the athlete motion pictures. The first posture, athlete shoulder angle, hip angle both are  $180^\circ$ , that is ideal "body fully stretching" state. Let athlete body each segment gravity center all in the horizontal bar same horizontal plane. The second posture, athlete body slight bends, both shoulder angle and hip angle are  $170^\circ$ ; To compare, let athlete gravity center also in the horizontal bar same horizontal plane. Compare athlete gravity torque generated angular accelerated values under the first posture and the second postures. The differences between the two postures are just that shoulder and hip angles are different [9].

**(1) Posture one:** In order to calculate human body rotational inertia and gravity center position, we adopt provided data, below are each posture human body rotational inertia calculation program and corresponding data Table 1:

Table 1: Both shoulder angle and hip angle are  $180^\circ$  data

Segment name	Mass center coordinate(cm)		Mass(g)	$J'_i(\text{g})\text{cm}^2$	$r_i^2(\text{cm}^2)$	$m_i r_i^2(\text{g})\text{cm}^2$	$J_i(\text{g})\text{cm}^2$
(double)Hand	x	5.170	742	10053	11287.622	8375415.52	8385468.52
	y	0					
(double)Forearm	x	28.021	1942	112302	6954.197	13505050.57	13617352.57
	y	0					
(double)upper arm	x	57.930	3498	255988	2860.663	10006599.17	10262587.17
	y	0					
Trunk	x	100.505	30150	8835714	118.944	3586161.60	12421875.60
	y	0					
(double)Thigh	x	146.957	11475	2057636	1263.610	14499924.75	16557560.75
	y	0					
(double)Shank	x	190.178	4757	705801	6203.782	29511390.97	30217191.97
	y	0					
(double)Foot	x	219.984	1582	55523	11787.453	18647750.65	18703273.65
	y	0					
Head	x	61.523	3763	152026	2489.131	9364110.82	9516136.82
	y	0					
Whole body	x	111.414	$r_1 = 111.414\text{cm}$			$J'_1 = 119681447\text{g} \cdot \text{cm}^2$	
	y	0					

By parallel axis theorem, it can calculate athlete body to horizontal axis rotational inertia:  $J_1 = J'_1 + m r_1^2 = 839640051\text{g} \cdot \text{cm}^2$

Gravity to horizontal bar axis torque:  $M_1 = mg \cdot r_1 = 6332771760\text{N} \cdot \text{cm}$

Gravity torque let human body generated surrounding horizontal bar angular accelerated speed

$$\alpha_1 = \frac{M_1}{J_1} = 7.542\text{rad} / \text{s}^2$$

is:

**(2) Posture two:** When athlete shoulder angle, hip angle both are  $170^\circ$ , use the same method solving athlete body correspond to mass center rotational inertia:  $J'_2 = 121497735\text{g} \cdot \text{cm}^2$

Athlete body mass center and horizontal bar axis distance:  $r_2 = 110.974\text{cm}$

Athlete body to horizontal bar axis rotational inertia:  $J_2 = J_2' + mr_2^2 = 835780998\text{g} \cdot \text{cm}^2$

Gravity to horizontal bar torque:  $M_2 = mg \cdot r_2 = 6307762160\text{N} \cdot \text{cm}$

Gravity torque let human body generated surrounding horizontal bar angular accelerated speed is:

$$\alpha_2 = \frac{M_2}{J_2} = 7.547\text{rad/s}^2, \quad \frac{\alpha_2}{\alpha_1} = \frac{7.547}{7.542} = 1.00066 > 1$$

That is when athlete both shoulder angle and hip angle are  $170^\circ$  the gravity torque generated accelerated speed values are larger than that on the condition shoulder angle and hip angle are  $180^\circ$ .

Results show, picked posture and stretched posture comparing, gravity torque generated horizontal bar accelerate speed are larger. To further test when horizontal bar downward swinging to bar horizontal plane, body keep certain picked posture and stretched posture, gravity torque generated angular accelerated speed to horizontal bar axis such problem; we make calculations on four cases that athlete shoulder keep  $165^\circ$  when horizontal bar downward swinging to horizontal plane, and hip angles are respectively  $165^\circ, 155^\circ, 145^\circ, 135^\circ$ , calculation results refer to Table 2.

**Table 2: Downward swinging to bar horizontal part each posture comparison**

Posture	1	2	3	4	5	6
Shoulder angle( $^\circ$ )	180	170	165	165	165	165
Hip angle( $^\circ$ )	180	170	165	155	145	135
$\alpha$ ( $\text{rad/s}^2$ )	7.542	7.547	7.646	7.713	7.803	7.917
$\frac{\alpha_i}{\alpha_1}$	1	1.00066	1.01379	1.02267	1.03461	1.04972

Results show:

- 1) When athlete is in horizontal bar downward swinging horizontal plane, picked posture comparing with stretched posture, its gravity torque generated angular accelerated speed to horizontal bar axis is larger;
- 2) On the condition that shoulder angle is fixed, the smaller hip angle is, its gravity torque generated angular accelerated speed to horizontal bar axis would become bigger.

### Giant circle force analysis model

The above model considers angular accelerated speed decisive two factors( that are gravity torque and rotational inertia), when athlete body lies in horizontal bar downward swinging horizontal plane, picked posture comparing with stretched posture, its gravity torque generated angular accelerated speed to horizontal bar axis is larger, which indicates postures have great impact on horizontal bar motion completion quality. By model two, make further analyze each phase pull force changes with giant circle as an example, so that finds out reasonable scheme.

Giant circle can approximate to people make circular movements in vertical plane, connection line from gripping point to mass center is rotational radius( $r$ ), when athlete lies in handstand position, gravity( $mg$ ) and pull force( $F$ ) both turn downwards, therefore their joint force is athlete centripetal force in handstand position. That is formula (2)

$$F + mg = m \frac{v^2}{r} \quad (2)$$

From formula (2), it is clear that centripetal force is larger than gravity, tension is in the front, athlete should pull upwards; when centripetal force is smaller than gravity, pull force is negative, athlete should have a downward push force so that can meet circular movement conditions; when centripetal force is the same as gravity, pull force is zero, athlete neither needs to use pull force nor push force, he can still make circular movements.

Sizes of centripetal force are mainly up to giant circle speed; centripetal force is in direct proportion to giant circle

speed quadratic. Generally, when athlete makes giant circle motion and arrives at handstand position, speed is relative smaller, so that athlete should act on a downward push force in handstand position; only when making high difficulty dismounting and flight motions, speed is relative larger when giant circle going through handstand position, horizontal bar will provide a downward pull force as Figure 1.

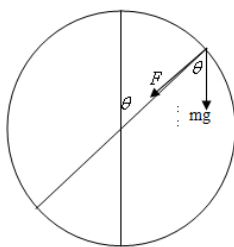


Figure 1: Random time people force schematic diagram

When athlete swings from handstand position to bar underneath vertical position, horizontal bar gives athlete pull force turn upwards, while gravity turn downwards so centripetal force is the gap between the two forces. That is formula (3):

$$F - mg = m \frac{v^2}{r} \quad (3)$$

From formula (3), it is clear that pull force is the sum of centripetal force and gravity, due to bar underneath vertical position speed is larger, and pull force is also larger. According to mechanical conservation that giant circle optional position starts from handstand position speed computational formula (4):

$$v_1 = \sqrt{2gr(1 - \cos\theta) + v_0^2} \quad (4)$$

Among them,  $v_1$  is optional position speed,  $g$  is gravity accelerated speed,  $r$  is rotational radius,  $\theta$  is optional position angle,  $\cos\theta$  is cosine,  $v_0$  is handstand position initial speed.

According to giant circle dynamic features, athlete in the whole giant circle process, optional position (angle) force sizes computational formula (5) as following:

$$F = m \frac{v^2}{r} - mg \cos\theta \quad (5)$$

From which horizontal component force is  $F_x = F \sin\theta$ , vertical component force is  $F_y = F \cos\theta$ .

By people handstand moment speeds are respectively  $0m/s$ ,  $1m/s$ , and rotational radius are respectively  $0.8m$ ,  $1m$  calculating each position force and mass ratio ( $g = 9.8$ ). Calculation results are as Table 3.

When athlete completing horizontal bar giant circle, his suffered pull force sizes is related to giant circle speed; With speed increasing, pull force would become large; according to calculation data, athlete in handstand position forward and backward  $40^\circ$ , due to speed is quite small, it needs a upward push force, therefore when athletes swings downward from handstand, it should have a shoulder withstanding motion, it not only can stretch radius increase force arm, enlarge torque, but also can let athlete continue to make circular movement.

Athlete arrives at bar underneath vertical position, due to speed is larger; it needs great pull force so that can provide its circular movement centripetal force. From figure, it is clear that when athlete lies in  $40\sim 50^\circ$  and  $310\sim 320^\circ$ , it near to  $x$  axis that force is minimum, at this time horizontal bar don't need to provide external force that can make circular movements, during the period people can complete some giant circle turn or grip change motion.

Table 3: Different conditions each position force

$\theta$	$\frac{F(0,0.8)}{mg}$	$\frac{F(0,1)}{mg}$	$\frac{F(1,0.8)}{mg}$	$\frac{F(1,1)}{mg}$
	$mg$	$mg$	$mg$	$mg$
0	-1.000	-1.000	-0.872	-0.898
10	-0.954	-0.954	-0.826	-0.852
20	-0.819	-0.819	-0.691	-0.717
30	-0.598	-0.598	-0.470	-0.496
40	-0.298	-0.298	-0.170	-0.196
50	0.072	0.072	0.200	0.174
60	0.500	0.500	0.628	0.602
70	0.974	0.974	1.102	1.076
80	1.479	1.479	1.607	1.581
90	2.000	2.000	2.128	2.102
100	2.521	2.521	2.649	2.623
110	3.026	3.026	3.154	3.128
120	3.500	3.500	3.628	3.602
130	3.928	3.928	4.056	4.030
140	4.298	4.298	4.426	4.400
150	4.598	4.598	4.726	4.700
160	4.819	4.819	4.947	4.921
170	4.954	4.954	5.082	5.056
180	5.000	5.000	5.128	5.102
190	4.954	4.954	5.082	5.056
200	4.819	4.819	4.947	4.921
210	4.598	4.598	4.726	4.700
220	4.298	4.298	4.426	4.400
230	3.928	3.928	4.056	4.030
240	3.500	3.500	3.628	3.602
250	3.026	3.026	3.154	3.128
260	2.521	2.521	2.649	2.623
270	2.000	2.000	2.128	2.102
280	1.479	1.479	1.607	1.581
290	0.974	0.974	1.102	1.076
300	0.500	0.500	0.628	0.602
310	0.072	0.072	0.200	0.174
320	-0.298	-0.298	-0.170	-0.196
330	-0.598	-0.598	-0.470	-0.496
340	-0.819	-0.819	-0.691	-0.717
350	-0.954	-0.954	-0.826	-0.852
360	-1.000	-1.000	-0.872	-0.898

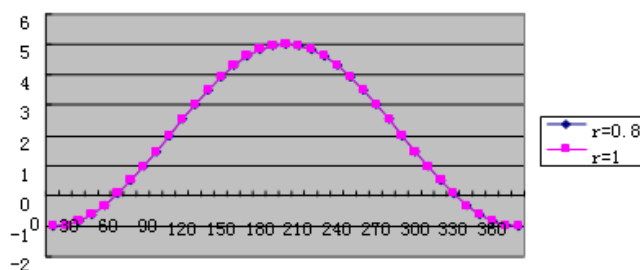


Figure 2: Each position people force changes when handstand initial speed is 0m/s

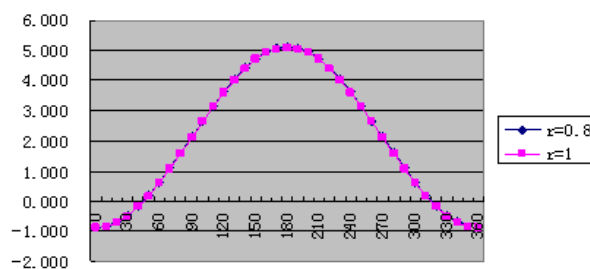


Figure 3: Each position people force changes when handstand initial speed is 1m/s

From Figure 2, Figure 3, it is clear that on the condition of handstand position initial speed is zero, rotational radius changing can only change speed, while push force and pull force sizes would not change. From Table 3 data

comparison, it is clear that on the condition handstand position has initial speed, rotational radius changing not only will change speed, but also force has changes, radius becomes smaller, then push force gets smaller and pull force gets bigger, therefore when completing short radius circle (such as stalder, endo), it should tightly fold body, trying to enlarge rotational radius, which can enlarge torque and also can save labor.

From Figure 4, Figure 5, it is clear that in comparison of athlete has initial speed and hasn't initial speed in handstand position, athlete with initial speed ,its push force is smaller than that from athlete without initial speed, and pull force is larger. Therefore, when completing pure one-arm giant circle, initial speed through handstand position should try to make smaller, the opportunity with one hand putting should be as later as possible in technique permissible range, so that can decrease one-arm pull force under the bar.

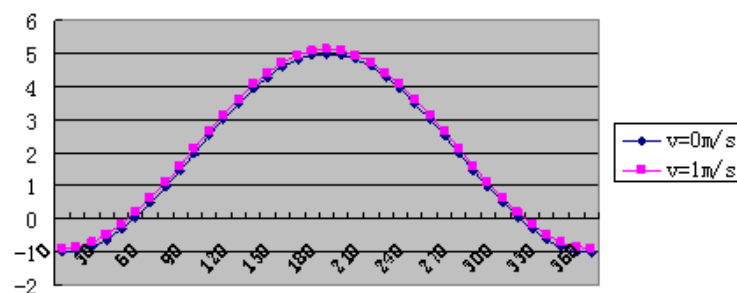


Figure 4: Each position force change when rotational radius is 0.8m

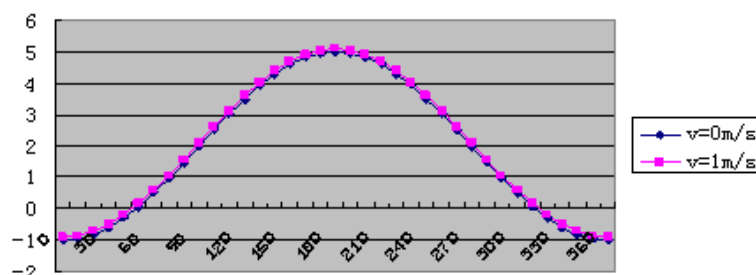


Figure 5: Each position force change when rotational radius is 1m

**Sports five links multi-rigid-body system model**

Considering human body horizontal bar swinging sports features, especially in technique concentrated practical movement, avoiding artificial research that let clock pendulum which makes double pendulum during swinging or simplifying handling with horizontal bar. The research focuses on link and uses it classifying trunk part, as Figure 6 shows, human body simplified into 5 links multi-rigid-body system model, from which, D1,D2, D3,D4 and D5 respectively represent arms( including hands), upper trunk( including head, neck, trunk), thigh( including foot), each link is defined according to concrete statuses human body measurement parameters from measuring or body system model.

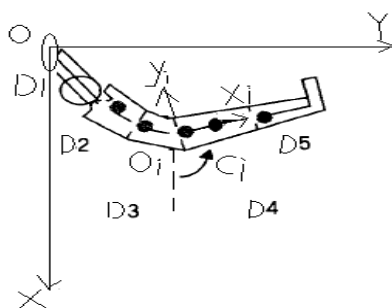


Figure 6: Model and coordinate system

Coordinate system as well as general coordinates' selection, as Figure 6 shows, it defines horizontal bar human body swinging motion as planar movement. Establish solidified horizontal bar O point inertia coordinate system OXY and segments link joint  $O_i$  as origin body-fixed coordinate system  $O_i X_i Y_i$  (Note:  $O$  is horizontal bar,  $O_i$  is each

segment link joint,  $x_i$  axis is link joint pointing to segment mass connection line) .Select each segment corresponding to X axis positive direction absolute angle  $\theta_i$  and define it as general coordinate.

Coordinate transform matrix, according to Figure 6, after establishing coordinate system, then  $O_i x_i y_i$  coordinate system corresponding to inertia coordinate system  $OXY$  coordinate transform matrix  $A_{io}$  as following formula (6):

$$A_{io} = \begin{pmatrix} \cos \theta_i & \sin \theta_i & 0 \\ -\sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad i=1,2,3,4,5 \quad (6)$$

**Model qualitative analysis:** Speed analysis: from general coordinate definition, it is clear that segment angular speed  $\omega_i$  in  $OXY$  can be expressed as formula (7):

$$(\omega_i) = \begin{pmatrix} 0 \\ 0 \\ \theta \end{pmatrix} \quad (i=1,2,3,4,5) \quad (7)$$

Given segment mass center to be  $C_i$ , then segment mass center  $C_i$  speed  $V_{ci}$  in  $OXY$  is expressed as formula (8):

$$(V_{ci}) = \sum_{j=1}^{i-1} A_{jo}^T \begin{pmatrix} 0 \\ l_j \theta_j \\ 0 \end{pmatrix} + A_{io}^T \begin{pmatrix} 0 \\ r_j \theta_j \\ 0 \end{pmatrix} \quad (i,j=1,2,3,4,5) \quad (8)$$

(Among them,  $l_i$  is the  $i$  segment length (that is two link joints distance),  $r_i$  is the  $i$  segment mass center  $C_i$  to the  $i-1$  segment link joint distance. )

Accelerated speed analysis: make derivation of segment angular speed  $\omega_i$  and segment mass speed  $V_{ci}$ , then it get segment angular accelerated speed  $\xi_i$  and segment mass center accelerated speed  $\partial_{ci}$  as following formula (9)(10):

$$(\xi_i) = \frac{d}{dt}(\omega_i) = \begin{pmatrix} 0 \\ 0 \\ \theta_i \end{pmatrix} \quad (i=1,2,3,4,5) \quad (9)$$

$$(\partial_{ci}) = \frac{d}{dt}(V_{ci}) = \sum_{j=1}^{i-1} A_{jo}^T \begin{pmatrix} 0 \\ l_j \theta_j \\ 0 \end{pmatrix} + A_{io}^T \begin{pmatrix} 0 \\ r_j \theta_j \\ 0 \end{pmatrix} \quad (i,j=1,2,3,4,5) \quad (10)$$

General active force: in horizontal bar human swinging motions, each active force, and torque as Figure 7 shows.

(From which,  $M_i$  and  $M_i'$  appear in pairs, is torque acting on each segment.) Regarding system general active force, it has computation formula as following (11):

$$F^{(r)} = \sum_{j=1}^n (\overline{R_j} \bullet \overline{V_j^{(r)}} + \overline{L_j} \bullet \overline{\omega_j^{(r)}}) = \sum_{j=1}^n ((V_j^{(r)})^T (R_j) + (\omega_j^{(r)})^T (L_j)) \quad (11)$$

Among them,  $n$  is number of rigid body,  $n=1,2,3,4,5$ ;  $r$  is freedom degree,  $r=1,2,3,4,5$ ;  $\overline{R}_j$  is principal vector acting on segment active force system;  $\overline{L}_j$  is principal active force system corresponding joint principal moment.

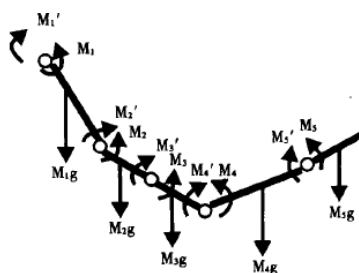


Figure 7: Horizontal bar human body swinging motion each principal active force and torque

General inertia force: with regard to rigid body system general inertia force  $F^{(r)}$ , it has following computational formula(12):

$$F^{*(r)} = \sum_{j=1}^n (\overline{R}_j^* \cdot \overline{V}_j^{(r)} + \overline{L}_j^* \cdot \overline{\omega}_j^{(r)}) \quad (12)$$

Among them,  $\overline{R}_j^* = -m_j \overline{\partial}_{cj}$ ,  $\overline{R}_j^*$  is segment inertial force system principal vector;

Establish system dynamics equations (13):

$$\begin{cases} F^{(r)} + F^{*(r)} = 0 \\ \theta_r = u_r \end{cases} \quad r=1,2,3,4,5 \quad (13)$$

## CONCLUSION

In horizontal bar swinging technique to human dynamics analysis model establishment, optimize and computer numerical simulation. Though giant circle is a basic motion, it is the technical basis to complete each kind of high difficulty motion, by dynamics analysis; it can make further understanding on circle technique and its relative motion. Circle motion is in handstand position forward and backward  $40^\circ$ , most of them should have a downward push force, only speed up circle can be possible to appear upward pull force. Under bar, the vertical position force is larger, calculation values are 5 times than weight. Actually it is nearly 4 times of weight. But when making high difficulty dismounting and flight motions, it should go beyond 5 times of weight. Giant circle turning or grip changing motion is more beneficial in  $40^\circ - 50^\circ$  or  $310^\circ - 320^\circ$ . In case handstand initial speed is zero, radius changes would not effect on force size, while in case initial speed is not Zero, radius changes and force would also change.

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