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Research Article

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Dividing geographical profile of serial criminals based on triangular distribution and cluster analysis

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ABSTRACT

This paper is based on the so called 1981 "Yorkshire ripper", which is the British criminal suspected of 13 murders. To crack the serial killing, the triangular distribution and cluster analysis are used to analyze the details of case. Combined with criminal psychology, we can find out the mass center of criminal attack sites and obtain the geographical contours, so as to eventually narrow the searching scope. These two different methods demonstrate that the criminal sites approximately concentrate in two regions, Leeds and Bradford, of which Leeds is the key areas of criminal perpetrators. The place where Peter Sutcliffe was arrested is closed to the location of our model given. It shows that this model is feasibility.

Key words: Serial Criminals, triangular distribution, cluster analysis, Yorkshire ripper

INTRODUCTION

1 Serial murder' characteristic

Serial murder is a kind of serious violent crime which the offenders kill many innocent people in a form of interval in a certain period of time. This kind of murders causes great fears to the local residents where the crime occurred. If the mobility of the offenders is great and a large number of people are killed, then it will even bring the whole country into a terror. They enjoy killing, remain indifferent to the suffering of victims, and even get pleasure from the torture to the victims. Offenders commit the crime recondite. They have strong capability of anti-reconnaissance and strive to avoid being captured; therefore, a detection rate of such crimes is low[1].

2 Criminal psychology and geographic profiling analysis

Recently, we mainly use criminal psychology profiling analysis and criminal geographic profiling analysis to study these issues. Criminal psychology can provide the unknown offenders' characteristics, including physiology, psychology, behavior, and social. Then, it can be used to narrow the scope of suspects, determine the key point of investigation, and improve the detection rate. Criminal geographic profiling is based on the links between the geographical information and the tendency of victim and criminal site target, to narrow the scope of sites and effectively help the police crack a criminal case as soon as possible.

"Geographical analysis", which has been focused on in recent years, is different from "dichotomy" and "behavioral evidence analysis". In the strict sense, it is not a psychological analysis method, but a more visualized statistic inductive method.

3 Current research

From the early 1990s, There are already some software is the most widely used in the prediction of crime and arrest consecutive[2], there are three geographic portrait mode: (1) Rassmo mode, with Rigel its software; (2) Canter mode , with Dragnet its software; (3) Lenine mode, and its software is Crime stat. Rassmo model criminal is geographic targeting model (CGT), abiding by the function of distance digressive from the criminal site to the

offender residence. Canter mode bases on the suppose that most offender crime near offender residence. In the strict sense, Lenine mode is not a psychological analysis method. It is based on the Rassmo mode, and uses the more strictly statistics to estimate the offenders' travel distance and the initial of crime[3].

For this situation, we use Topology; Probability and Statistics; Geometric with Criminal psychology and Crime Graphics[4-5], zhong Established area overlap weighted model, this methods could prediction the series crimes.peng chen has researched Timing of criminal act invitees during the day, and researched Long-term time-series forecasting of social interventions for narcotics use and property crime[6],liu applied the Grey system theory to prediction of dynamic tendency of crimes, The optimal GM(1,1)model was applied to the prediction of figure of Serial Criminals[7], Chandra use the clustering approach for crime trends prediction[8],pangchen applied ARMA model to forecast the crime[9].

In this paper, we take the British criminal suspected of 13 murders, known as "Yorkshire ripper," for example, to find the "center "of the activities of criminals. In this paper, we use two mathematical statistics' methods which include the triangular distribution and cluster analysis.

THE TRIANGLE DISTRIBUTIVE MODEL 1 Model hypothesis

A serial murderer usually has three criminal tendencies, i.e. stick to familiar crime locations; to avoid familiar people; to not be too far away from dwelling place.

2 Triangle distribution speculative knowledge

2.1 Triangle distribution function and density function

Triangle distribution function:
$$F(x) = \begin{cases} 0, & x < 0; \\ \frac{x^2}{2}, & 0 \le x < 1; \\ -\frac{x^2}{2} + 2x - 1, 1 \le x < 2; \\ 1, & x \ge 2. \end{cases}$$
(1)

Triangle distribution density function:
$$p(x) = \begin{cases} x, & 0 \le x < 1; \\ 2 - x, 1 \le x < 2; \\ 0, & other. \end{cases}$$
 (2)

2.2 Examine if it obeys the triangle distribution theory

As the general distribution only picks finite single values and each P_i is known, now we examine the fitting of distribution; the procedures are as following:

$$\chi^2 = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i} \Box \chi^2(k-1)$$

calculate and examine the statistic quantity

to the given significance level $\alpha(0 < \alpha < 1)$, the rejection region of examination is $W = \{\chi^2 \ge \chi^2_{1-\alpha}(k-1)\}$ Compare the results of the examination. If the value of the statistic quantity is in the rejection region, the original assumption is refused; if it is not in the rejection region, the original assumption is accepted.

2.3 Test if Peter William Sutcliffe's serial murders obey the triangle distribution

The geographic locations of the 13 serial murders are shown as below [11]



Fig. 1: Location map of crime

By using software, we can calculate the differences of latitudes and longitudes of each of the 13 crime locations to the 8th crime location, which is shown as below:

Number	1	2	3	4	5	6	7
Difference of longitude	-27323.6	-27031.4	-30674.9	-1710.88	-15136	49913.14	40777.88
Difference of latitude	18134.62	17258.96	20881.57	17565.75	17996.8	-20087.9	-13233.1
Number	8	9	10	11	12	13	
Difference of longitude	0	48934.97	35026.03	10369.39	-11786.1	-25252.3	
Difference of latitude	0	-21124.2	-10822.2	11819.34	17293.8	16777.96	

Table 1: The differences of latitudes and longitudes of the crime locations

From the table above we can get the coordinates of the points on side EB and BA on triangle ABE. Shown as below:

Table 2: coordinates of crime locations

Number	1	2	3	4	5
Coordinate	(1.74, 0.22)	1.65, 0.23)	(2, 0)	(1.1, 1.01)	(1.92, 0.2)
Number	7	8	11	12	13
Coordinate	(1.03, 0.97)	(0, 0)	0.86, 0.78)	(1.46, 0.48)	(1.87, 0.23)

As the rectangular coordinate system in Fig. 1 above, we choose interval [0, 0.3], [0.3, 0.5], [0.5, 0.7], [0.7, 1.0] on the axis *y*, the possibilities of the cases happening on the side *EB* $\$ *BA* are: 0.5, 0, 0.1, 0.4. Each case' s possibility of happening in different intervals is shown below:

Table 3: possibility of cases happening in different intervals

Interval	[0,0.3]	[0.3,0.5]	[0.5,0.7]	[0.7,1.0]
Possibility	0.5	0.1	0.1	0.3

Proceed the following steps:

a. Establish hypothesis and its alternative hypothesis.

 H_0 : the distribution obeys triangle distribution

 H_1 : the distribution disobeys the triangle distribution

b. Calculate and examine the statistic quantity and rejection region.

This is a test of goodness of fit of distributions which has 4 types as a whole; its possibilities of happening are 0.5, 0.3, 0.1, 0.1 respectively. As n = 13, we examine the statistic quantity

$$\chi^{2} = \sum_{i=1}^{k} \frac{(n_{i} - np_{i})^{2}}{np_{i}}$$

= $\frac{(5 - 13 * 0.5)^{2}}{13 * 0.5} + \frac{(1 - 13 * 0.1)^{2}}{13 * 0.1} + \frac{(1 - 13 * 0.1)^{2}}{13 * 0.1} + \frac{(3 - 13 * 0.3)^{2}}{13 * 0.3}$
= 0.243693

As k = 4, we get the examined rejection region $W = \{\chi^2 \ge \chi^2_{1-\alpha}(3)\} = \{\chi^2 \ge 7.8147\}$. c. Come to a conclusion.

The examined statistic quantity $\chi^2 = 0.243693$ is not in the rejection region, so the original assumption is accepted. The geographical location of this serial murder obeys the triangle distribution.

3 Model solution

3.1 Solving the geographical profile of crime

Taking E as the origin, EA is axis x, axis y perpendicular to EA; we set the rectangular coordinate system as below:



From Fig.2: the possible crime locations are distributed on sides BE and BA on triangle ABE. So that sides BE and BA and the areas nearby create the crime geographical distribution map.

Suppose θ is a parameter of the population, its parameter space is Θ , x_1, x_2, \dots, x_n is a sample from the population. To the given α ($0 < \alpha < 1$), if there are two statistics

$$\hat{\theta}_{L} = \hat{\theta}_{L}(x_{1}, x_{2}, \dots, x_{n}) \text{ and } \hat{\theta}_{U} = \hat{\theta}_{U}(x_{1}, x_{2}, \dots, x_{n}), \text{ if to the random } \theta \in \Theta, \text{ there will be}$$

$$P_{\theta}(\hat{\theta}_{L} \leq \theta \leq \hat{\theta}_{U}) \geq 1 - \alpha, \text{ then the stochastic } [\hat{\theta}_{L}, \hat{\theta}_{U}] \text{ is the interval of confidence } \theta^{[3]}$$
Given α ($0 < \alpha < 1$) in the question, from $P\{|x| < k\} = 1 - \alpha, \text{ we get}$

$$P = P\{x < k\} - P\{x < -k\} = F_x(k) - F_x(-k)$$
(3)

According to equation (1), we get $k = \frac{1-\alpha}{4} (1 < k < 2)$. When $\alpha = 0.05$, k = 0.2375.

Crime sequence	longitude	latitude	The distance to the 1st point (meter)	The distance to the 8th point (meter)
1	1.529958	53.81588	0	53815.88
2	1.532939	53.80799	927.1343	31252.96
3	1.498125	53.84062	4222.307	30572.28
4	1.763831	53.81075	23880.04	35234.3
5	1.641267	53.81463	11344.06	17652.99
6	2.236572	53.47153	87105.43	22881.49
7	2.149433	53.53328	75081.24	54436.11
8	1.779778	53.65239	32448.37	42798.67
9	2.2284	53.46219	86795.49	0
10	2.096494	53.555	68608.87	54018.03
11	1.873389	53.75898	36337.65	36413.24
12	1.671956	53.8083	14534.29	15333.72
13	1.549317	53.80365	2401.851	20515.44

Table 4: longitude and latitude of the 13 crime locations

Because of event $A = \{1, 2, 3, 5, 13\}$, event $E = \{8\}$, using the average distance method, according to the Euclidean distance formula $d_{ij} = \sqrt{\sum_{t=1}^{m} |x_{it} - x_{jt}|^2} (i, j = 1, 2, \dots, n)$, we can get the Euclidean distance between

event A and event E as below:

Crime sequence		Longitude	Latitude	The distance to the 8th point (meter)
	1	1.529958	53.81588	53.81588
	2	1.532939	53.80799	31252.96
Event A	3	1.498125	53.84062	30572.28
	5	1.641267	53.81463	17652.99
	13	1.549317	53.80365	20515.44
Average distance				20009.5
Event E	8	1.779778	53.65239	0

Table 5: the Euclidean distance between event A and event E

From Table 5 we can know the distance between event A and event E is 20009.5 meters.

In the triangle distribution density function graph (Fig. 2), the numeral value from A to L is 2. According to the proportion relations:

act ual	di st ance a_1	act ual di st ance a_3	actual distance 20009.5	actual distance L
axi s	number a_2	axis number a_4 ,	axis number 2	axis number 0.2375

Value k represents the actual distance L =2376.128 meters. Thus, the crime geographical profile is formed with each point on the 2 sides of the triangle distribution as centers and 2376.128 meter as radius. Its "geographical profile" is as below:



Fig. 3: Geographical profile of crime

Fig. 4: Geographical profile of crime

3.2 Solving the criminal's living place

3.2.1 rationale of geographic drawing:

"Rationale of geographic drawing" refers to the detecting method that marks each location of crime scenes in the serial murders on the map, and analyzes the space distribution graphs of each location of crime so as to determine the territorial scope where the suspects hide. Using the geographic drawing to analyze the spatial relations of point, line and surface which are formed in each location of crime in the serial murders, we can comparatively accurately deduce the most likely position of the criminals. In other words, if we can make clear every location of attacks of the criminals, we are most likely to find out the "center of mass" of the criminals' crime activities; and the "center of mass" is mostly the criminals' home or living place.

Criminology believes that the criminals will consider many problems when they choose the locations of serial crime, but generally they mainly have three tendencies: one is that they will choose familiar locations. Most criminals will plan how to escape from the crime scene. Two is that they will try to avoid their familiar people. They usually have the thought that "a villain doesn't harm his next-door neighbors"; This thinking indicates that when the criminals are choosing the crime location, they will keep away from their familiar environment. Three is the crime location cannot be too far from their living place. If it is too far, it is not easy for them to escape fast; and they also lack the familiarity. But if the location is too near, it will also be easy for them to encounter familiar people, which lead to problems committing their crimes. In most cases, the criminals will unconsciously take their own living place into account when they are choosing the locations of attacks. This offers the possibility that the police determine the detecting scope through geographic drawing [12].

3.2.2 Use the geographic drawing to deduce the criminal' s living place

a. The criminal will be dominated by the thinking of "a villain doesn't harm his next-door neighbors", and chooses the place that is either too far nor too near from his living or working place to commit a crime.



Fig. 5: criminal' s living place (1)

Fig. 6: criminal's living place (2)

b. The criminal will be dominated by the thinking of "not being too far away from home"; his living place is usually located in the center of the crime scope.

3.3 Solving the next crime location

The criminal in this case attempted 23 crimes, succeeded 13 times and failed 10 times. The basic situation is as below:



Fig. 7: criminal' s locations of crime

Analyzing Fig. 7, we can predict the next crime direction with every 2 neighboring points in random.

E.g.: take the 1st, 2nd and 3rd locations of crime as an example, the direction from the 1st to 2nd is opposite to that from the 2nd to the 3rd.

Take the 11th, 12th and 13th locations of crime as an example, the direction from the 11th to the 12th is opposite to that from the 12th to the 13th.

Take the 21st, 22nd and 23rd locations of crime as an example, the direction from the 21st to 22nd is opposite to that from the 22nd to the 23rd.

In the map, every neighboring 3 points of crime all satisfy the above-mentioned law. Therefore, only knowing the neighboring 2 points of crime, we can deduce the next crime direction. With the geographical profile of crime (Fig. 3), we can comparatively determine the next crime direction accurately.

If we know the former 2 locations of crime like point n and point n+1, we can more deeply predict the scope of the next location of crime. The shadow in the graph below is the scope of the next location of crime.



Fig. 8: next location of crime

THE SHORTEST DISTANCE CLUSTERING

1 Clustering Model

It is one of the important work of the police to solve the case finding out the region of the criminal often committed a crime. Through investigation of these areas, we can get a lot of information about offenders, such as the offender of the living environment, working environment, and even physical appearance characteristics. Detailed case information will help the police to solve the case timely and accurately.

During the period between 1974 and 1980, Peter Sutcliffe committed 22 crimes in total, which killed 13 people. We observed all sites in Fig. 1 and found these sites have some certain concentration regions. In order to find out the

region of the offender often committed a crime with a scientific method accurately; we conducted cluster analysis based on the shortest distance through using the longitude and latitude (Annex1) of 22 crimes. Then the approximate sites will be together. The program of the shortest distance cluster analysis can be seen in Annex 2. Fig. 2 and 3 below show the clustering results. Given these regions, we can see more clearly where "center of mass" of the locations of the attacks is.

					C	luster His	tory						
												Norm	Т
												Min	i
NCL	C	lusters	Joined		FREQ	SPRSQ	RSQ	ERSQ	CCC	PSF	PST2	Dist	е
21		7		13	2	0.0000	1.00					0	т
20	CL21			20	3	0.0000	1.00	-	-	-	-	0	
19		4		8	2	0.0000	1.00			53E5		0.001	
18		10		18	2	0.0000	1.00		-	7E4	-	0.0108	
17	CL19		CL20		5	0.0000	1.00			16E3	1522	0.0151	
16		15		21	2	0.0000	1.00			1E4	-	0.0265	
15		9		14	2	0.0001	1.00	-	-	4297	-	0.0516	
14	CL17			5	6	0.0002	1.00			2136	43.3	0.0566	
13	CL15		CL 18		4	0.0004	.999	-	-	1170	8.8	0.0644	
12	CL14			11	7	0.0006	.999	-	-	718	16.6	0.0738	
11		2		17	2	0.0002	.999			754		0.0818	
10		12		16	2	0.0003	.998	-	-	746	-	0.1066	
9	CL12			6	8	0.0027	.995			359	20.2	0.1376	
8	CL9			22	9	0.0010	.994	-	-	359	2.0	0.1406	
7	CL13			19	5	0.0033	.991		-	279	23.2	0.2543	
6		1		3	2	0.0025	.989			278	-	0.2946	
5	CL8		CL7		14	0.0677	.921	-	-	49.5	97.3	0.2986	
4	CL11		CL16		4	0.0125	.908	.870	1.92	59.6	117	0.4377	
3	CL4		CL5		18	0.1627	.746	.805	-1.6	27.9	29.4	0.5148	
2	CL6		CL3		20	0.1068	.639	.667	43	35.4	7.6	0.5427	
- 1	61.9		0140		99	0.420.0	000	000	0 00		9E h	4 6607	





Pseudo-statistic t^2 (PST2) is used to evaluate the combined effect of clustering process indicators. From the statistic knowledge, we can know if the pseudo-value t^2 is large, it is showing the two categories of the last mergers are separated, namely, the last clustering effect is quite good. Let us have a look at the clustering process of Fig. 4; the biggest value of pseudo-statistics t^2 (PST2) is 1522, sub-big as 117 while the third as 97. So, it is more appropriate can to divide it into 5,6 or 18 categories . Combining with Fig. 5 Pedigree Chart and the practical condition in the place of occurrences in West Yorkshire, we believe it is most appropriate to divide into six categories. Specific categories are as follows: CL1 = (1,3), CL2 = (2,17), CL3 = (15,21), CL4 = (4,5,6,7,8,11,13,20,22), CL5 = (9,10,14,18,19), CL6 = (12,16), of which 1 to 22 mean committing crimes locations. After clustering, we marked these areas out in the coordinate axis, regionalization results shown in Fig. 6.

Since we can get the committing times of the offender did in each region, we can easily count the committing rate of

the offender did in different kind of regions are

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P (CL1) =2/22=9.09%, P (CL2) =2/22=9.09%, P (CL3) =2/22=9.09%,
P (CL4) =9/22=40.91%, P (CL5) =5/22=22.73%, P (CL6) =2/22=9.09%.
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Set Peter Sutcliffe is not arrested after his 22 crimes, we have reason to predict the crime place of his next crime will probably be in $CL4_{\sim}$ CL5.



Fig. 11Coordinate Diagrams of Case High-Occurring Regions

Usually, offenders commit a crime in a particular area. We call it "Crime Zone". On the serial murder case, victims are not often selected randomly. In order to show the directions of the serial killer committing the crime, we calculate the percentage of the number of the case in each Crime Zone (the number of each Crime Zone is CL1, CL2, CL3, CL4, CL5 CL6, Fig. 4).From Table 6 below, we can see the probability of the direction of the offender committing the crime next time. For example, assume the last crime site is CL1. Because CL2 and CL4 have the same probability, 50%, we can predict the offender will go to CL2 or CL4 next time.

Region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	0	1	0	1	0	0
percentage	0	50%	0	50%	0	0
region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	1	0	0	0	1	0
percentage	50%	0	0	0	50%	0
Region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	0	0	0	1	0	1
percentage	0	0	0	50%	0	50%
Region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	0	0	1	4	2	1
percentage	0	0	12.5%	50%	25%	12.5%
Region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	0	0	1	2	2	0
percentage	0	0	20%	40%	40%	0
Region	CL1	CL2	CL3	CL4	CL5	CL6
The rate of direction of committing a crime	0	1	0	1	0	0

Table 6 Predictions on th	e Direction of th	e Crime of the O	ffender
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Note: Table 6, the blue regions with underlines are the regions we predicted that the offender would commit his next crime in. From Table 6, we can see that, as the 22nd crime place is in the regional CL4, if he was not arrested after the 22nd crime, then the rate of his next crime will occur in the regional from CL1 to CL6 probably are as follows: 0%, 0%, 12.5 %, 50%, 25%, 12.5%.

2 Solution and Result

Combining the method of round covering face and the shortest distance clustering method, we can get a geographical outline shown in Fig. 12(A combination of geographic map). Fig. 12 shows the place distribution of

committing crime between getting few information and more information the two situation in the case of serial murder. Finally, we predict the perpetrator will do his 23rd crime still in District CL4. So the police should pay more attention to Zone CL4.



Fig. 12 Geographical Profile

CONCLUSION

When we looked for the geographical distribution map of the serial killers commit crimes, this paper adopts two methods. One is the triangle distributive model based on the geographical locations, while the other one is the shortest distance clustering method.

Based on the past crime data and geographical location, we can probably determine that they obey the triangular distribution. Using the knowledge of mathematical statistics to be tested, we can determine the perpetrators of crime region. Of course, when crime information is little, perhaps it is not so accurate. Scheme one uses the knowledge on possibility and mathematical statistics to construct the Simpson distribution model, and test and verify that the crime location obeys the Simpson distribution. Then according to the interval, we can estimate that the interval of confidence of the "geographical profile" is [-2376.128m, 2376.128m] under the condition of the confidence level,

 $1-\alpha$. Taking all the points on the edges of the Simpson distribution as the centers and 2376.128m as radius, we draw circles and connect all the circles' edges, which form an area that is the "geographical profile" of the serial criminal.

However, using the shortest distance clustering method, the presupposition is already getting more information on the case, especially, the information of geographical areas where the offender often come and go. In the case of serial murder, only by making sure a more accurate history crime area can narrow the detection range, saving time and cost of police, and improving the successful rate of capturing the criminals.

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