Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(7):361-370

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Design and reliability analysis of large-scale multi-level sampling plan

Lihong Li, Yan Li*, Jie Sun and Qinghua Liu

College of Science, Hebei United University, Tangshan, Hebei, China

---

ABSTRACT

In order to establish the additional method of single objective layer composite PPS sample the permanent random technique was used in a set of multi-objective hierarchical design scheme. Yves g. Berger (2005) research was used instead of Hajek estimator for HT estimate variance estimators. Simple analysis was carried out on the sampling error.

Keywords: Sampling survey; multilevel; stratified sampling; error analysis; additional PPS sample method

---

INTRODUCTION

Hierarchical sampling survey method versatilely uses a sample and improved the utilization rate of it, which is a major tool in large-scale data collection and analysis [1]. China has implemented hierarchical management system on the government survey in economy, population, agriculture, etc. More and more cases are inseparable from the stratified sampling, and therefore a suitable design of the hierarchical sampling program is necessary [2-5].

Currently, investigating solutions of the problem in hierarchical estimation methods are mainly direct and indirect estimation methods. Direct estimation methods are layers of sampling method, ABC method and the sample addition method [6-7]. Layers of sampling and sample design using ABC method have great limitations in practical applications because they require that all levels should be investigated which contrary to consume large sample survey economic advantage [8-11]. Additional samples' sampling method is a bottom-up design idea, and it is to meet the level of the target variable estimation sample additional ways to the next level of the target variable estimation accuracy required on the basis of need, which can satisfy sample survey of actual demand. Indirect estimation method is to fully exploit the existing sample information, make full use of indirect information and data improve all levels of the estimated amount of the target variable [12-15].

This paper introduced the basics of stratified sampling, followed by an additional method using sample design, and the design of the program in evaluation and optimization.

STRATIFIED SAMPLING

The definition of stratified sampling

Stratified sampling first stratify the unit according to certain general sign, and then several layers according to the principles of randomly selected sample units into the sample. Sample units drawn from the layers constitute the total sample. Suppose there are N units overall $\pi_N = \{Y_1, Y_2, \ldots, Y_N\}$, stratify the unit k layers according to certain general sign, the $h(h = 1,2,\ldots k)$ layer π_{Nh} has N_h units, that is

$$N = \sum_{h=1}^{k} N_h \,,\quad \pi_{N_h} = \{Y_{h1}, Y_{h2}, \ldots, Y_{hNh}\}$$

From $h(h=1,2,...k)$ layer siphon n_i units to constitute a sub-sample of the h layer:

$$y_{hi}=(y_{h1},y_{h2},...,y_{hni}),\quad (i=1,2,...,n_h)$$

All the sub-samples are the overall sample:

$$y=\bigcup_{i=1}^{k}y_{(i)}=(y_1,y_2,...,y_n),n=\sum_{i=1}^{k}n_i$$

Sample append method

Process of additional sample is divided in three steps: first we need to determine the overall sample, followed by determining the sample size, and the third to determine additional sampling methods.

Suppose overall units are U, in certain initial sample S_p, initial sample size is n_p, there are two additional strategies:

（1）Replace the additional. Additional sampling during the initial samples have been returned to the sampling frame, additional sampling during the whole sampling remains U;

（2）Additional without replacement. Additional sampling during the initial sample without replacement sampling frame, additional sampling during the whole sampling is U/S_p .

For multi-phase sampling, the situation is more complex, the type of additional stages may have different policies.

Sample estimation method

Different additional sampling and sample methods can take two estimation methods, namely the weighted estimate, another is estimated HT.

If we obtain parameter estimates $\hat{\theta}_p$ of parameter θ using initial sample S_p, and unbiased estimates $\hat{\theta}_c$ of parameter θ using additional sample S_c, then the final estimated is $\hat{\theta}=a\hat{\theta}_p+(1-a)\hat{\theta}_c,(0\le a\le 1)$ which is weighted average of $\hat{\theta}_p$ and $\hat{\theta}_c$.

HT estimates are generally used in sampling without replacement unit will not be repeated when pumped. When there is an additional sampling besides sampling is not put back twice, and the strategy is not added back in, the unit will not be able to get repeated in other cases, cells are likely to be repeated pumped.

In fact, regardless of whether the unit may be able to get repeated, we can take the HT estimates. Using HT estimate for overall Y:

$$\hat{Y}_m=\sum_{i=1}^{n'}\frac{y_i}{\pi_i}$$

Among them n' represents the effective sample size, π_i represents probability eventually being able to get of unit i, called final probability sample. We use π_{pi} said in the initial probability sampling unit i is pumped, called it into initial probability, π_{ci} called chase join probability. When additional sampling and initial sampling are independent:

$$\pi_i=1-(1-\pi_{pi})(1-\pi_{ci})$$
$$=\pi_{pi}+\pi_{ci}-\pi_{pi}\pi_{ci}$$

When additional strategy is not put back when, according to the probability of incompatible events additive:

$$\pi_i=\pi_{pi}+\pi_{ci}$$

When additional strategy is back, but additional sampling and initial sampling are not independent, the relationships of $\pi_i and \pi_{pi}, \pi_i and \pi_{ci}$ are not clear.

Define the indicator function:

$$I_i = \begin{cases} 1(i \in S) \\ 0(i \notin S) \end{cases}$$

。

When additional sampling and initial sampling are independent,

$$\begin{aligned} \pi_{ij} &= (\pi_i + \pi_j - 1) + P(I_i = 0, I_j = 0) \\ &= (\pi_{pi} + \pi_{ci} - \pi_{pi}\pi_{ci} + \pi_{pi} + \pi_{ci} - \pi_{pi} - \pi_{ci} - 1) \\ &\quad + (1 - \pi_{pj} - \pi_{pj} + \pi_{pij})(1 - \pi_{ci} - \pi_{cj} + \pi_{cij}) \\ &= (\pi_{pj} + \pi_{cj} + \pi_{pj} + \pi_{ci} + \pi_{pij} + \pi_{cij}) \\ &\quad + \pi_{pij}(1 - \pi_{ci} - \pi_{cj}) + \pi_{cij}(1 - \pi_{pi} - \pi_{pj}) \end{aligned}$$

When additional sampling and initial sampling are not independent, there is no clear formula of π_{ij} .

In different methods of sampling and sample additional, two estimation methods are considered. As to which method is better depends on the general characteristics and overall or domain-specific sampling and sample additional methods.

PROGRAM DESIGN

Sample survey targets

The case is based on 2012 statistics, Hebei Province, rural food production as the original survey data. In the survey, we chose Hebei Province, Chengde City, Qinhuangdao City as a sub-population surveys and investigations in general to form a hierarchical sampling. All the survey sample size is at 95% confidence level, and the target amount for each research domain absolute error does not exceed 5% of the limit determined by the premise.

The overall sample and the cell

In order to facilitate convergent with national sample surveys, we selected 171 counties in Hebei Province as the survey sampling design of the survey period. 171 samples of all county towns are the overall design of the sample, and each township is a investigation unit.

Sampling methods

The whole process will use the PPS random systematic sampling method. First part of the 11 municipalities in Hebei Province is the first level; each survey counties and municipalities as a second level; each county under the jurisdiction of the township streets as the third level. After determining the sample size n, we take M_0（$M_0 = \sum M_i$，M_i measure the size of each unit）divide n get the sampling interval K. We draw a random number R between 1--K, the range in which the amount of the sample is drawn, the rest of the sample interval of time plus the selected K. The probability into a sample is $Z_i = M_i / M_0$.

Qinhuangdao city with a select sample as an example, withdrawn from the sample size n =62 PPS sample. Sampling interval $K = M_0 / n$, $R \in (1, K)$,we suppose R=30000. In the harbor town harbor area code range, the sample is drawn, followed by adding intervals to get the rest of the sample. Which Qinglong Manchu Autonomous County Qinglong Town, Funing Liuying Town, Shimenzhai town, Lulong County Lulong town, Liutianzhuang town is drawn twice. Which Funing Township, Mujing town in Lulong County is drawn three times. The town of Changli County Changli be drawn four times.

Table 1 Qinhuangdao municipal sample selection process table

Qinhuangdao municipal sample	The total population of the township	$\sum M_i$	Code range		Probability of being a sample
Donggang Town in Harbor District	19786	19786	1	19786	0.009021352
Harbor Town in Harbor District	26858	46644	19787	46644	0.012245804
West harbor Town in Harbor District	22056	68700	46645	68700	0.01005635
Haiyang Town in Harbor District	18733	87433	68701	87433	0.008541241
North Harbor Town in Harbor District	24929	112362	87434	112362	0.011366284
First Town in Shanhaiguan District	15868	128230	112363	128230	0.007234955
Shihe Town in Shanhaiguan District	19983	148213	128231	148213	0.009111174
Mengjiang Town in Shanhaiguan District	17436	165649	148214	165649	0.007949879
Bohai Town in Shanhaiguan District	9301	174950	165650	174950	0.004240756
Haibin Town in Beidaihe District	13404	188354	174951	188354	0.006111503
Daihe Town in Beidaihe District	23189	211543	188355	211543	0.010572937
Qinglong Town in Qinglong Manchu Autonomous County	78086	289629	211544	289629	0.035603019
Zushan Town in Qinglong Manchu Autonomous County	23574	313203	289630	313203	0.010748477
Mutoudeng Town in Qinglong Manchu Autonomous County	33524	346727	313204	346727	0.015285142
Shuangshanzi Town in Qinglong Manchu Autonomous County	23434	370161	346728	370161	0.010684644
Majuanzi Town in Qinglong Manchu Autonomous County	25404	395565	370162	395565	0.011582858
Xiaoyingzi Town in Qinglong Manchu Autonomous County	34724	430289	395566	430289	0.015832277
Dawulan Town in Qinglong Manchu Autonomous County	35349	465638	430290	465638	0.016117244
Tumenzi Town in Qinglong Manchu Autonomous County	26907	492545	465639	492545	0.012268146
Badaohe Town in Qinglong Manchu Autonomous County	30507	523052	492546	523052	0.013909552
Gehetou Town in Qinglong Manchu Autonomous County	26853	549905	523053	549905	0.012243525
Louzhangzi Town in Qinglong Manchu Autonomous County	24412	574317	549906	574317	0.01113056
Fenghuangshan Town in Qinglong Manchu Autonomous County	11262	585579	574318	585579	0.005134867
Longwangmiao Town in Qinglong Manchu Autonomous County	17181	602760	585580	602760	0.007833612
Sanxingkou Town in Qinglong Manchu Autonomous County	14714	617474	602761	617474	0.006708793
Gangou Town in Qinglong Manchu Autonomous County	9738	627212	617475	627212	0.004440005
Dashiling Town in Qinglong Manchu Autonomous County	12399	639611	627213	639611	0.005653278
Guanchang Town in Qinglong Manchu Autonomous County	12780	639611	639612	639611	0.005826993
Ciyushan Town in Qinglong Manchu Autonomous County	19460	671851	639612	671851	0.008872714
Pingfagnzi Town in Qinglong Manchu Autonomous County	11964	683815	671852	683815	0.005454941
Anziling Town in Qinglong Manchu Autonomous County	15673	699488	683816	699488	0.007146046
Zhuzhangzi Town in Qinglong Manchu Autonomous County	15116	714604	699489	714604	0.006892083
Caonian Town in Qinglong Manchu Autonomous County	11805	726409	714605	726409	0.005382445
Qidaohe Town in Qinglong Manchu Autonomous County	9560	735969	726410	735969	0.004358846
Sanbozi Town in Qinglong Manchu Autonomous County	12414	748383	735970	748383	0.005660117
Liangshuihe Town in Qinglong Manchu Autonomous County	20426	768809	748384	768809	0.009313158
Changli Town in Changli County	121489	890298	768810	890298	0.055392453
Jing'anTown in Changli County	42858	933156	890299	933156	0.019540944
Anshan Town in Changli County	48114	981270	933157	981270	0.021937398
Longjiadian Town in Changli County	43687	1024957	981271	1024957	0.019918924
Nijing Town in Changli County	26821	1051778	1024958	1051778	0.012228934
Dapuhe Town in Changli County	11787	1063565	1051779	1063565	0.005374238
Xinji Town in Changli County	32266	1095831	1063566	1095831	0.014711562
Liutaizhang Town in Changli County	23418	1119249	1095832	1119249	0.010677349
Ruhe Town in Changli County	16131	1135380	1119250	1135380	0.007354869
Zhugezhuang Town in Changli County	31558	1166938	1135381	1166938	0.014388752
Huangdianzhuang Town in Changli County	29786	1196724	1166939	1196724	0.013580815
Tuanlin Town in Changli County	7022	1203746	1196725	1203746	0.003201655
Getiaogang Town in Changli County	21730	1225476	1203747	1225476	0.009907712
Matuodian Town in Changli County	38420	1263896	1225477	1263896	0.017517455
Liangshan Town in Changli County	19830	1283726	1263897	1283726	0.009041414
Shilipu Town in Changli County	13308	1297034	1283727	1297034	0.006067733
Funing Town in Funing County	111679	1408713	1297035	1408713	0.050919621
Liushouying Town in Funing County	55540	1464253	1408714	1464253	0.025323254
Yuguan Town in Funing County	37850	1502103	1464254	1502103	0.017257565
Niutouya Town in Funing County	40846	1542949	1502104	1542949	0.01862358
Shimenzhai Town in Funing County	48861	1591810	1542950	1591810	0.02227799
Taiying Town in Funing County	45535	1637345	1591811	1637345	0.020761512
Daxinzhai Town in Funing County	35697	1673042	1637346	1673042	0.016275913
Zhucaoying Town in Funing County	24810	1697852	1673043	1697852	0.011312026
Duzhuang Town in Funing County	23308	1721160	1697853	1721160	0.010627195
Chapeng Town in Funing County	40425	1761585	1721161	1761585	0.018431627
Shenhe Town in Funing County	8251	1769836	1761586	1769836	0.003762012
Lulong Town in Lulong County	70591	1840427	1769837	1840427	0.032185701
Panzhuang Town in Lulong County	26088	1866515	1840428	1866515	0.011894726
Yanhe Town in Lulong County	35871	1902386	1866516	1902386	0.016355248
Shuangwang Town in Lulong County	31085	1933471	1902387	1933471	0.014173089
Liutiangezhuang Town in Lulong County	45258	1978729	1933472	1978729	0.020635215

Shimen Town in Lulong County	44380	2023109	1978730	2023109	0.020234894
Xiazhai Town in Lulong County	21642	2044751	2023110	2044751	0.009867589
Liujiaying Town in Lulong County	17235	2061986	2044752	2061986	0.007858234
Chenguantun Town in Lulong County	28349	2090335	2061987	2090335	0.01292562
Yinzhuang Town in Lulong County	28856	2119191	2090336	2119191	0.013156785
Habo Town in Lulong County	29331	2148522	2119192	2148522	0.013373359
Mujing Town in Lulong County	44719	2193241	2148523	2193241	0.02038946

Determination of sample size
In the survey, the sample size of the survey is all at 95% confidence level, the target amount for each research domain absolute error limit does not exceed 5% of the premise determined.

The population census in 2010 as auxiliary variables, we use PPS systematic sampling.
Using the formula: $n = P(1-P)/\left(e^2 / Z + P(1-P)/N\right)$

Calculated sample size of Hebei Province, Chengde and Qinhuangda.
Hebei Province-level sample size =475
Chengde municipal sample size =124
Qinhuangdao municipal sample size =55
Sampling ratio: 24.25%

Determine the estimated amount
In this paper, using traditional Hansen - Hurwitz (HH) estimator，

The overall amount was estimated: $$\hat{Y}_{HH} = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{Z_i}$$

Sampling variance: $$\hat{v}\left(\hat{Y}_{HH}\right) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{Z_i} - \hat{Y}_{HH}\right)^2$$

Confidence level of 95% confidence interval: $$\hat{Y}_{HH} \pm Z_{\frac{a}{2}}\sqrt{\hat{v}\left(\hat{Y}_{HH}\right)}$$

The overall mean estimate of: $$\hat{Y}_{PPS} = \frac{1}{Nn}\sum_{i=1}^{n}\frac{y_i}{Z_i}$$

Variance estimator is: $$\hat{v}\left(\hat{Y}_{PPS}\right) = \frac{1}{N^2 n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{Z_i} - \hat{Y}_{PPS}\right)^2$$

PROGRAM IMPLEMENT
Progress of program implement
Step one：Hebei general confer permanent random number. Using random starting point equidistant PPS sampling method chooses the initial sample of Hebei Province, and calculates first-order inclusion probabilities.

Step two: The same sample using the above method to remove Chengde and Qinhuangdao City, in this selection, we need to remove selected samples in first step. And calculate the probability enrolled in the overall sample.

Step three: Chengde, Qinhuangdao selected samples combined with the first step as the final stage of the cell sample of the beginning of Hebei Province.

Step Four: Using random starting point, equal probability sampling methods draw samples of sample households in the village and household survey to calculate the average value of the target amount, multiplied by the total number of households in the sample villages, as the statistical value of the sample villages.

Step Five: Calculate the Chengde, Qinhuangdao, Hebei Province, the total estimated amount of food production and value of variance estimators.

Stratified sample survey

Detailed results of the various indicators are list：

Tables 2 2012 Hebei province's total grain output sample data

Unit: tonne

	Chengde City	Qinhuangdao City	Hebei Province
Estimates	1310701	816959	32845843
Average variance estimates	123485	13177	820582
Actual statistics	1336367	786108	37889069
Test value	0.02702	0.01938	0.00365

To illustrate the effectiveness of this sampling program, we will combine the province's grain output actual food production and the province with the kind of probability sampling and test for comparison chart（Fig1, Fig2, Fig3）:

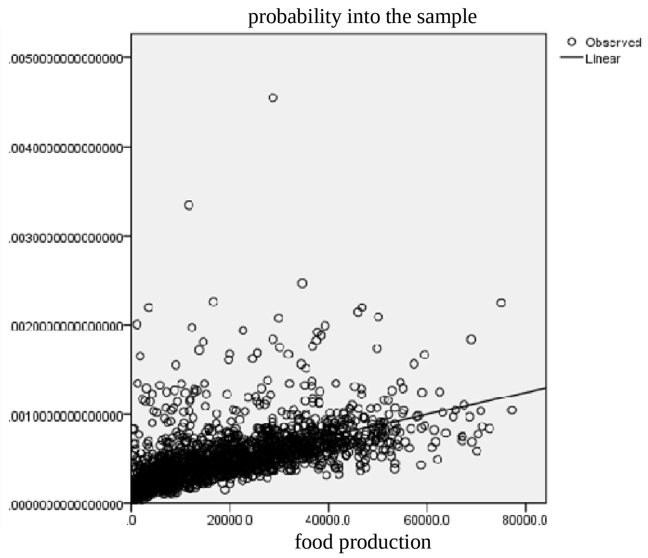


Fig. 1: Actual food production

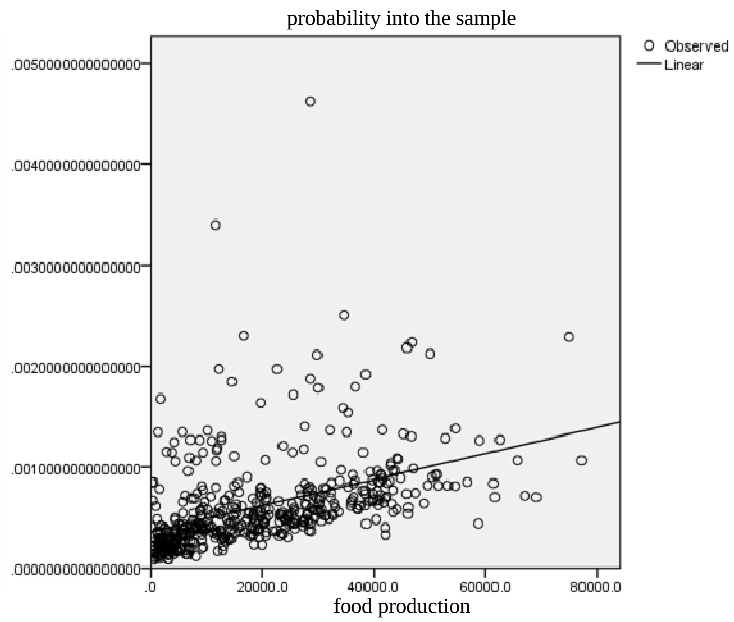
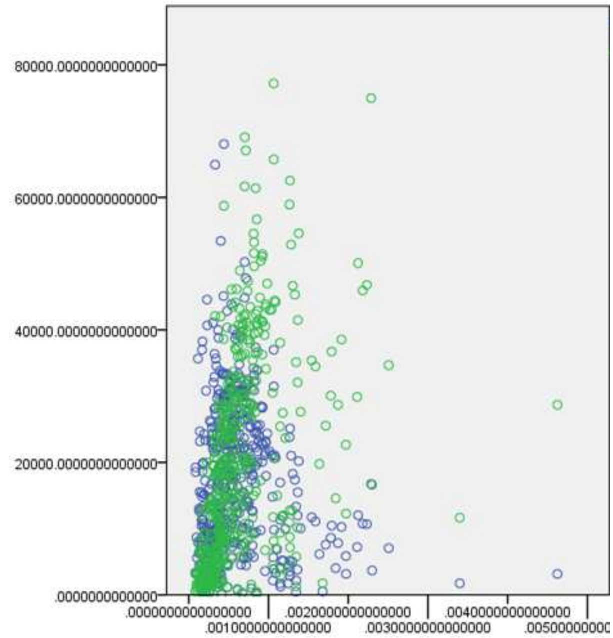


Fig. 2: Probability sampling



Blue point：probability into the sample- food production of sample estimates
Green point：probability into the sample- food production of sample

Fig. 3: Test for comparison chart

Figure 1 for the province in which the actual food production in the scatter diagram into a kind of total probability, figurer 2 for the province's grain output sample distribution probability samples in total income. By comparing the two figures can be found the sample roughly match the actual number of distribution. Figure 3 is scatter plot in which the sample under the same probability sample estimate of actual food production and food production sample comparison. This figure can be observed that the estimated value and the actual value of roughly the same, but there are some differences. In order to more accurately estimate the reliability of the samples obtained, we further analyzed.

In many cases, the reliability is defined：

$$R = P(Z_{\min} \leq Z \leq Z_{\max})$$

So that, for X～N (μ, σ^2), its reliability is：

$$R = P(X \leq x) = \Phi\left(\frac{X-\mu}{\sigma}\right) \qquad (4.8)$$

μ and σ are not clear, we use $\frac{X - X'}{S'}$ replace $\frac{X-\mu}{\sigma}$

Above it：

$$\overline{X}' = \frac{1}{n-1}\sum_{\substack{i=1 \\ i \neq k}}^{n} X_i$$

$$S'^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(X_i - \overline{X}'\right)^2$$

According to estimates of the actual yield and grain samples (Figure 4, Figure 5), the two sets of data can be observed approximately normal:

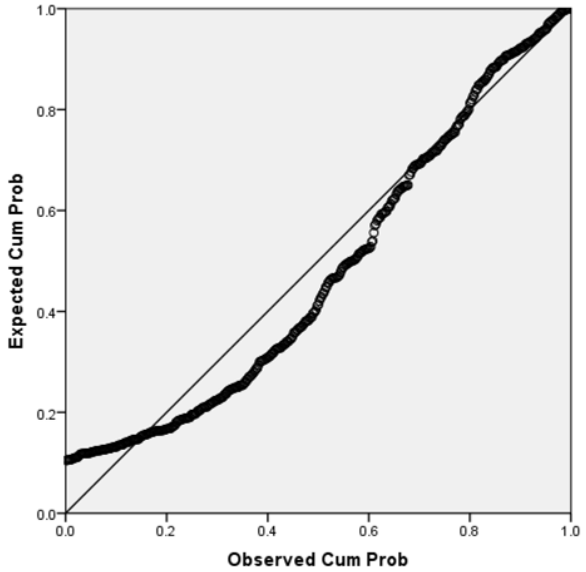


Fig. 4 Estimates of the actual yield

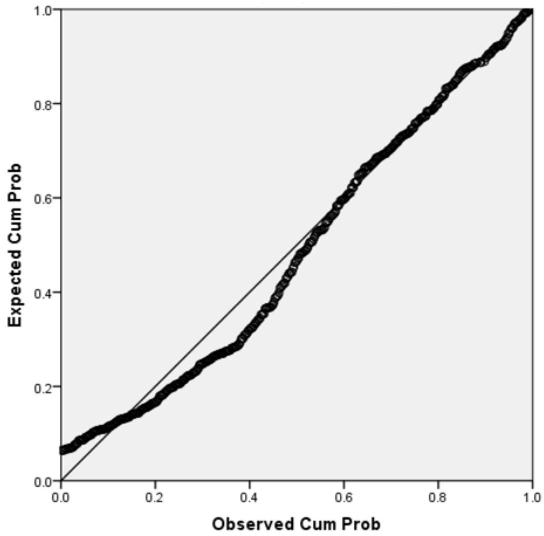


Fig. 5 Estimates of the grain samples

R=0.789。After calculation, the reliability of the findings R = 0.789, which proves the credibility of the findings between 75% and 80%. The result has some representation, but its accuracy needs improving.

PROGRAM IMPROVING

Improvement program using Hajek estimator method[14] instead of variance estimator method.

The total estimated amount is：

$$\hat{T} = \sum_{i \in s} y_i \pi_i^{-1}$$

Variance estimator is:

$$\hat{\sigma}^2 = \frac{n}{n-1}\left[\sum_{i \in s} y_i^2 \left(\pi_i^{-1}\right)\pi^{-1} - \hat{d}\hat{G}^2\right]$$

Among it:

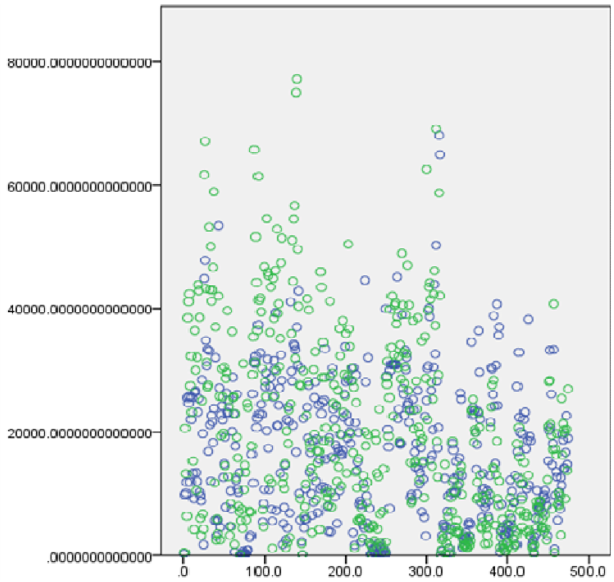
$$\hat{d} = \sum_{i \in s}(1-\pi_i) \qquad \hat{G} = \frac{1}{\hat{d}}\sum_{i \in s} y_i (1-\pi_i)\pi_i^{-1}$$

Where S is the sample collection, y_i is statistical variables, π_i is a first-order inclusion probabilities.

Table 3: Improved statistical results and actual statistical results table
Unit: tonne

	Hebei Province	Chengde City	Qinhuangdao City
Estimates	39975816	1306819	660034
Average variance estimates	1835920.84	110074.33	32643.52
Actual statistics	37889069	1336367	786108

Using Hajek estimation method the improved sample food production estimates and actual food production sample comparison chart (figure 6) and sample food production estimates figure (figure 7) are as follows:



Blue point：Improved sample estimates
Green point：Improved food production of sample

Fig. 6: Comparison chart

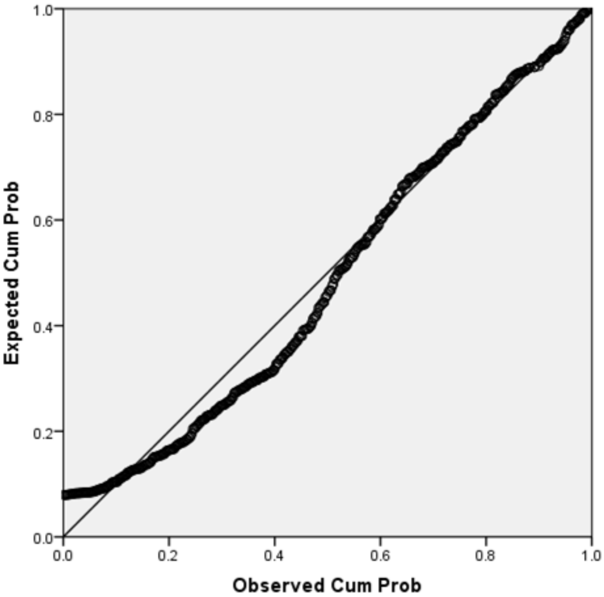


Fig. 7: Sample food production estimates

The improved reliability of the sample R = 0.839, its credibility in the range of 80% to 85%, slightly higher than the HH estimation.

CONCLUSION

Through statistical results, the conclusions were as follows:

All the survey results were within the permissible error range, which explained that additional PPS sampling method of random sample was feasible.

The conclusion that when using HH estimation sampling the results of its reliability was 75% to 80% was obtained through reliability analysis of the samples. While when estimating the reliability of estimated amount by using Hajek the reliability was 80% to 85%. The precision was slightly higher.

Acknowledgement
Supported by National Science Fund of China (NO.61370168), National Science Fund of Hebei Province China (NO.A2012209030)

REFERENCES

[1]Zhou Q Y. *Statistics and Decision*, Vol 1, 14-15, **2014**.
[2]You X M. S*cience and Technology Information*, Vol 20, 191-213, **2007**.
[3]Liu H Y. On Multi-objective Stratified Sampling Method and Its Application. Xi'an: Xi'an University of Finance, **2010**: 7-13.
[4]Jin Y J，Zhao X H. *Statistical Research*, Vol 12, 55-56, **2003**.
[5]Fan H K, Zhang F M. A. *Journal of Nankai University*. v 37, n 4, 44-47, **2004**.
[6]Lu Z H, He C Y, Tao H. *Number of Economic Research Technology*, vol 4, 60-65, **2005**.
[7]Zhang Y, Zeng Y P, Wang F X. *National Bureau of Statistics Education Center*, v 20, n 2, 28-29, **2005**.
[8]Jin Y J. Non-sampling Errors' Analysis. China Statistics Press, **1996**: 228-233.
[9]Feng S Y, Sample Survey Theory and Methods, China Statistics Press, **1998**:25-30.
[10]Lv P. *Statistics and Decision*. **2012**: 9-11.
[11]Cao Z G, Yang J X, Niu J L. *Hebei Rural Statistical Yearbook*, **2013**.
[12]Lin C S. Complex Multi-objective Hierarchical Sampling Design Research. Xiamen: Xiamen University, **2006**.47-57.
[13]Zhao J K. Survey Sampling Theory and Methods. China Statistics Press, **2002**:212-236.
[14]Hajek, *Ann. Math. Statist.* Vol 35, 1491-1523, **1964**.
[15]Berger.Y.G. *Statist. Plann. Inference*, Vol 127, 253-277, **2005**.