



Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Corrugated paperboard property modeling under transient impulse condition and response analysis

Dapeng Zhu

School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou, P.R. China

ABSTRACT

A mass loaded corrugated paperboard system is used to simulated a real package. The viscoelasticity of the paperboard is taken into account, the model for the corrugated paperboard dynamic property under the transient impulse excitation condition is formulated. The free response of a viscoelastic system is expressed as the sum of complex exponentials. The residues and poles of the system are identified by use of extended Prony method. An experimental system is set up for carrying out impulse experiments under different load conditions. Rounded impulse function is used to simulate the real half-sine impulse, the acceleration response analysis method of the system is presented. The model and the response analysis method provide theoretical basis for the proper use the corrugated paperboard.

Keywords: Corrugated paperboard, Dynamic model, Parameter identification, Response analysis

INTRODUCTION

Corrugated paperboard comprises of two layers of liner between which is the corrugated medium, because of its specific structure, it has many advantages over other material, such as the light weight, high strength to weight ratio, good thermal insulation property, etc. For long time, corrugated paperboard is mainly used to manufacture the corrugated box. In recent years, because of the good cushioning property and environmental performance, corrugated paperboard has been gaining increasing attention as a replacement for polymeric materials in protective packaging of product. Therefore, it's essential to study the properties of corrugated paperboard for its engineering application.

Navaranjan N et al[1] evaluates the humidity effect on the elastic properties and failure mechanism of two types of layered corrugated paperboard by a series of experiments. E and Wang[2] presents a mathematical model to predict the stress plateau of multilayered corrugated paperboard under flatwise compression in various humidity environments, the proposed model can be used for practical application of the optimum design and material selection of multilayered corrugated paperboard. Wang[3] analyses the compressive behavior of multi-layer corrugated sandwich paperboard, the research results can be used to optimize and design corrugated sandwich pads. The cushioning property of corrugated paperboard pad is tested by use of high-speed video equipment, a acceleration-deformation model of corrugated paperboard is presented, this model can be used to reproduce the drop behavior of corrugated paperboard, and can be used in protective packaging design[4-5].

During the transportation of packages, the main factors causing the failure of the products are shock and vibration. Because of increase of the distribution velocity, the dynamic loads during the distribution is becoming greater, especially the transient shock energy in the random vibration is becoming greater. According to the analysis results of the obtained transport environment[6-9], it is the transient impulses that cause the non-Gaussian distribution of the random vibration during the distribution. The motion status of package varies greatly in very short time if excited by the transient impulse, therefore, under the transient impulse condition, the dynamic property differs

greatly from the properties under other load conditions. If the package is excited by transient impulse, the damage probability of package is much more greater. Therefore, the dynamic property of corrugated paperboard analysis and modeling as well as the response analysis is important for the assurance of products security in packages. In this paper, the viscoelastic property is taken into account, the dynamic property of corrugated paperboard under transient impulse condition is modeled, a parameters identification method is presented using the free acceleration data. The model and the identified parameters presented in this paper can be used to simulate the response of package under transient impulse condition.

1. Corrugated paperboard property

Because the main component of corrugated paperboard is the paper, which is mainly made of linear macromolecules and extenders. Therefore, similar to many polymeric material, corrugated paperboard possesses viscoelastic property. The experimental results indicate that the mechanical behavior of polymeric material neither obey the Hooker's law, nor obey the Newton's fluid law, its property is between the ideal Hooker's elasticity and ideal Newton's viscosity. The stress depends not only on the strain, but also on the strain rate, this property is called the viscoelasticity. Physical manifestation of the corrugated paperboard viscoelastic properties can be seen in its force relaxation behavior and the creep phenomenon, which are shown in Figure 1(a) and 1(b) respectively. If the corrugated paperboard specimen is subjected to a constant compressive strain, the force in the material decreases over time, logarithmically approaching its steady state value. Also, if corrugated paperboard specimen is loaded with a given mass, the compressive strain increases over time from the initial value. A typical force relaxation measurement for the corrugated paperboard specimen is shown in Figure 1(a). A constant deformation of 3mm was applied to the honeycomb paperboard specimen, the resulting force was measured over a period 2000 seconds. In Figure 1(b), a typical creep of honeycomb paperboard specimen is shown, a mass is loaded on the specimen, the deformation increases exponentially with time, the applied constant force is 2000N.

In addition to these static and quasi-static effects, viscoelasticity also influences the behavior of honeycomb paperboard under dynamic conditions. When subjected to a dynamic excitation, a mass loaded honeycomb paperboard system is seen to display additional dynamic creep beyond its static equilibrium[10]. It has also been shown that the stiffness and damping characteristics of honeycomb paperboard undergoing forced vibration depend on the length of time that the material has been exercised. Therefore, it's important to observe the corrugated paperboard viscoelasticity under dynamic condition.

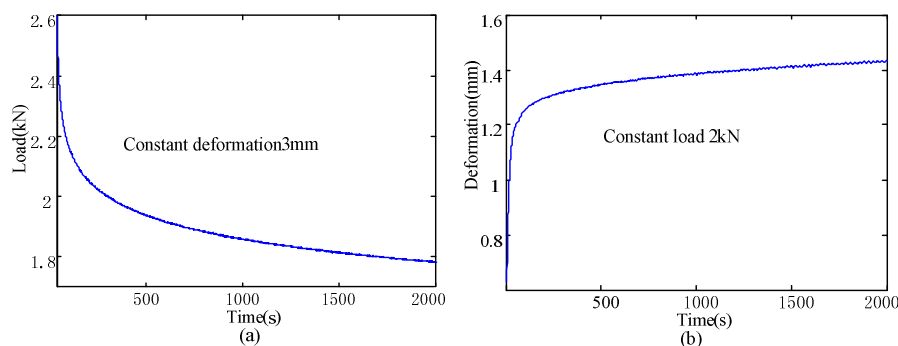


Figure 1. Static viscoelasticity of corrugated paperboard: (a) Stress relaxation (b) Creep

There are several approaches to modeling the viscoelastic nature of material. Bagley et al[11] model viscoelasticity of material by fractional derivative, which can describe the complex viscoelasticity by relatively small amount parameters both in time and frequency domains. But with this model, it's difficult to identify the model parameters by use of the experimental data. In [12], a multiplication decomposition method is deduced and used to model the material viscoelastic property, the nonlinear elastic and creep properties are described and explained by this model. Yu et al[13] used a dynamic system identification technique to describe the viscoelastic properties, according to the dynamic testing results, creep and relaxation equations of viscoelastic material are obtained, the discrete time domain signal analysis method is used to identify the system parameters. According to the viscoelastic theory[14], the relation between the stress and strain of viscoelastic material can be modeled as:

$$a_0\sigma + a_1 \frac{d\sigma}{dt} + \dots + a_p \frac{d^p\sigma}{dt^p} = b_0\varepsilon + b_1 \frac{d\varepsilon}{dt} + \dots + b_q \frac{d^q\varepsilon}{dt^q} \quad (1)$$

Assume $t=0$, the corrugated paperboard specimen is unstressed and undeformed, taking Laplace transformation of (1), one can obtain:

$$(a_0 + a_1s + \dots + a_p s^p)\sigma(s) = (b_0 + b_1s + \dots + b_q s^q)\varepsilon(s) \quad (2)$$

$\sigma(s)$ and $\varepsilon(s)$ are the counterparts of σ , ε in Laplace domain respectively. Because of the considerably big stiffness coefficient of honeycomb paperboard, the deformation of the honeycomb paperboard specimen is very small if excited by a shock, one can obtain the basic relationship between the stress σ and strain ε

$$\sigma = E\varepsilon$$

Where E is the instantaneous Young's modulus. Thus, we can assume $p=q$ in (1). Rewriting (2), one can obtain:

$$\frac{\sigma(s)}{\varepsilon(s)} = E - E \sum_{i=1}^p \frac{d_i}{s + r_i} \quad (3)$$

Using the inverse Laplace transformation and the convolution theorem, one obtains from (3)

$$\sigma(t) = E[\varepsilon(t) - \int_0^t \varepsilon(\tau) \sum_{i=1}^p d_i e^{-r_i(t-\tau)} d\tau] \quad (4)$$

(4) is the stress-strain relationship of the viscoelastic corrugated paperboard. When excited by a transient impulse, the amplitude of the response is very small, therefore, the nonlinearity of the stiffness of the specimen is ignored, i.e., we assume that the stiffness coefficient keep constant during a single transient impulse process. A linear viscous damping term is included in this model to account for the viscous losses in the material. Therefore, according to (4), the motion equation of the mass loaded corrugated paperboard system can be expressed as:

$$m\ddot{x} + c\dot{x} + kx - k \int_0^t \sum_{i=1}^p d_i e^{-r_i(t-\tau)} x(\tau) d\tau = f(t) \quad (5)$$

Where m is the mass loaded on the corrugated paperboard specimen, c is the viscous damping coefficient, k is the stiffness coefficient, $f(t)$ is the force exerted on the mass, x is the specimen deformation. In the free response process, $f(t)=0$, the motion equation of the mass loaded corrugated paperboard can be expressed as:

$$m\ddot{x} + c\dot{x} + kx - k \int_0^t \sum_{i=1}^p d_i e^{-r_i(t-\tau)} x(\tau) d\tau = 0 \quad (6)$$

Assume at the beginning of the free response, the initial velocity and deformation is \dot{x}_0 and x_0 respectively, taking Laplace transform of (5), one can obtain:

$$m[s^2 x(s) - s\dot{x}_0 - sx_0] + c[sx(s) - x_0] + kx(s) - kx(s) \sum_{i=1}^p \frac{d_i}{s + r_i} = 0$$

The equation above can be rewritten as

$$x(s) = \frac{m(\dot{x}_0 + sx_0) + cx_0}{ms^2 + cs + k \left(1 - \sum_{i=1}^p \frac{d_i}{s + r_i} \right)} = \frac{R(s)}{T(s)}$$

From the equation above, the numerator $R(s)$ is a polynomial of order p , while the denominator $T(s)$ is a polynomial of order $p+2$. Knowing the roots $p_i(i=1,2,\dots,n+2)$ of polynomial $T(s)$, and assuming p_i are simple(multiplicity of 1), one can express (7) as the sum of fractions:

$$x(s) = \sum_{j=1}^{p+2} \frac{f_j}{s - g_j}$$

Taking inverse Laplace transform of the equation above, the following equation can be obtained:

$$x(t) = \sum_{j=1}^{p+2} f_j e^{g_j t} \quad (7)$$

From (7), we can learn that the free response of the mass loaded viscoelastic material system can be expressed as the sum of exponentials.

2. Identify the poles and residues

Because in the experiments, the free response data of the mass loaded corrugated paperboard system are recorded by the digital equipments, one can obtain the following equation by discretizing (7)

$$x[i] = x(i\Delta t) = \sum_{j=1}^{p+2} f_j e^{g_j i \Delta t} = \sum_{j=1}^{p+2} f_j z_j^i \quad i = 1, 2, \dots, N \quad (8)$$

Where N is the number of samples, Δt is the sampling interval, z_j is called the system pole, $z_j = e^{g_j \Delta t}$, f_j is the system residue. To obtain the model parameters expressed in (5), one should record the free response data of the mass loaded corrugated paperboard system, then identify the system poles z_j and residues f_j of the system.

Since the free response of the system express in (7) and (8) is damped oscillation, therefore the system poles and residues can be obtained by use of the extended Prony method[15] based on the recorded response data. The identifying procedure can be outlined as the follows:

Step 1: Record the free response data $x[1], x[2], \dots, x[N]$, let $p_e \gg p+2$, form matrix

$$\mathbf{B} = \begin{bmatrix} u(1,0) & u(1,1) & \cdots & u(1, p_e) \\ u(2,0) & u(2,1) & \cdots & u(2, p_e) \\ \vdots & \vdots & \vdots & \vdots \\ u(p_e,0) & u(p_e,1) & \cdots & u(p_e, p_e) \end{bmatrix}$$

Where

$$u(i, j) = \sum_{k=p_e+1}^N x(k-j)x(k-i)$$

Step 2: Find the effective rank of matrix \mathbf{B} by use of SVD-TLS method[15,16], this rank is the number of free response order $p+2$ in (7).

Step 3: Construct matrix

$$\mathbf{A} = \begin{bmatrix} x[p+2] & x[p+1] & \cdots & x[1] \\ x[p+3] & x[p+2] & \cdots & x[2] \\ \vdots & \vdots & \cdots & \vdots \\ x[N-1] & x[N-2] & \cdots & x[N-p-2] \end{bmatrix}$$

and construct vector

$$\mathbf{b}^T = [-x[p+3], -x[p+4], \dots, -x[N]]$$

The superscript T denotes the transposition of the matrix. Find the least square solution of vector \mathbf{a} by

$$\mathbf{a} = [\mathbf{A}^T \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{b}$$

Where the superscript -1 denotes the inverse of a matrix and let $\mathbf{a}^T = [a_{n+2}, a_{n+1}, \dots, a_1]$

Step 3: According to vector \mathbf{a} , form polynomial

$$1 + a_1 z^{-1} + \dots + a_{n+2} z^{-(n+2)} = 0$$

The roots of the polynomial above is the system poles $z_i (i=1, 2, \dots, p+2)$.

Step 4: Using (8), one can construct the matrix

$$\mathbf{M} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{p+2} \\ z_1^2 & z_2^2 & \cdots & z_{p+2}^2 \\ \vdots & \vdots & \cdots & \vdots \\ z_1^N & z_2^N & \cdots & z_{p+2}^N \end{bmatrix} \quad (9)$$

and the vector

$$\mathbf{x}^T = [x[1], x[2], \dots, x[N]] \quad (10)$$

the residues $\mathbf{f}^T = [f_1, f_2, \dots, f_{p+2}]$ can be obtained by the following equation

$$\mathbf{f} = [\mathbf{M}^T \mathbf{M}]^{-1} \mathbf{M}^T \mathbf{x} \quad (11)$$

The system poles and residues of the mass loaded viscoelastic system can be obtained by the 4 steps outlined above.

3. Experiment system and model parameter identification

In present research, a corrugated paperboard-mass system is used to simulate a real package. Put the system onto the vibration table, the corrugated paperboard-mass system is excited by a transient impulse which is generated by the vibration table. Record the excitation and response data of the system, the data can be used to identify the model parameters.

The experiment system is shown in Figure 2. In this system, the corrugated paperboard specimen is provided by Lanzhou Tongsheng packaging technology CO LTD, the specimen is made up of B flute and E flute corrugated paperboard to ensure the cushioning and compression ability of the specimen. The basis weight of the outer and inner linerboard are 220g/m² and 160g/m² respectively, the basis weight of the corrugated medium is 140g/m². The

vibration table is controlled by the vibration controller, generate a transient half-sine impulse, the excitation and response data are recorded by the accelerometers, the excitation and response data are passed through a low-pass filter, the charge amplifier is used to transform the charge signal from the piezoelectric accelerometers into voltage signal. A dynamic data acquisition equipment is used to transform the voltage signal from charge amplifier into digital data, and transmit the data to the computer. The sampling frequency is set to be 5000Hz, the cut-off frequency of low-pass filter is set to be 500Hz, the roll-off is 36dB/OCT.

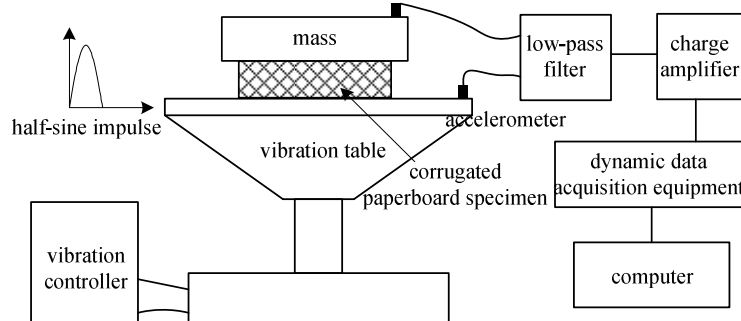


Figure 2. the experiment system

Record the excitation and response data, the time instant can be considered as the beginning of the free response of the mass loaded corrugated paperboard system. Record the free response part of the system, because the free response can be expressed by (7), substitute (7) into (6), set all the coefficient of the exponential functions to be zero, one can obtain:

$$\sum_{j=1}^{p+2} \frac{r_i}{g_j + r_i} f_j = 0 \quad i = 1, 2, \dots, p \quad (12)$$

$$mg_j^2 + cg_j + k - k \sum_{i=1}^p \frac{d_i}{g_j + r_i} = 0 \quad j = 1, 2, \dots, p+2 \quad (13)$$

(12) is a polynomial of r , if the system poles and residues p and f are obtained. Find the roots of (12), parameter r can be obtained. Equation (13) in essence are $p+2$ polynomials of parameters c, k, d_1, \dots, d_p , find the solutions of this equation set, parameters c, k, d_1, \dots, d_p can be obtained.

As viscoelastic material performs quite different under different load condition, it's very necessary to identify different mass loaded system parameters. The mass loaded corrugated paperboard system is excited by transient impulse under different load condition, using the algorithm outlined above, one can obtain the model parameters under different load condition. These parameters are shown in Table 1.

Table 1 identified parameters under different load conditions

Parameters	Load m (kg)				
	5	7	10	12	15
k (N/m)	1349900	1084700	930140	1427500	1791700
c (Ns/m)	637.9	575.8	490.8	416.4	339.5
$r_{1,2}$	52.4±799.1i	65.9±801.7i	82.3±802.1i	167.2±548.3i	230.2±231.8i
$d_{1,2}$	48.9±111.5i	86.9±107.3i	128.8±99.3i	102.6±59.4i	77.7±14.9i
$r_{3,4}$	-	-	74.2±524.9i	44.9±517.0i	14.3±523.0i
$d_{3,4}$	-	-	-32.1±64.6i	7.1±32.5i	36.7±5.1i

From Table 1, the following can be concluded: (1) Under the low load condition ($m=5\text{kg}, m=7\text{kg}$), in the viscoelastic model expressed by (3), 2 orders of viscoelastic parameters can account for its property. With the increase the load, the viscoelastic property of corrugated paperboard becomes more complex, 4 orders of parameters are needed to describe the viscoelastic property accurately. (2) In the low load phase ($m=5\text{kg}, 7\text{kg}, 10\text{kg}$), the stiffness coefficient decrease with the increase of load, but with the increase of the load, the specimen is densified by the load, with the increase of the load, the stiffness coefficient increase.

4. Response analysis of the mass loaded corrugated paperboard system

The mass-loaded corrugated paperboard system is used to simulate a real package, after obtaining the model parameters, the system response can be simulated based on the system model express in (5).

In this paper, the rounded impulse acceleration function [17,18] is used to simulate the half-sine impulse, which is expressed by

$$\ddot{x}(t) = \frac{e^2}{4} \ddot{x}_{\max} \omega^2 t^2 e^{-\alpha t} \quad (14)$$

Where \ddot{x}_{\max} is the amplitude of the impulse, this function can be used to simulate the real impulse by properly choosing the value of ω . A rounded impulse acceleration function is shown in Figure 3, a real half-sine impulse is also shown in the figure for comparison.

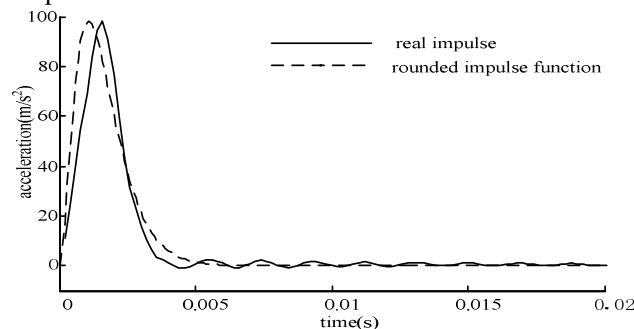


Figure 3. Simulate the real half-sine impulse wave by use of the rounded impulse function

If the mass loaded corrugated paperboard system is base excited by impulse, then based on the (5) and (14), we can express the response of the system as

$$\ddot{x}_e + 2\xi\omega_n(\dot{x}_e - \dot{x}_r) + \omega_n^2(x_e - x_r) - \omega_n^2 \int_0^t [x_e(\tau) - x_r(\tau)]\Gamma(t-\tau)d\tau = 0 \quad (15)$$

Where x_e and x_r are the displacement of the excitation and response of the base excited system shown in Figure 2 respectively, $\omega_n = \sqrt{k/m}$, $\xi = c/(2\sqrt{km})$, (15) can be rewritten as

$$\ddot{x} + c\dot{x} + kx - k \int_0^t [y(\tau) - x(\tau)]\Gamma(t-\tau)d\tau = -\frac{e^2}{4} \ddot{x}_{\max} \omega^2 t^2 e^{-\alpha t} \quad (16)$$

Where, $x = x_r - x_e$. Taking Laplace transform of (16), one can obtain

$$s^2 x(s) + csx(s) + kx(s) - kx(s) \sum_{i=1}^p \frac{d_i}{s+r_i} = -\frac{e^2}{2} m \ddot{x}_{\max} \omega^2 \frac{1}{(s+\omega)^3} \quad (17)$$

From the identified parameters in Table 1, when $p=2$, $r_{1,2}=r_r \pm r_i$, $d_{1,2}=d_r \pm d_i$, (17) can be rewritten as

$$x(s) = -\frac{1}{2} \ddot{x}_{\max} e^2 \omega^2 \frac{s^2 + 2r_r s + |r|^2}{(s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0)(s+\omega)^3}$$

According to the differentiation theorem of Laplace transform, the Laplace transform of the acceleration \ddot{x} can be expressed as

$$\ddot{x}(s) = -\frac{1}{2} \ddot{x}_{\max} e^2 \omega^2 \frac{s^4 + 2r_r s^3 + |r|^2 s^2}{(s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0)(s+\omega)^3} \quad (18)$$

Where, $A_3 = 2(\xi\omega_n + r_r)$, $A_2 = (\omega_n^2 + 4\xi\omega_n r_r + |r|^2)$, $A_1 = 2\omega_n(\omega_n r_r - \omega_n d_r + \zeta |r|^2)$, $A_0 = \omega_n^2 |r|^2 - 2\omega_n^2(d_r r_r + d_i r_i)$.

Because when $t=0$, $\ddot{x} = 0$, (18) can be rewritten as the sum of the fractions

$$\ddot{x}(s) = -\frac{1}{2} \ddot{x}_{\max} e^2 \omega^2 \left[\frac{b_1}{(s-p_1)(s-p_2)} + \frac{b_2}{(s-p_3)(s-p_4)} + \frac{b_3}{(s+\omega)^2} + \frac{b_4}{(s+\omega)^3} \right] \quad (19)$$

One can obtain the coefficients b_1, b_2, b_3, b_4 by use of the residue theorem and obtain the acceleration response by the inverse Laplace transform of (19)

$$\begin{aligned} \ddot{x} = & \frac{b_1}{\omega_{n1} \sqrt{1-\xi_1^2}} e^{-\xi_1 \omega_{n1} t} \sin(\omega_{n1} \sqrt{1-\xi_1^2} t) \\ & + \frac{b_2}{\omega_{n2} \sqrt{1-\xi_2^2}} e^{-\xi_2 \omega_{n2} t} \sin(\omega_{n2} \sqrt{1-\xi_2^2} t) + b_3 t e^{-\alpha t} + \frac{1}{2} b_4 t^2 e^{-\alpha t} \end{aligned} \quad (20)$$

Where, $\omega_{n1}^2 = p_1 p_2$, $\xi_1 = -(p_1 + p_2)/(2\omega_{n1})$, $\omega_{n2}^2 = p_3 p_4$, $\xi_2 = -(p_3 + p_4)/(2\omega_{n2})$, the acceleration response of the system can be expressed as:

$$\ddot{x}_r = \ddot{x} + \ddot{x}_e \quad (21)$$

According to (20) and (21), we can simulate the acceleration response of the mass loaded corrugated paperboard system excited by transient impulse. In Figure 4, the simulated acceleration response of the system loaded with 5kg mass excited by impulse with peak value of 10g is shown, the recorded experiment data are also shown in the figure

for comparison. If the system is in high load phase ($m=10\text{kg}$, 12kg , 15kg), $p=4$, the response analysis method resemble the method outlined above.

From Figure 4, it can be concluded that the model presented in this paper can be used to predict the dynamic response of the mass loaded corrugated paperboard system with accuracy.

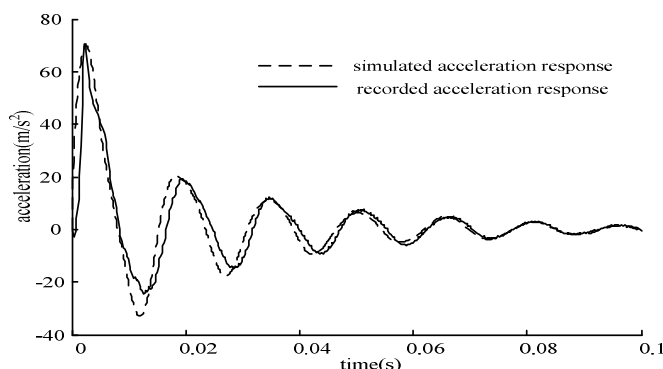


Figure 4 Comparison between the simulated response with the recorded response of the system

CONCLUSION

Because of the good cushioning property and environmentally friendly performance of corrugated paperboard, this material began to be adopted in the transportation packaging as the substitution for the traditional polymeric foam. During the transportation, the transient impulse caused by the road unevenness, vehicle velocity variation is one of the main factors causing the failure of the package.

In this paper, a mass loaded corrugated paperboard system is used to simulate a real package, the viscoelasticity of the paperboard is taken into account, the dynamic property model of the paperboard under transient impulse condition is presented, the free response of the system is expressed as the sum of complex exponentials, the residues and poles of the system are identified by use of the extended Prony method. The experiment system for recording the free response of the mass loaded corrugated paperboard system is set up. Under different load conditions, impulse experiments are performed, model parameters are identified. The model and the identified parameters can be used to analyze the acceleration response of the system. The rounded acceleration function is used to simulate the real half-sine impulse, the system response method is presented. Acceleration response of the system loaded by 5kg mass is presented and compared with the recorded response data. The comparison result indicates that the model presented in this paper is accurate to account for the dynamic property of corrugated paperboard. The model and the response analyzing method provide design basis for the packaging design.

REFERENCES

- [1] N Navaranjan, A Dickson, J Paltakari, K Ilmonen, *Composites part B-engineering*, vol.45, no.1, pp.965-971, **2013**.
- [2] YP E, ZW Wang, *Packaging Technology and Science*, vol.25, no.4, pp.187-202, **2012**.
- [3] DM Wang, *Journal of Sandwich Structures and Materials*, vol.11, no.1, pp.57-66, **2009**.
- [4] MA Sek, V Rouillard, *Applied Mechanics and Materials*, vol.3, no.4, pp.383-388, **2005**.
- [5] ME Biancolini, C Brutti, *Packaging Technology and Science*, vol.16, no.2, pp.47-60, **2003**.
- [6] MA Garcia-Romeu-Martinez, SP Singh, *Packaging Technology and Science*, vol.21, no.8, pp.439-451, **2008**.
- [7] GO Rissi, SP Singh, G Burgess et al, *Packaging Technology and Science*, vol.21, no.4, pp.231-246, **2008**.
- [8] F Lu, Y Ishikawa, T Shiina et al., *Packaging Technology and Science*, vol.21, no.8, pp.479-489, **2008**.
- [9] SP Singh, APS Sandhu, J Singh et al, *Packaging Technology and Science*, vol.20, no.6, pp.381-392, **2006**.
- [10] K Vorspohl, J Mertes, R Zschiesche, *Journal of Cellular Plastics*, vol.30, no.3, pp.361-373, **1994**.
- [11] RL Bagley, PJ Torvik, *AIAA Journal*, vol.21, no.5, pp.741-748, **1983**.
- [12] S Govindjee, S Reese, *Journal of Engineering materials and technology*, vol.119, no.2, pp.251-255, **1997**.
- [13] P Yu, YM Haddad, *International Journal of Pressure Vessels and Piping*, vol.61, no.1, pp.87-97, **1994**.
- [14] RM Christensen, "Theory of Viscoelasticity: Second Edition", DOVER PUBN INC, **2010**.
- [15] XD Zhang, "Modern signal process", Beijing: Tsinghua University Press, **1995**.
- [16] GH Golub, CF VanLoan, *SIAM Journal of Numerical Analysis*, vol.17, no.6, pp.883-893, **1980**.
- [17] NC Shekhar, H Hatwal, AK Mallik, *Journal of Sound and Vibration*, vol.214, no.4, pp.589-693, **1998**.
- [18] NC Shekhar, H Hatwal, AK Mallik, *Journal of Sound and Vibration*, vol.227, no.2, pp.293-307, **1999**.