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Compute method of transition probability of internetware system in chemical and pharmaceutical industry

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ABSTRACT

Internetware system are widely used in the chemical and pharmaceutical industry, effective acquisition of transition probability matrix is directly related to internetware reliability computation based on Markov chain model. The construction of Markov chain and the acquisition of transition probability of internetware are studied; the Markov chain of internetware reliability is constructed. Transition probability matrix computation based on the smallest quadratic difference is presented by using the occupancy of component executing the transition as the sample statistics to calculate transition probability. The approximation algorithm based on projection gradient is put forward, and it effectively guarantees the transition law of Markov chain and the characteristics of transition probability matrix. The experiment proves that the presented method and the designed algorithm can effectively compute transition probability with great value in internetware reliability computation.

Key words: Chemical; Pharmaceutical industry; Internetware; Transition probability; Compute; Method

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INTRODUCTION

In the chemical and pharmaceutical industry, Internetware, as a new software application mode, has the remarkable characteristics of adaptability and dynamic characteristic based on open network. Its reliability is different from traditional software system. More and more people start studying its reliability on software design, development, test and operation. Great result has been achieved on internetware reliability study by using Markov chain[1,2,3], which has been used in reliability calculations and performance evaluation. Transition probability matrix is one of the main problems that need to be solved on reliability computation by Markov. Transition probability of traditional software can be acquired by testing, but because of the change of network environment and the adaptation of internetware, the sample got by direct test is difficult to make sure the accuracy of the acquired testing data, so it is difficult to test transition probability directly. But it is possible to calculate transition probability matrix by the occupancy of the internetware executing transition as the sample data, and then to implement internetware reliability calculation.

There are three commonly used methods of reliability study of software system[4, 5]: the model based on state, the model based on route and the additive model. 1) The model based on state usually uses the control flow graph to mean the system structure, and also assumes that the control transfer among software modules has Markov characteristics, that is, the future action of the system is independent from the action in the past, which is in accordance with the actual situation of the operation. 2) The model based on route computes the related reliability of the software system by considering all the possible execution routes and the related frequency of the system. It does not take it into consideration that the component reliability has difference in different network environment. Its accuracy is not high. It is just the estimation of the system reliability, and is not suitable for the system with infinite routes, so it has limitations. 3) The additive model does not take the structure of the software system into consideration, but focuses on the modeling of system reliability growth and predicts the reliability of the whole system through the failed data of the components. Usually it acquires the reliability through testing. The acquisition and the accuracy of the testing data in the internetware environment can hardly be guaranteed. Comparatively speaking, the model based on state has higher adaptation and accuracy. The model based on state can be used in the software system with infinite routes. In the open environment, the components are loose coupled and they are highly independent from each other, and the transition among the components is in accordance with Markov characteristics. It accords with internetware operation when state change and the software system structure are taken into consideration. So the model based on state is much more suitable to be used in the open environment.

Document [6] puts forward the reliability computation by using execution route information and component transition probability under certain testing condition. Documents [1,2] propose internetware reliability computation based on Markov chain and achieve reliability computation based on structure relationship and transition probability. Document [3] proposes specific reliability computation based on structure and transition probability and reliability computation of composite members. Document [7] puts forward convex programming optimized transition probability computation based on maximum a posteriori and maximum likelihood by using Bayesian estimation. Document [8] puts forward the method to estimate transition matrix based on maximum likelihood. Document [9] puts forward approximate recursion method of posterior probability density function by using Bayesian theory on Markov jump systems, and then puts forward four mean square deviation estimation computation of transition probability matrix. Then, the acquisition and computation of transition probability, the guarantee of Markov chain transition law and the characteristics of transition probability matrix are the important and basic parts of internetware reliability computation based on Markov chain. So this paper proposes optimization calculation to approximatively calculate transition probability based on sample data.

TRANSITION PROBABILITY MATRIX

Internetware Markov chain

Internetware components have their corresponding Markov chain state. When it operates to a certain component, the system is in that state corresponding to the component. The transfer after the state is relative to the transfer of the component. When the operation of one component goes to another, there is the corresponding transfer of the state to another.

State set is E={1,2,…,N}, transition probability matrix among states is P=[p_{ij}]. When the system is on the state of i at k point of time, the probability is $x_i(k)$， $x_i(k) \geq 0$， $\sum_{i=1}^{n} x_i(k)=1$; when at k+1 point of time, probability distribution is $x(k+1)$, then $x(k+1)=x(k)P$, that is:

$$x_j(k+1)=\sum_{i=1}^{n} x_i(k)p_{ij} \tag{1}$$

change $x_i(k)$ into $(x_{i1}, x_{i2},..., x_{im})$， $x_i(k+1)$ into $(y_{i1}, y_{i2},..., y_{im})$， $i=1,2,...,m$， the abbreviation of formula（1）is：

$$Y = XP \tag{2}$$

Reliability computation model

Reliability computation model is constructed by using DTMC[5]. Internetware DTMC is a two-tuples (S,Q), in which S is finite state, $s=\{s_1, s_2,..., s_n\}$, $P: S \times S \to [0,1]$; Q is the state transition probability, $Q=\{q_{ij}\}, i, j \in [1,2,...,n]$.

The reliability of state s_i is r_i, the transition probability is q_{ij}, then the reliability of s_j is:

$$R_j=\begin{cases} r_i \times q_{ij} & if \quad c_i \to c_j \\ 0 & otherwise \end{cases}$$

$Q_{1,n}^{k}$ means the transition probability after the system executes k steps from state s_1 to state s_n, r_n is the reliability of state s_n, then the reliability is:

$$R = Q_{1,n}^{k} \times r_n \tag{3}$$

According to
$\lim_{k\to\infty}\sum_{i=0}^{k} Q^i = \sum_{i=0}^{\infty} Q^i = I + Q + Q^2 + ... = (I-Q)^{-1}$, the matrix is $T = (I-Q)^{-1}$, then:

$$T = I + Q + Q^2 + ... = \sum_{i=0}^{\infty} Q^i \quad (4)$$

Because the determinant of full rank matrix is not zero, $|I-Q| \neq 0$.

$T(1,n) = I(1,n) + Q(1,n) + Q^2(1,n) + Q^3(1,n) + ... = \sum_{0}^{\infty} Q^n(1,n) = (-1)^{n+1}\frac{|(I-Q)_{n,1}|}{|(I-Q)|}$, then:

$$T(1,n) = (-1)^{n+1}\frac{|(I-Q)_{n,1}|}{|(I-Q)|} \quad (5)$$

In formula (4), I is characteristic matrix $n \times n$, $(I-Q)_{n,1}$ means matrix $(I-Q)$ deleting the first line of row n. So, the reliability is:

$$R = T(1,n) \times r_n \quad (6)$$

Internetware transition probability

Definition 1: transition probability

Suppose Markov chain is composed of m states, and according to software operation, it changes into the sequence composed of these m states. Start from any state, pass any transfer, certainly there will be one state of 1, 2…m. There is a state i with a certain probability in the states, $i \in [1,2,...,m]$, and the probability of this transfer among the states is called transition probability.

When at m point of time it is on the state of a_i, the transition probability of transferring to state a_j at m+n point of time is:

$$p_{ij}(m, m+n) = p\{x_{m+n} = a_j \mid x_m = a_i\} \quad (7)$$

Transition probability forms transition probability matrix:

$$P = \{p_{ij}(m, m+n)\} \quad (8)$$

Transition probability matrix has the following characteristics:

1) non-negative condition, $0 \le p_{ij} \le 1$;

2) row sum condition $\sum_{j=1}^{n} p_{ij} = 1$, that is, the sum of transition probability of each row in matrix P is 1.

According to 2), transition probability is not negative, then 1) can be written as $0 \le p_{ij}$.

The execution state of internetware system component forms a Markov chain. Usually transition frequency is used to replace execution transition probability. Transition frequency means the occupancy of transition execution times, and it is:

$$ptf = \frac{\text{transition time}}{\text{transition totality}} \quad (9)$$

When theoretical distribution of state probability is not known, if the sample capacity is large enough, sample distribution can be used approximately to describe the theoretical distribution. So transition frequency ptf is used as sample to approximately estimate transition probability p.

Transition probability matrix computation

Formula (1) can be written as $Y(t) = Y(t-1)P$.

The error is:

$$e_j(t) = y_j(t) - \sum_{i=1}^{m} p_{ij} y_i(t-1)$$

$i, j = 1,2,...,m$, $t = 0,1,2,...,n$

The sum of squares of error(SSE) is：

$$Q = \sum_{j=1}^{m}\sum_{t=1}^{n}(e_j(t))^2 \quad (10)$$

Least square method has:

$$\begin{aligned} &\min \quad Q = \sum_{j=1}^{m}\sum_{t=1}^{n}(e_j(t))^2 \\ &s.t. \quad \sum_{j=1}^{n} p_{ij} = 1 \quad i = 1,2,..., m \\ &\quad\quad p_{ij} \geq 0 \end{aligned} \quad (11)$$

Introduce lagrange parameter $\xi_i (i = 1,2,...,m)$, obtain lagrange function:

$$f = \text{-ñ}\sum_{j=1}^{m}\sum_{t=1}^{n}(e_j(t))^2 + \sum_{i=1}^{m}\xi_i \times (\sum_{t=1}^{n} p_{ij} - 1)$$

The condition of minimum value is： $\partial f / \partial p_{ij} = 0$ ， $\partial f / \partial \xi_i = 0$ 。

$\partial f / \partial p_{ij} = -2\sum_{t=1}^{n} e_j(t) y_i(t-1) + \xi_i = 0$ ， obtain $\xi_i = 2\sum_{t=1}^{n} e_j(t) y_i(t-1)$

According to occupancy $\sum_{j=1}^{m} y_j(t) = 1$, t is unrelated.

$$\sum_{j=1}^{m} e_j(t) = \sum_{j=1}^{m} y_j(t) - \sum_{j=1}^{m}(\sum_{i=1}^{m} p_{ij} y_i(t-1)) \quad = \sum_{j=1}^{m} y_j(t) - \sum_{i=1}^{m} y_i(t-1)$$

$=0$

By $\partial f / \partial \xi_i = \sum_{i=1}^{n}\xi_i = 0$

Then： $\sum_{t=1}^{n}\xi_i = 2\sum_{t=1}^{n}(\sum_{j=1}^{n} e_j(t)) y_i(t-1) = 0$

And then： $\sum_{t=1}^{n} e_j(t) y_i(t-1) = 0$

So: $\sum_{t=1}^{n}(y_j(t) - \sum_{i=1}^{m} p_{ij} y_i(t-1)) y_i(t-1) = 0$

Furthermore： $\sum_{t=1}^{n}(y_j(t) - \sum_{i=1}^{m} p_{ij} y_i(t-1)) y_i(t-1) = 0$

So：

$$[\sum_{t=1}^{n} y_i(t-1) y_1(t-1),...,\sum_{t=1}^{n} y_i(t-1) y_m(t-1)][p_{ij},...,p_{mj}]^T = \sum_{t=1}^{n} y_i(t-1) y_j(t-1) \quad]$$

Now define matrix X_1 ， X_2 :

$$X_1=\begin{bmatrix} y_1(0) & y_2(0) & ... & y_m(0) \\ y_1(1) & y_2(1) & ... & y_m(1) \\ ... & ... & ... & ... \\ y_1(n-1) & y_2(n-1) & ... & y_m(n-1) \end{bmatrix} \quad X_2=\begin{bmatrix} y_1(1) & y_2(1) & ... & y_m(1) \\ y_1(2) & y_2(2) & ... & y_m(2) \\ ... & ... & ... & ... \\ y_1(n) & y_2(n) & ... & y_m(n) \end{bmatrix} \tag{12}$$

Then: $X_1^T X_1 P = X_1^T X_2$

$$P = (X_1^T X_1)^{-1}(X_1^T X_2) \tag{13}$$

According to formula (13), transition probability matrix P can be obtained.

In addition, according to formula (10), $Y = XP + [\varepsilon_{ij}]$, in which ε_{ij} is the error.

The minimal value: $g(P) = \| XP - Y \|^2$

$$g(P) = \| XP - Y \|^2 = (XP - Y)^T (XP - Y)$$
$$= P^T X^T XP - 2Y^T XP + Y^T Y$$
$$= (X^T XP - X^{T'} Y)^T (X^T X)^{-1} (X^T XP - X^T Y)$$
$$+ (X^T X)^{-1} - YX (X^T X)^{-1} X^T Y$$

According to method of completing the square, the minimum value is: $X^T XP = X^T Y$, then:

$$P = (X^T X)^{-1}(X^T Y) \tag{14}$$

According to (2), $X_2 = X_1 P$. Suppose $X = X_1$, $Y = X_2$, then (13) and (14) have the same meaning.

Suppose E_m is a m-dimensional vector, $E_m = [1,1,...,1]^T$, then, $YE_m = E_m$, $XE_m = E_m$.

Furthermore: $X^T YE_m = X^T XE_m$, then:

$E_m = (X^T X)^{-1}(X^T Y)E_m$.

That is: $E_m = PE_m$.

It meets the row sum condition.

According to the above analysis, the computation model of transition probability matrix is achieved from formula (14).

The above minimum square computation method is used to obtain transition probability matrix. The row sum is 1, but it does not have non-negative condition, so non-negative condition should be introduced as constraints to compute best approximation.

APPROXIMATE COMPUTATION METHOD OF TRANSITION PROBABILITY

Computation model of transition probability

In formula (2), X and Y may not meet Markov chain model, and there would not be the solution to P in the above equations, so the minimum square method is used to compute best approximation of formula (2). Suppose $ptf = p$, D means the random matrix set of total order n, which is bounded closed set of Euclidean space, f(p) is continuous function on D, the minimum p is regarded as the estimation of transition matrix determined by X and Y on D, and the minimum p exists.

Let be: $$f(p) = \sum_{i=1}^{m} \sum_{j=1}^{n} (\sum_{k=1}^{n} x_{ik} p_{kj} - y_{ij})^2 \tag{15}$$

$$f(p)=\sum_{j=1}^{n}(Xp_j-y_j)^T(Xp_j-y_j)$$

$$=\sum_{j=1}^{n}(p_j{}^T X^T p_j-2p_j{}^T X^T y_j+y_j{}^T y_j)$$

For the sake of simplicity, remember it as: $H=diag(X^T X,...,X^T X)$, $q=(p_1,p_2,...,p_n)^T$, $C=(X^T y_1,X^T y_2,...,X^T y_n)^T$,in which， $A=[I,I,...,I], e=(1,1,...,1)^T$, H is a square matrix of n^2 order, q,c is a n^2 -dimensional column vector, I is a unit matrix of n order, A is a $n\times n^2$ matrix, e is n-dimensional vector. Formula (15) changes into:

$$f(p)=q^T Hq-2C^T q+\sum_{j=1}^{n}y_j{}^T y_j \tag{16}$$

So formula (16) is transformed to the solution of the following question. The model is as follows:

$$\begin{cases}\min & f=q^T Hq-2C^T q\\ s.t. & Aq=e\\ & q\geq 0\end{cases} \tag{17}$$

The above computation models satisfy the two conditions, that is, the row sum is 1, and it is non-negative.

So, when the rank of matrix X is n, and H is positive definite, there is the only minimum point.

The improved gradient projection method can be used in formula (17). Because of the time consuming of iteration, the added conditions can guarantee the iteration in the relative interior in feasible region, and then it can save time when using gradient projection method.

Transition probability computation method

Gradient projection method is an optimization algorithm first proposed by Rosen in 1961 aiming at the optimization of linear constraints. Generalize this algorithm to non-linear constraints, the model is as follows:

$$\begin{cases}\min & f(x)\\ s.t. & Ax\geq b\\ & Ex=e\end{cases} \tag{18}$$

In which $f(x)$ is differentiable, $A\in R^{m\times n}, E\in R^{l\times n}, b\in R^m, e\in R^l, x\in R^n$, $D=\{x\in R^n \mid Ax\geq b, Ex=e\}$. The basic idea of gradient projection method is: when iteration point x_k is the interior point of feasible region D, take $d=-\nabla f(x_k)$ as the search direction; otherwise, when x_k is the boundary point of feasible region D, take the projection of $d=-\nabla f(x_k)$ on these boundary surface intersection as the search direction.

Definition 2: Projection matrix $P\in R^{m\times n}$ satisfies the condition: $P=P^T, P^2=P$.

Then there is the following lemma 1.

Lemma 1: the properties of projection matrix $P\in R^{m\times n}$

1) $P\in R^{m\times n}$ is positive semidefinite;

2) $P\in R^{m\times n}$ is projection matrix. When and only when $I-P$ is also projection matrix, I is a unit matrix of n order;

3) suppose $Q = I - P$, then $L = \{y = Px \mid x \in R^n\}$, $L^+ = \{z = Qx \mid x \in R^n\}$ is orthogonal linear subspace, and $\exists\ x \in R^n$ is only expressed as $x = y + z$, in which $y \in L, z \in L^+$.

Lemma 2: suppose $\bar{x}$ is the feasible solution to formula (18), $A = [A_1 \ \ A_2]^T$, $b = [b1 \ \ b2]^T$, meet $A_1 x = b1$, $A_2 x \ge b2$, then the necessary and sufficient condition of $d \in R^n$ on the descent direction on $\bar{x}$ is: $A_1 d \ge 0,\ Ed = 0$, $\nabla f(\bar{x})^T d < 0$.

Proof: sufficiency. $A_1 d \ge 0$, $Ed = 0$, $\bar{x}$ is the feasible point, and $A_1 x = b1$, $Ex = e$, arbitrary $a > 0$, then $A_1(\bar{x} + ad) = A_1 \bar{x} + a(A_1 d) \ge A_1 \bar{x} = b1$, $E(\bar{x} + ad) = E\bar{x} + a(Ed) = e$. Because $A_2 x \ge b2$, there is certainly a $\bar{a} > 0$ which makes arbitrary $a \in [0, \bar{a}]$ have $A_2(\bar{x} + ad) = A_2 \bar{x} + a(A_2 d) \ge A_{12} \bar{x} \ge b2$.

So, there is $\bar{a}$, $a \in [0, \bar{a}]$, then: $A(\bar{x} + ad) \ge b$, $E(\bar{x} + ad) = e$. So, $\bar{x} + ad$ is the feasible point, d is the feasible direction on $\bar{x}$.

The necessity can be proved in the same way.

Theorem 2: suppose $\bar{x}$ is the feasible solution to formula (18), $A = [A_1 \ \ A_2]^T$, $b = [b1 \ \ b2]^T$, meet $A_1 x = b1,\ A_2 x \ge b2,$ suppose $M = [A_1 \ \ E]^T$ is full rank, $P = I - M^T (MM^T)^{-1} M$, $P\nabla f(\bar{x}) \ne 0$. If $d = -P\nabla f(\bar{x})$, d is on the descent direction of problem solving.

Proof: if $P\nabla f(\bar{x}) \ne 0$, then $\nabla f(\bar{x})^T \times d = -\nabla f(\bar{x})^T P \nabla f(\bar{x}) = -\|\nabla f(\bar{x})\|^2 < 0$, so d is on the descent direction.

$$Md = -MP\nabla f(\bar{x}) = -M(I - M^T (MM^T)^{-1} M)\nabla f(\bar{x})$$
$$= (-M + m)\nabla f(\bar{x}) = 0$$

Then $A_1 d = 0,\ Ed = 0$. So, according to lemma 2, d is on the feasible direction of $\bar{x}$.
If $P\nabla f(\bar{x}) = 0$, then:
Suppose $w = (MM^T)^{-1} M \nabla f(\bar{x}) = [\lambda \ \ \upsilon]^T$, then:

$$P\nabla f(\bar{x}) = (I - M^T (MM^T)^{-1} M)\nabla f(\bar{x})$$
$$= \nabla f(\bar{x}) - M^T (MM^T)^{-1} M)\nabla f(\bar{x})$$
$$= \nabla f(\bar{x}) - M^T w = \nabla f(\bar{x}) - [A_1^T \ E^T][\lambda \ \ \upsilon]^T$$
$$= \nabla f(\bar{x}) - A_1^T \lambda - E^T \upsilon$$

So $P\nabla f(\bar{x}) = 0$.

1) when $\lambda \ge 0$, and $\nabla f(\bar{x}) - A_1^T \lambda - E^T \upsilon = 0$, $\bar{x}$ is just the condition point of KT.

2) when $\lambda_j < 0$, delete the corresponding row of λ_j in A_1, and get the new matrix $\tilde{A}_1$ and the corresponding matrices $\tilde{M}$, $\tilde{P}$, $\tilde{w}$, and also $d = -\tilde{P}\nabla f(\bar{x})$. The following is the proof of $\tilde{P}\nabla f(\bar{x}) \ne 0$.

Proof: if $\tilde{P}\nabla f(\bar{x}) = 0$, $\tilde{P}\nabla f(\bar{x}) = \nabla f(\bar{x}) - \tilde{M}^T \tilde{w}$.

Suppose in $A1$, r_j is the corresponding row of λ_j, $A_1^T + E^T \upsilon$ becomes $\tilde{A}_1^T \tilde{\lambda} + \lambda_j r_j^T + E^T \upsilon = \tilde{M}^T \tilde{w} + \lambda_j r_j^T$. When $\tilde{P}\nabla f(\bar{x}) = 0$, then $\nabla f(\bar{x}) - (\tilde{M}^T \tilde{w} + \lambda_j r_j^T) = 0$, which is a linear combination, and at least there is $r_j \ne 0$. M's row vector is linear correlation, and is contradict to M being full rank. So $\tilde{P}\nabla f(\bar{x}) \ne 0$

Because
$$\tilde{M}\,d = -\tilde{M}\,\tilde{P}\nabla f(\bar{x}) = -\tilde{M}(I - \tilde{M}^T(\tilde{M}\,\tilde{M}^T)^{-1}\tilde{M})\nabla f(\bar{x}) = -(\tilde{M} - \tilde{M})\nabla f(\bar{x}) = 0$$

Then $\tilde{A}_1 d = 0$， $Ed = 0$

$\nabla f(\bar{x}) - (\tilde{M}^T \tilde{w} + \lambda_j r_j^T) = 0$. There is：

$r_j \tilde{P}(\nabla f(\bar{x}) - (\tilde{M}^T \tilde{w} + \lambda_j r_j^T)) = 0$。

$r_j \tilde{P}\nabla f(\bar{x}) - r_j \tilde{P}\tilde{M}^T \tilde{w} - r_j \tilde{P}\lambda_j r_j^T = 0$， because $\tilde{P}\tilde{M}^T = 0$， $d = -\tilde{P}\nabla f(\bar{x})$，

Then： $r_j d + \lambda_j r_j \tilde{P} r_j^T = 0$

$\tilde{P}$ is positive semidefinite, $r_j \tilde{P} r_j^T \geq 0$ and $\lambda_j < 0$, then $r_j d = -\lambda_j r_j \tilde{P} r_j^T \geq 0$

So: $A_1 d \geq 0$， $Ed = 0$.

So, according to lemma 2, d is feasible on the $\bar{x}$ direction.
According to the above, the effectiveness of the solution to formula (18) can be guaranteed.

Suppose $D = \{q : Aq = e, q \geq 0\}$, and $D_1 = \{q : Aq = e, q > 0\}$, $D_2 = \{q : Aq = 0\}$, project on space D2, projection matrix is $S = (I - A^T(AA^T)^{-1}A) = I - \frac{1}{n}A^T A$, then transition probability matrix computation method is as follows:

(1) the initial feasible solution to q is $q^{(1)}$, letbe each component of $q^{(1)}$ is $1/n$, $k = 1$;

(2) compute the gradient of f, $\nabla f = 2Hq^{(k)} - 2C$;

(3) compute the projection of negative gradient $-\nabla f$, $d = S(-\nabla f)$;

(4) compute the minimum point t^* of $f(t) = f(q + td)$, $t^* = (c^T d - d^T Hq^{(k)})/(d^T Hd)$;

(5) search the upper bound τ^*, $\tau^* = \min\{-q_i / d_i ; d_i < 0)$;

(6) get the new approximate solution $q^{(k+1)}$, suppose $\tau_1 = 0.9\tau^*$, $t_1 = \min\{\tau_1, t^*\}$, $q^{(k+1)} = q^{(k)} + t_1 d$;

(7) letbe $k = k + 1$, go to step (2).

Iteration condition, $\min(|q^{(k+1)} - q^{(k)}|) < \rho$, take $\rho = 0.0001$.

In the end, test the model computation result.

$X_{m\times n}$ transfers to $Y_{m\times n}$ with probability $P_{n\times n}$. Besides Markov chain transition, other factors may also influence the production of random error ε, so $Y = XP + \varepsilon$, $\varepsilon = [\varepsilon_{ij}]_{m\times n}$.

Suppose: ε is independent, $E[\varepsilon] = 0$, the variance is σ^2, the fitting residual error is $\Delta_{ij} = y_{ij} - \hat{y}_{ij}$. According to regression analysis theory, the unbiased estimation of the variance σ^2 ：

$$\sigma = \sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta_{ij})^2 / (mn - n^2 + n - 1)$$

Take a threshold $\sigma_\Delta = 0.001$, if $\sigma < \sigma_\Delta$, the model is Markov chain.

ILLUSTRATION

There is a Internetware system in some chemical and pharmaceutical industry, when the system is run, the state transition is occurred. Data obtained by a system is as follows:

x=[0.2 0.2 0.3 0.1 0.2; 0.4 0.2 0.1 0.3 0; 0.1 0.3 0.2 0.3 0.1;0 0.1 0.2 0.3 0.4; 0.2 0.3 0.4 0.1 0; 0 0.2 0.3 0.3 0.2; 0.4 0 0.2 0.1 0.3; 0.6 0 0 0.4 0.1];

y=[0.26 0.20 0.24 0.23 0.07; 0.12 0.14 0.36 0.28 0.10; 0.19 0.18 0.29 0.22 0.12; 0.25 0.19 0.20 0.25 0.11; 0.21 0.23 0.25 0.22 0.08; 0.25 0.18 0.23 0.23 0.11; 0.24 0.16 0.24 0.30 0.16; 0.10 0.10 0.35 0.33 0.12];

According to formula (14), compute and get transition matrix P .

P1=[0.0933 0.2113 0.3503 0.0109 0.4243;
0.1083 0.2961 0.2649 0.0320 0.2140;
0.3900 0.5468 -0.0288 0.2704 0.1863;
0.3443 0.1343 0.2113 0.2646 0.2832;
0.1240 0.0774 0.0591 0.0993 0.1873];

But, the value on row 3 line 3 is negative. This value is not transition probability, because it does not make sure that transition probability is non-negative.

According to transition probability approximate computation model, formula (17) and computation method based on projection gradient, compute and get the following result:

P2=[0.0814 0.0963 0.3774 0.3327 0.1122;
0.1588 0.2432 0.4895 0.0830 0.0255;
0.3784 0.2933 0.0033 0.2384 0.0867;
0.0751 0.0964 0.3369 0.3283 0.1632;
0.3656 0.1551 0.1251 0.2252 0.1290];

$\sigma = \sum_{i=1}^{8}\sum_{j=1}^{5}(y_{ij} - \hat{y}_{ij})^2/19 = 4.2632\text{e-}004$, $\sigma < \sigma_\Delta = 0.001$ (the given threshold)，so it is Markov chain model.

Compare P1, P2, combine the lines successively to form a 1×25 matrix. The comparison of computation results as in Figure 1 shows that these two have the approximate trend result. But P2 is the best result. It makes sure Markov chain transfer law and the characteristics of transition matrix.

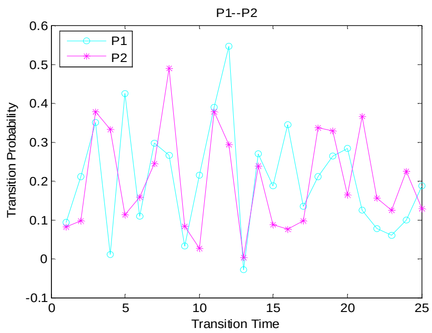

Fig. 1. Comparison of computation results

CONCLUSION

To compute internetware reliability based on Markov chain, the acquisition of transition probability and quantum chemistry calculation is the important basic factors. This paper has analyzed internetware reliability computation method, defined reliability computation model and transition probability, and studied the construction of Markov chain of internetware reliability. It has also analyzed the acquisition of component transition probability, proposed the method to calculate transition probability by using the occupancy of component executing transition as sample data, studied transition probability matrix computation method based on the minimum quadratic difference, and then put forward the method to prove its efficiency. In order to prove Markov chain law and that the row sum of transition probability matrix is 1 and non-negative, and to improve the efficiency of transition probability matrix computation, the approximation algorithm based on projection gradient has been designed. The experiment result proves that the presented method and design computation can effectively calculate internetware transition probability matrix with a good practical value on internetware reliability computation.

The study on transition probability has caught great attention and been widely used. Document [10] proposes transition probability estimation method by using ordered probability to select model based on Markov chain degradation model. Document [11] puts forward condition transition probability computation method by using the decline of related information in information theory as time passes by under the influence of certain conditional probability in credit model. Document [12] explains the non-reasonability about transition probability independence hypothesis from section length distribution probability and gets transition probability computation method, which is more effective than classic HMM's Baum-Welch iterative algorithm re-estimating transition probability. Document [13] estimated and predicts future state based on Markov chain transition matrix on secondary loan and puts forward the innovated transition matrix estimation. Document [14] puts forward the application study of Markov transition probability on medicine. Document [15] computes transition probability by using Monte-Carlo sampling and Bayesian method on asynchronous vector Markov process. These studies improve the efficiency and accuracy of transition probability computation and the application on special occasions. Many things about transition probability need further study. The method to acquire transition probability sample, the influence of the amount of samples on transition accuracy, and the influence of internet environment on transition probability and so on are to be studied further.

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