Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2013, 5(12):648-653

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Calculating extreme value of unimodal function using bisection method

Hao-ran Shi, Xiaoqing Liu*, Yao Yang and Zhiqin Li

School of Energy and Environment, Xihua University, Chengdu, P. R. China

---

ABSTRACT

Based on the features of unimodal fun ction, select a practical value of an independent variable within the entire value range of the unimodal function; then, evaluate the function and conduct a reverse calculation of the function value; if there is a unique solution, make a new value range using the solution; carry on this calculation until we obtain the extreme value of the function. Compared with the Fibonacci method and the 0.618 method, bisection method is simpler and more effective.

Key words: Fibonacci method, 0.618 Method, one-dimensional search, unimodal function bisection method

---

INTRODUCTION

There are several ways to calculate extreme value using unimodal function. The Fibonacci method and the 0.618 method are two most frequently used methods. The philosophy of these methods is to carry out repeated test and iteration to obtain the proximate optimum solution that meets the requirement of measurement accuracy. Iteration is slow because: the initial interval and calculation accuracy should be determined first; the calculation procedure is complex; the solution is a proximate optimum value. This paper proposes bisection method, which is an improved version based on the previous two methods. Bisection method is quick, simple, with fewer iteration and obtains the optimum solution instead of the proximate optimum solution.

Concepts [1]

If f(x) has only one maximum/minimum point C within the range of [a, b], and f(x) moves upwards/downwards on the left side of C; f(x) moves downwards/upwards on the right side of C, f(x) is called a unimodal function within the range of [a, b]. for example, f(x) and g(x) in figure 1 are two unimodal functions.

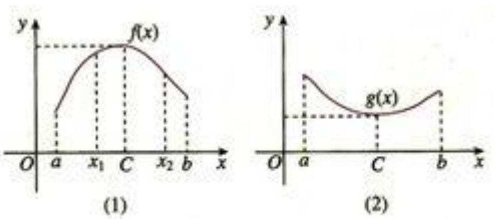

Fig. 1 Unimodal function

We define a monotonic function within the range of [a, b] as a unimodal function.

F(x) is a unimodal function within the range of [a, b] and C is the extreme value:

* Corresponding author email: xiaoqingliou1999@163.com

If $f(x)$ is a unimodal function within the range of $[a, b]$ and $\bar{x}$ is the minimum point of f(x) within$[a, b]$, for any two points

a_1 and b_1, $a_1 < b_1$,compute function values $f(a_1)$ and $f(b_1)$,

(1) when $f(a_1) < f(b_1)$, $\bar{x}$ is within $[a, b_1]$

(2) when $f(a_1) \geq f(b_1)$, $\bar{x}$ is within $[a_1, b]$

For any unimodal function, select any two points within the range of$[a, b]$ and calculate their function values to narrow the range to $[a, b_1]$ or $[a_1, b]$ (extreme value is within the narrowed range). In this way, by calculating the function value of the narrowed value range, we can further narrow the search range. When the range meets the requirements of measurement accuracy, we can obtain a proximate or accurate extreme value.

Figure 2 shows common unimodal functions

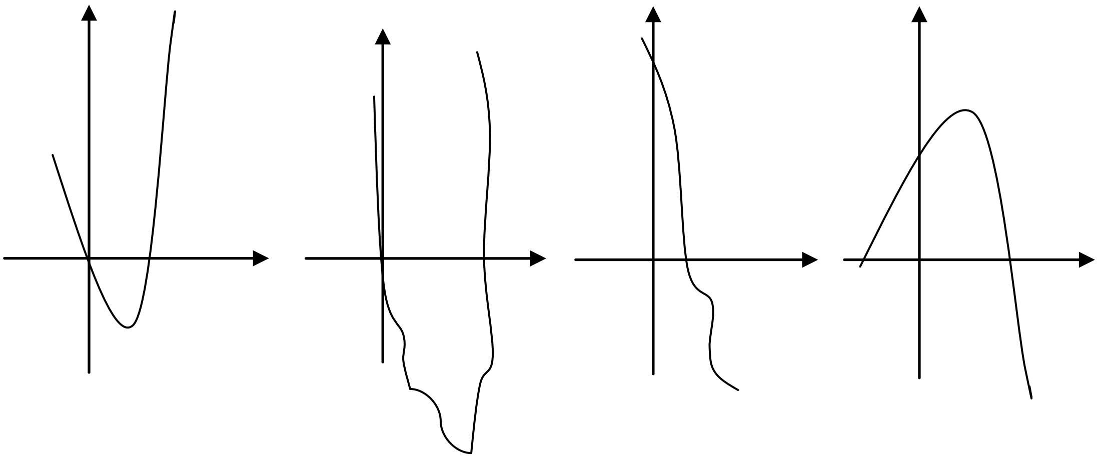

Fig.2 Shapes of unimodal function

FIBONACCI METHOD AND 0.618 METHOD

Based on the aforementioned theory, we continue to narrow the value range by selecting test point until it meets the measurement accuracy and the function value of each point is close to the minimum value.

There is no fundamental difference between the two methods. As classic one-dimensional algorithms, their principle and application are elaborated in many literature[2]~[6]. Therefore, we will skip that. They don't have to take the derivative of function but is complicated in calculation. Below are two cases of the Fibonacci method and the 0.618 method, as shown in Table [1].

Application of 0.618 method: $\min f(x) \stackrel{\text{def}}{=} 2x^2 - x - 1$, the initial value range is $[a_1, b_1] = [-1, 1]$,

accuracy $L \leq 0.16$,Computation is shown in Table 1[1]:

Table 1. Iteration of 0.618

k	a_k	b_k	λ_k	μ_k	$f(\lambda_k)$	$f(\mu_k)$
1	−1	1	1	0.236	−0.653	−1.125
2	−0.236	1	1	0.528	−1.125	−0.97
3	−0.236	0.528	0.056	0.236	−1.050	−1.125
4	0.056	0.528	0.236	0.348	−1.125	−1.106
5	0.056	0.348	0.168	0.236	−1.112	−1.125
6	0.168	0.348	0.236	0.279	−1.125	−1.123
7	0.168	0.279				

After 6 iterations, $b_7 - a_7 = 0.111 < 0.16$, which meets the requirement of accuracy.

Minimum point
$\bar{x} \in [0.168, 0.279]$.

In fact, the optimum solution $\bar{x} = 0.25$.

The proximate optimum solution $\bar{x} = \frac{1}{2}(0.168 + 0.279) \approx 0.23$

Application of Fibonacci method:

Initial value range is $[[-1,1]$, accuracy is $L \le 0.16$. Constant $\delta = 0.01$. When

$F_n \ge \frac{b_1 - a_1}{L} = 12.5, n = 6$. After six iterations, minimum point $\bar{x} \in [0.23077, 0.38461]$.

Iterations are shown in Table 2[1]

Table 2. Iterations of Fibonacci

k	a_k	b_k	λ_k	μ_k	$f(\lambda_k)$	$f(\mu_k)$
1	-1	1	-0.23077	0.23077	-0.66272	-1.12426
2	-0.23077	1	0.23077	0.53846	-1.12426	-0.95858
3	-0.23077	0.53846	0.07692	0.23077	-1.06509	-1.12426
4	0.07692	0.53846	0.23077	0.38461	-1.12426	-1.08876
5	0.07692	0.38461	0.23077	0.23077	-1.12426	-1.12426
6	0.23077	0.38461	0.23077	0.24077	-1.12426	-1.12483

From these two cases, it is clear that the calculation of Fibonacci method and 0.618 method is complicated. What's more, they can only obtain a proximate value. Based on the features of unimodal function, we can find a simpler method. Bisection method is an improved and simpler version of the Fibonacci method and the 0.618 method.

PHILOSOPHY OF THE BISECTION METHOD

Calculate extreme value of $f(x) = ax^2 + bx + c, x \in R$ between the value range [a, b].

Because f(x) is an unimodal function, it has only one extreme value $f(x^*)$ and only one extreme point x^*; otherwise, if $f(x)$ has two x, it is not the extreme value of the function. Therefore, if we use x to calculate $f(x)$ and use $f(x)$ to conduct a reverse calculation of x, the calculation stops when there is only one x and we obtain the optimum solution; if not, take the mean of the two x into the function $f(x)$. After repeated calculation, we can obtain an optimum solution and optimum value.

Calculation is shown below:

(1) Take $x_k = \frac{a_k + b_k}{2}$ into function to calculate the function value $f(x_k)$.

(2) Calculate x. If there is only one x, or $x''_{k+1} = x'_{k+1} = \frac{a_k + b_k}{2}$, the calculation stops and we obtain the extreme value.

If there are more than one x, or there are $x'_{k+1} = x_k = \frac{a_k + b_k}{2}$ and $x''_{k+1} \ne \frac{a_k + b_k}{2}$: when $x''_{k+1} \notin [a_k, b_k]$, apparently the extreme value is on the endpoint, calculate $f(a_k)$ or $f(b_k)$ and we can find the extreme value point; when

$x''_{k+1} \in [a_k, b_k]$,, continue step (3).

(3) The new value range is $[a_{k+1}, b_{k+1}] = [x'_{k+1}, x''_{k+1}]$ or $[a_{k+1}, b_{k+1}] = [x''_{k+1}, x'_{k+1}]$.

Take $x_{k+1} = \frac{a_{k+1}+b_{k+1}}{2}$ to calculate $f(x_{k+1})$ and turn to step（2）.

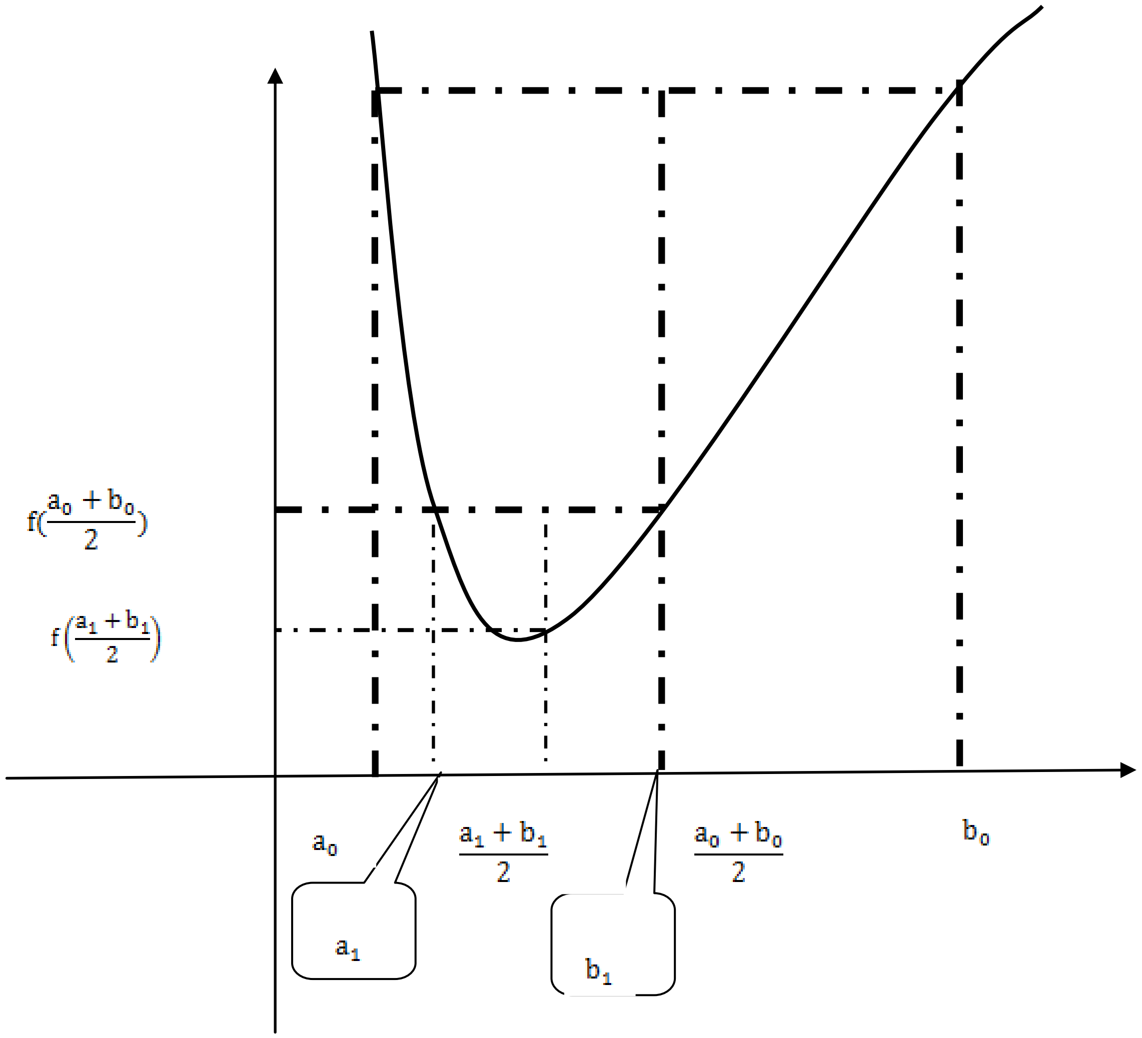

Fig. 3. Calculation process of bisection method

In bisection method, there is only one iteration. Repeat the above process in the new value range and continue to narrow the range infinitely close to 0 and obtain the minimum point $\bar{x}$.

CASE STUDY

Case 1: use bisection method to solve the function $\min f(x) \overset{\text{def}}{=} 2x^2 - x - 1$, the initial range is $[a_1, b_1] = [-1,1]$,Take $x_1 = \frac{a_1+b_1}{2}$ into the function to obtain function value f.

k	a_k	b_k	x_1	$f(x_1)$
1	-1	1	0	-1

(2)Take $f(x_1) = -1$ into the function to obtain a_k andb_k.

k	$f(x_1)$	a_k	b_k

2	-1	0	$\frac{1}{2}$

(3) Take $x_2 = \frac{a_2+b_2}{2}$ into the function to obtain function value $f(x_2)$.

k	a_k	b_k	x_2	$f(x_2)$
2	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{9}{-8}$

(4) Take $f(x_2) = \frac{9}{-8}$ into the function to calculate a_k and b_k.

k	$f(x_{k-1})$	a_k	b_k
3	$\frac{9}{-8}$	$\frac{1}{4}$	$\frac{1}{4}$

For $a_3 = b_3 = \frac{1}{4}$, we have obtained the optimum solution and the optimum value is $\frac{9}{-8}$.

Case 2:
Use Fibonacci method to calculate the proximate minimum point and minimum value of $f(t) = t^2 - t + 2$, and the narrowed value range should be no more than 0.08 times the length of $[-1,3]$.

After 6 iterations, the narrowed value range is $[0.231,0.545]$, the length of the range is 0.314, the minimum proximate value is 1.752.

The accurate solution is $t^* = 0.5 \quad f(t^*) = 1.75$.

Use bisection method to solve the problem.

Take $x_1 = \frac{a_1+b_1}{2}$ into the function, and turn to the previous step. Iteration process is shown below:

k	a_k	b_k	$\frac{a_k+b_k}{2}$	$f(\frac{a_k+b_k}{2})$
1	-1	3	1	2

k	$f(\frac{a_{k-1}+b_{k-1}}{2})$	a_k	b_k	$\frac{a_k+b_k}{2}$	$f(\frac{a_k+b_k}{2})$
2	2	0	1	$\frac{1}{2}$	1.75

k	$f(\frac{a_{k-1}+b_{k-1}}{2})$	a_k	b_k
3	1.75	$1/2$	$1/2$

CONCLUSION

Based on the features of unimodal function, many literature successfully discussed and improved methods to calculate extreme value of unimodal function. However, many methods requires the calculation of derivatives. By studying two cases, we have demonstrated the advantages of bisection method. Compared with the Fibonacci method and the 0.618 method, it is simpler, with easier calculation, can obtain an accurate solution, and is applicable to a wider range of problems. It is known to all that the Fibonacci method and 0.618 method requires the identification of the range that contains the minimum value. Bisection method, on the other hand, can calculate the function value with any feasible solution to the unimodal function and conduct a reverse calculation in exactly the same way. Bisection method can also assign a value to the unimodal function, calculate its solution, and obtain a unique optimum solution after several iterations. Bisection method simplifies calculation of extreme value. What's more, it can also be applied to calculate extreme value of convex function of convex set.

Acknowledgments
This work was supported by the Scientific Research Fund of Sichuan Provincial Education Department (Grant No.13204684), Key Scientific Research Foundation of Xihua University (Grant No. Z1120412) and Academic Cultivation Project of Key Laboratory of Fluid and Power Machinery Engineering, Xihua University (Grant No. SBZDPY-11-10).

REFERENCES

[1] Chen Baolin, Optimization Theory and Algorithm, Tsinghua University Press. pp.256
[2] Operations Research Teaching Materials Writing Group, Operations Research. Tsinghua University Press. pp.152
[3] Yuan Yaxiang , Sun Wenyu: Optimization Theory and Method [M] Beijing: Science Press, **1997**
[4] Xie Zhen, Li Jianping, Tang Zeying. Non-linear Optimization [M]. Beijing: National university of defense technology press, **2003**: 1.
[5] XU Wang -bao, CHEN Xue -bo, LI Xiao -hua, SU Xiao –ying, *Journal of Anshan University of Science and Technology*, Vol. 29 No. 4,pp.356-359,
[6] Xue Jiaqing. Optimization Theory and Method [M]. Beijing: Metallurgical Industry Press, **1983**: 1.