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B-spline surfaces on circle and fan-shaped domain

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ABSTRACT

Rotated a B-spline curve we can get rotating B-spline basis functions and the definition of B-spline surfaces on cylindrical coordinate system, gives the nature of B-spline basis functions and B-spline surfaces with de Door algorithm. As the rotating B-spline surfaces is not conducive to geometric modeling. In this paper, we improve the rotating B-spline method, and then we defined B-spline basis functions, tensor B-spline surfaces under the cylindrical coordinate system which has it nature. All this provides a new method in geometric modeling.

Keywords: B-spline, Cylindrical coordinate system, Surfaces, de Door algorithm.

INTRODUCTION

In 1971, French car manufacturer Renault's engineers Bezier proposed a new approach design the curve by control polygon, The method by manipulating control vertices to modify the shape of the curve and shape changes completely expected. Bezier method is simple and easy to operate, successfully resolved the overall shape of the control problem, make a success on curve and surface modeling technology, but this method can not solve connection problems and partial modification. In 1972, de-Boor [1] set the standard method of the B-spline, On this basis 1974 General Motors manufacturing company Gordon [2] and Riesenfeld [3] use the B-spline theory to describe the shape and give the B-spline methods design curve and surface. This method inherits all the advantages and overcomes the deficiencies, not only solve the problem of local control, but also on the basis of parametric continuity solve connection problems which get a better solved description free-form curves and surfaces shaped.

B-spline method successfully describe the shape of freedom curves and surfaces but not accurate representation of conic section and elementary analytic surfaces, so that there is no uniform mathematics manifestations descript curve and surface, which can not meet the requirements of most industrial products exterior design. So Versprille [4] in Syracuse University USA first proposed a rational B-spline method in 1975, then, due to Piegl, Tiller [5] and other people's work, making this method becomes popular mathematical methods represent NURBS curves surfaces, but NURBS method also has its deficiencies, such as how to choose the right factor and to increase the number of the derivative times makes the calculation increase and so on. Based on these aspects, we propose a B-spline surfaces under cylindrical coordinate system.

The paper is organized as follows: The second part of the article describes an element of B-spline curves; The third part of the article describes the rotating B-spline basis, rotating B-spline surfaces definition and its nature; The fourth part of the article describes the definition and nature of tensor B-spline basis and surfaces under cylindrical coordinate system.

2. B-spline curve

2.1 B-spline basis

The concept of B-spline [6] was originally developed by Schoenberg in 1946, but now there are several equivalent definitions. From geometric concepts Clark proposed B-spline defined [7] which is visual image and easy to accept; B-spline defined by power function with truncated difference quotient apply to the theoretical analysis of the B-spline; The de Boor and Cox were proposed the recurrence B-spline [8] which making the relevant calculation is simple and stable, most widely used in CAGD.

Use the recursive definition of B-spline proposed by de Boor and Cox, B-spline can be expressed as

$$N_{i,0}(x)=\begin{cases}1, & t_i \le x \le t_{i+1}\\ 0, & else\end{cases}$$

$$N_{i,k}(x)=\frac{x-t_i}{t_{i+k}-t_i}N_{i,k-1}(x)+\frac{t_{i+k+1}-x}{t_{i+k+1}-t_{i+1}}N_{i+1,k-1}(x), k>0 \tag{1}$$

Among them $0/0=0$. k said as B-spline power law, t as node, subscript i as B-spline serial number. From (1) arbitrary k B-spline can be composed of adjacent two $k-1$ B-spline .

When the spline node is uniformly segmented called uniform B-spline, and they have the same graphics within the range, B-spline basis on any interval of node can be obtained from the B-spline pan on another node interval.

The following given the uniform node of cubic spline basis functions formula on $[0,1]$

$$N_{0,3}=(1-x)^3/6, N_{1,3}=(4-6x^2+3x^3)/6$$
$$N_{2,3}=(1+3x+3x^2-3x^3)/6, N_{3,3}=x^3/6$$

2.2 Uniform B-spline curve

Given $N_{i,k}(x), x\in[x_j,x_{j+1}], i=0,1,\cdots,k$ said as k uniform B-spline basis $V_i, i=0,1,\cdots,k$ said as control vertices of the polygon, so the uniform B-spline curve can be expressed as

$$P(x)=\sum_{i=0}^{k}N_{i,k}(x)V_i, x\in[x_j,x_{j+1}]$$

cubic uniform B-spline curve on $[0,1]$ can be expressed as

$$P(x)=\sum_{i=0}^{3}N_{i,3}(x)V_i, x\in[0,1]$$

Written in matrix form as

$$P(x)=\frac{1}{6}[x^3 \quad x^2 \quad x \quad 1]\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 0 & 3 & 0\\ 1 & 4 & 1 & 0\end{bmatrix}\begin{bmatrix}v_0\\ v_1\\ v_2\\ v_3\end{bmatrix}$$

Among them v_0, v_1, v_2, v_3 is the control vertex of the polygon.

3. The rotating uniform B-spline surfaces

3.1. Rotating B-spline basis

In order to accurately represent the shape of a circle, many scholars improved the B-spline. This paper rotates B-spline curve located in the positive x to B-spline surfaces. First we given the rotating B-spline basis, given a B-spline basis function if it exists the branch on positive axle, then we take this part of the B-spline basis rotated along the axis of z , then (1) is

$$N_{i,0}(x,y)=\begin{cases}1, & r_i \le \sqrt{x^2+y^2} \le r_{i+1} \\ 0, & else\end{cases}$$

$$N_{i,k}(x,y)=\frac{\sqrt{x^2+y^2}-r_i}{r_{i+k}-r_i}N_{i,k-1}(x,y)+\frac{r_{i+k+1}-\sqrt{x^2+y^2}}{r_{i+k+1}-r_{i+1}}N_{i+1,k-1}(x,y),k>0 \qquad (2)$$

If the (2) corresponds to the cylindrical coordinate system we can obtained when taking $r=\sqrt{x^2+y^2}$

$$N_{i,0}(r)=\begin{cases}1, & r_i \le r \le r_{i+1} \\ 0, & else\end{cases}$$

$$N_{i,k}(r)=\frac{r-r_i}{r_{i+k}-r_i}N_{i,k-1}(r)+\frac{r_{i+k+1}-r}{r_{i+k+1}-r_{i+1}}N_{i+1,k-1}(r),k>0 \qquad (3)$$

So we obtain a recursive rotating B-spline basis under the cylindrical coordinate system.

The figure below shows the three rotating B-spline image on a circular area with a radius of 4

Fig1 evenly rotating cubic B-spline basis

3.2 The character of rotating B-spline

Character 1, local support. Each rotating B-spline basis nonzero in the local area but in other locations are 0

$$N_{i,k}(r)=0, r\notin[r_i,r_{i+k+1}]$$

Character 2, Symmetry. Uniform B-spline graphical is symmetry.

Character 3, Non-negative. $N_{i,k}(r)\ge 0$,for any i,k and r

Character 4, Decomposition of the unit. $\sum_{j=i-k}^{i} N_{i,k}(r)=1, r\in[r_i,r_{i+1}]$

Character 5, Derivative. We can get rotating B-spline derivative formula as

$$N'_{i,k}(r)=\frac{p}{r_{i+k}-r_i}N_{i,k-1}(r)-\frac{p}{r_{i+k+1}-r_{i+1}}N_{i+1,k-1}(r)$$

Character 6, Continuous bands. In each region $[r_i,r_{i+1}]$, $N_{i,k}(r)$ is polynomial function, and in each node r_i , $N_{i,k}(r)$ is $k-m_j$ continuously and differentiable, m_j is the multiplicities for the node. So increase the number, increase continuity, and increase nodes, then reduce the continuity.

3.3 Rotating uniform B-spline surfaces

Given $N_{i,k}(r), r \in [r_j, r_{j+1}], i = 0,1,\cdots,k$ said as k uniform B-spline basis, $V_i, i = 0,1,\cdots,k$ as the control curve, then a period of k time rotating uniform B-spline surface can be expressed as

$$P(r) = \sum_{i=0}^{k} N_{i,k}(r) V_i, r \in [r_j, r_{j+1}]$$

The three rotating uniform B-spline surfaces on the unit circle can be expressed as

$$P(r) = \sum_{i=0}^{3} N_{i,3}(r) V_i, r \in [0,1]$$

Written in matrix form as

$$P(r) = \frac{1}{6}[r^3 \quad r^2 \quad r \quad 1]\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Among them v_0, v_1, v_2, v_3 is control curve to control the rotating B-spline surfaces, the ruled surface generated by the control curve is rotating B-spline surface control surfaces.

3.4 Character of the rotating uniform B-spline surfaces

Character 1, Geometric invariance and affine invariant. By the decomposition of the unit of rotating B-spline basis we can be easily proved.

Character 2, Piecewise smooth polynomial surface. Rotating B-spline surfaces in each sub-region is on the parameter polynomial surface. Rotating B-spline surfaces of k polynomial surface of parameter r in each sub-region $[r_j, r_{j+1}]$. So the rotating B-spline surfaces is a segmented polynomial surface. The increase each curves for corresponding to increase the surfaces.

Character 3, Partial adjustment. As the rotating B-spline basis has nature of collection of local support functions, each control curve V_i only contribute to centralize $\left[r_i, r_{i+k+1}\right]$. So the adjustment V_i will only affect B-spline surfaces corresponding to section $\left[r_i, r_{i+k+1}\right]$.

Character 4, Convex hull. By the rotating B-spline basis non-negative and decomposition of the unit, rotating B-spline surfaces are included in its control polygon convex hull. And by the local branch of rotating B-spline basis, rotating B-spline surfaces have stronger convex hull than the rotating Bezier surfaces.

Character 5, Continuous bands. When the node r_i of multiplicity $m_i \geq 1$, then that branch within point r_i of the k time B-spline basis $N_{i,k}(r)$ order is $k - m_j$, so the corresponding B-spline surfaces rotating in continuous of order $k - m_j$.

3.5 de Boor algorithm of rotating B-spline surfaces

By the recursive formula of rotating B-spline surface and definition of rotating B-spline basis, on the nodes interval $[r_i, r_{i+1}]$

$$P(r)=\sum_{i=0}^{k}p_iN_{i,k}(r)$$
$$=\sum_{i=0}^{k}p_i(\frac{r-r_i}{r_{i+k}-r_i}N_{i,k-1}(r)+\frac{r_{i+k+1}-r}{r_{i+k+1}-r_{i+1}}N_{i+1,k-1}(r))$$
$$=\sum_{i=1}^{k}(\frac{r-r_i}{r_{i+k}-r_i}N_{i,k-1}(r)p_i+\frac{r_{i+k}-r}{r_{i+k}-r_i}p_{i-1})N_{i,k-1}(r)$$

This shows that the k times rotating B-spline surfaces can be seen as two $k-1$ rotating B-spline surfaces, the new control point is the average of the original two adjacent control points weighted, and the weight coefficient depends on the parameters r . So wecan be obtained de Door algorithm of rotating B-spline surfaces as

$$P(r)=\sum_{i=0}^{k}p_iN_{i,k}(r)=\sum_{i=1}^{k}p_i^1N_{i,k-1}(r)$$
$$=\cdots=\sum_{i=s}^{k}p_i^sN_{i,k-s}(r)=\cdots=p_i^k(r)$$

$p_i^k(r)$ is just as $p(r)$.

4. Tensor uniform B-spline surfaces under cylindrical coordinate system

While rotating B-spline surfaces under the cylindrical coordinate system is obtained from the B-spline curve, and have many similar properties with B-spline curves. But the rotating B-spline surfaces biggest drawback is the rotation of lead-based function has a good symmetry, so constructed the rotating B-spline surfaces also has good symmetry. More lines curved in the shape of geometric modeling is irregular, which leads to the rotating B-Spline surface modeling method is not very good application in geometric modeling. In order to solve this problem we introduce two quantities radius and angle to represent B-spline surfaces. So we can get the tensor B-spline surfaces under cylindrical coordinate system.

4.1Tensor uniform B-spline basis under cylindrical coordinate system

B-spline basis are piecewise smooth polynomial defined on vector nodes, it can be said to be split on a given region. To get the tensor product type B-spline basis, this paper split along the two-dimensional r and θ under cylindrical coordinate system, so we can get node vector

$$\begin{cases} R:\{r_i\},r_i<r_{i+1},i=0,\pm1,\cdots \\ \Theta:\{\theta_j\},\theta_j<\theta_{j+1},j=0,\pm1,\cdots \end{cases}$$

So by the definition of B-spline basis functions were obtained B-spline basis about parameters r , θ is $N_{i,k}(r)$ and $N_{j,s}(\theta)$. Combinative them in accordance with the tensor product we obtained the $k\times s$ B-spline basis

$$N_{i,k}(r)N_{j,s}(\theta)$$

When given the B-spline equally split, r split length is 1 , θ split length is $\frac{\pi}{2}$,then we can get $N_{3,3}(r)N_{3,3}(\theta)=\frac{2r^3\theta^3}{9\pi^3}$. So we can get the figure in the first quadrant of unit circle

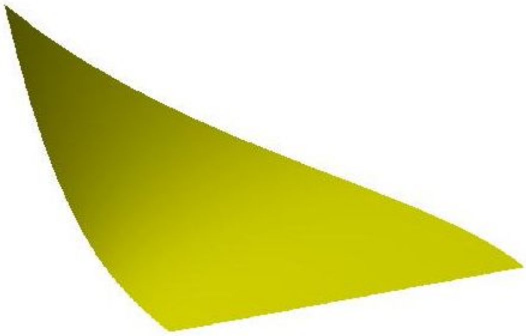

Fig 2 $N_{3,3}(r)N_{3,3}(\theta)$ **on the first quadrant**

If the image is not defined on the unit circle, such as $r \in [1,2]$, The tensor B-spline surfaces defines the fan-shaped domain, it has the same definitions and properties with the B-spline basis on the circular domain.

4.2 Character of tensor uniform B-spline basis

Character 1, Local support and non-negative. Support local of binary B-spline basis $N_{i,k}(r)N_{j,s}(\theta)$ is the tensor product of support local $N_{i,k}(r)$ and $N_{j,s}(\theta)$, as

$$N_{i,k}(r)N_{j,s}(\theta)=0,(r,\theta)\notin\left[r_i,r_{i+k+1}\right]\times\left[\theta_j,\theta_{j+s+1}\right]$$

and non-negative in support.

Character 2, Unit decomposable

$$\sum_{i=0}^{k}\sum_{j=0}^{s}N_{i,k}(r)N_{j,s}(\theta)=1,(r,\theta)\in\left[r_i,r_{i+1}\right]\times\left[\theta_j,\theta_{j+1}\right]$$

Character 3, Derivative

$$\frac{\partial}{\partial r}N_{i,k}(r)N_{j,s}(\theta)=\frac{k}{r_{i+k}-r_i}N_{i,k-1}(r)N_{j,s}(\theta)-\frac{k}{r_{i+k+1}-r_{i+1}}N_{i+1,k-1}(r)N_{j,s}(\theta)$$

$$\frac{\partial}{\partial \theta}N_{i,k}(r)N_{j,s}(\theta)=\frac{s}{\theta_{j+s}-\theta_j}N_{i,k}(r)N_{j,s-1}(\theta)-\frac{s}{\theta_{j+s+1}-\theta_{j+1}}N_{i,k}(r)N_{j+1,s-1}(\theta)$$

Character 4, Continuous bands of heavy nodes. In each cell chamber $[r_i,r_{i+1}]\times[\theta_j,\theta_{j+1}]$. $N_{i,k}(r)N_{j,s}(\theta)$ is $k\times s$ times tensor product polynomial function,, therefore the derivative exists. On isoparms $r=r_i$, $N_{i,k}(r)N_{j,s}(\theta)$, Similarly, on isoparms $\theta=\theta_j$, $N_{i,k}(r)N_{j,s}(\theta)$ is $s-m_j$ times continuously differentiabl about θ, and m_j is the multiplicity of nodes θ_j .

4.3 Tensor uniform B-spline surfaces

given $k\times s$ vector $P_{i,j}$ under the cylindrical coordinate system, $N_{i,k}(r)$ and $N_{j,s}(\theta)$ was k times and s times B-spline basis on the node vector $R=\{r_0,r_1,\cdots,r_k\}$ and $\Theta=\{\theta_1,\theta_2,\cdots,\theta_s\}$, Then the tensor product surface of cylinder coordinates corresponding to

$$P(r,\theta)=\sum_{i=0}^{k}\sum_{j=0}^{s}P_{i,j}N_{i,k}(r)N_{j,s}(\theta),(r,\theta)\in\left[r_i,r_{i+1}\right]\times\left[\theta_j,\theta_{j+1}\right]$$

called $k\times s$ B-spline surfaces. $P_{i,j}$ called control points, connected each other control vertex grid in turn called the control grid.

4.4 Character of tensor uniform B-spline surfaces
Character 1, Geometric invariance and affine invariant. It can easily proved by decomposition units of the tensor B-spline basis.

Character 2, Piecewise polynomial surface. B-spline surfaces is $k \times s$ polynomial surface of the parameters r and θ on each cell chamber $[r_i, r_{i+1}] \times [\theta_j, \theta_{j+1}]$, Therefore B-spline surfaces is a slice polynomial surface.

Character 3, Border. Four boundary line of B-spline surfaces are B-spline curves, The control points were combination by linear control mesh.

Character 4, trong convex hull. As tensor B-spline basis is non-negative, partition of unity and local support. Each piece of B-spline polynomial surface are located in convex hull of the control mesh.

Character 5, Continuous bands. From the continuous bands of binary B-spline basis , When the node multiplicity $m_i \geq 1$ along the direction r , the B-spline surfaces continuous bands is $k - m_i$ on the parameters line of $r = r_i$ .Similarly When the node multiplicity $m_j \geq 1$ along the direction θ , the B-spline surfaces continuous bands is $s - m_j$ on the parameters line of $\theta = \theta_j$.

Character 6, De Door algorithm of tensor B-spline surfaces. The de Door algorithms B-spline curve along the isoparms can expanded to tensor B-spline surfaces under cylindrical coordinate system.

CONCLUSION

In this paper, rotated the B-spline curve we can obtained under rotating B-spline basis and B-spline surfaces under the cylindrical coordinate system, and given the character of rotating B-spline basis, B-spline surfaces and it de Door algorithm. However, due to the rotating B-spline surfaces have good symmetry which not conducive to a wide applications in geometric modeling. Therefore, this paper established the tensor B-spline basis and surfaces under cylindrical coordinates, and given the character of the tensor B-spline basis and surfaces on circle and fan-shaped domain. However, if the shape of the surface is closed then the surface will appear intermittently near closed position, only the non-closed shape of fan-shaped domain the surface segments has good results. Tensor B-spline surfaces has some applicability in the shape and it can express simple arc, which provides a new approach for geometric modeling.

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