



Research Article

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## Basketball player field-goal percentage influence factors quantitative analysis and research

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### ABSTRACT

In order to improve basketball player's field-goal percentage, combining with mathematical, mechanical relative aspects principles, by establishing basketball motion trajectory mathematical model when player shoots, utilize variation method, projection method and so on; from minimum angle with angular deviation, optimal incident angle, and minimum angle with speed variation these three aspects, respectively research player basketball shooting angle size influences on field-goal percentage, and get respectively field-goal percentage highest interval is  $\frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} < \alpha < \frac{\pi}{2}$  ( $\alpha \geq 55^\circ 40'$ ). Through three models comparison, it respectively gets shooting angle optimal interval, make optimization on shooting angle optimal interval, on the condition that guarantee player shooting angular deviation and shooting speed variation have smaller influences on hoop deviation when entering basketball, meanwhile ensure basketball get optimal incident angle so that provide reference for effective improving basketball player's field-goal percentage.

**Key words:** Angular deviation, optimal incident angle, speed variation, field-goal percentage

### INTRODUCTION

It is well known that shooting is a very difficult while especially skillful technique; it is not only with multiple motion segments but also with high accuracy requests. Basketball playing antagonistic center is layup, shoot and slam dunk. Among them, shoot is the supremely key technique in basketball playing, field-goal percentage plays crucial roles in competition, and shoot tends to be two parties teams attack and defense major focus. Basketball has been quickly caught on around the world with its own terrific ornamental [1, 2]. With basketball quickly development, basketball players' attack and defense techniques have been continuously improved. People start more deeply thinking about basketball player field-goal percentage influence key factors, which lead to basketball relative documents quantities, are quickly increasing [3]. Relative documents proposed many methods play proactive roles in improving basketball player field-goal percentage and maintain player stable playing [4].

So far, lots of documents have put forward different methods in improving basketball player field-goal percentage. But, until now most of relative documents are qualitative pure technical researches [5]. Though some relative documents apply physical mechanical knowledge, it still has not deep comprehensive applied mathematical and physical mechanical knowledge that caused their results on field-goal percentage optimization lack of accuracy and quantitative feature [6-8].

In order to make basketball player daily training more scientific and reasonable, this paper carries out accurate quantitative researching on how to improve basketball player field-goal percentage by comprehensive applying

mathematical and physical sports mechanical principles.

### BASKETBALL MOVEMENT TRAJECTORY MODEL ESTABLISHMENTS

In match, it is always seen that some shooters still can make shooting when suddenly affected by outside force and lose body balance, which indicates that shooters have good body coordination; In the moment that ball is out of hand, body and hands are relative stable, shooters' strong space and time sense, good hand feeling, strong confidence make whole shoot motion with balance and gentle strength, and the motions are natural, coherent, fluency. Science and practice have proved that ball takeoff angle influences ball flight route, the ball flight routes are normally low arc, middle arc and high arc such three kinds, generally it is best with middle arc. But due to shooting distances and players' heights and jumping qualities are different, ball flight route would also be different when shooting, it is up to practical situations in training. Meanwhile, stable psychological factors are also of great importance, learning self-control and self psychological hint without affected by referee, court, audience, atmosphere and scores, adopting reasonable, decisive actions to make shooting. Motions of shooting have single-hand, two hands. No matter what way it adopts, it should strictly follow standardization motions. Cultivate and seize muscle sense when shooting is the prerequisite that prior to any others, which is required to strengthen standardization shooting motions training, finally achieve dynamic stereotype. Then we make quantitative research on shooting motions so as to explore optimal shooting mathematical model [9].

#### Minimum angular deviation angle

When basketball player shoots with angle  $\alpha$ , due to shooting angular deviation  $\partial\alpha$ , caused basketball drop point deviation  $\partial x$ . To determine deviation minimum angle, it needs to get minimum basketball drop point deviation that caused by shooting angular deviation. Regard basketball movement trajectory as oblique upcast.

$$x = V_0 t \cos \alpha \quad (1)$$

$$y = V_0 t \sin \alpha - \frac{1}{2} g t^2 \quad (2)$$

Convert formula (1) get:

$$t = \frac{x}{V_0 \cos \alpha} \quad (3)$$

Input formula (3) into formula (2), it can get:

$$y = x \tan \alpha - \frac{g x^2}{2 V_0^2 \cos^2 \alpha} \quad (4)$$

Convert formula (4), it can get:

$$y \cos^2 \alpha = \frac{1}{2} x \sin 2\alpha - \frac{g x^2}{2 V_0^2} \quad (5)$$

Simultaneously determine partial derivatives of  $\alpha$  with two sides of formula (5), it can get:

$$\frac{\partial x}{\partial \alpha} = \frac{x \cos 2\alpha + y \sin 2\alpha}{\frac{g x}{V_0^2} - \frac{1}{2} \sin 2\alpha} \quad (6)$$

When basketball arrives at top point:

$$t = t_h = \frac{V_0 \sin \alpha}{g} \quad (7)$$

Input formula (7) into formula (1), it can get:

$$x = x_h = \frac{V_0^2 \sin 2\alpha}{2g} \quad (8)$$

Let formula (6)  $\frac{\partial x}{\partial \alpha} = 0$ , it can get:

$$2\alpha = -\arctan \frac{x}{y} \quad (9)$$

From  $x > 0, y > 0$ , it can get:

$$-\frac{\pi}{2} < -\arctan \frac{x}{y} < 0 \quad (10)$$

Known that  $0 < \alpha < \frac{\pi}{2}$ , it can get  $0 < 2\alpha < \pi$ , from formula (9), it can further get:

$$2\alpha = \pi - \arctan \frac{x}{y} \quad (11)$$

Convert formula (11), it can get minimum deviation angle:

$$\alpha = \frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} \quad (12)$$

With original point  $O$  as shooting point, hoop lies in point  $A$ , make straight line  $OB$  as  $\angle yOA$  angular bisector, as Figure 1 shows.

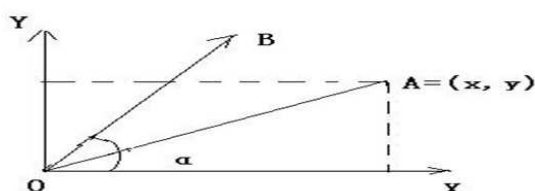


Figure 1: Speed analysis schematic figure when shooting

Known that  $\arctan \frac{x}{y} = \angle yOA$ , it can get  $\frac{1}{2} \arctan \frac{x}{y} = \frac{1}{2} \angle yOA = \angle yOB$ , then get  $\alpha = \frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} = \frac{\pi}{2} - \angle yOB = \angle BOx$ , as above Figure 1 shows.

From above model, it is clear that when basketball player shooting direction is along vertical direction and player arrives at hoop direction angular bisector directions, due to basketball player shooting angular deviation causes basketball drop point get minimum deviation.

### Optimal incident angle

Basketball movement trajectory is oblique upcast:

$$h = h_0 + V_0 t \sin \alpha - \frac{1}{2} g t^2 \quad (13)$$

$$l = V_0 t \cos \alpha \quad (14)$$

Basketball inverse movement trajectory:

$$h = H - V_1 t \sin \beta - \frac{1}{2} g t^2 \quad (15)$$

$$l = L - V_1 t \cos \beta \quad (16)$$

Respectively input formula (13)、(14) into formula (15)、(16), it can get through sorting:

$$\frac{H - h_0}{L} = \frac{1}{2} (\tan \alpha - \tan \beta) \quad (17)$$

Convert formula (17), it can get:

$$\tan \alpha = \frac{2H - 2h_0}{L} + \tan \beta \quad (18)$$

With regard to basketball that makes oblique upcast movement, hoop actual shape is called incident cross section. Incident cross section shape is oval that got by hoop projection on basketball movement speed vertical plane.

Oval semi-major axis:  $a = \frac{D}{2}$ , oval semi-minor axis:  $b = \frac{D \sin \beta}{2}$ .

To enter basketball into hoop, it should meet:

$$b \geq \frac{d}{2} \quad (19)$$

Input values into formula (19) can get:

$$45 \sin \beta \geq 24.6 \quad (20)$$

Determine formula (20), it can get  $\beta \geq 33^\circ 8'$ .

When basketball incident angle  $\beta$  becomes bigger, incident cross section area becomes bigger, basketball is more easily to hit. Different basketball incident angle permissible deviations along basketball movement trajectory plane direction are as below Table 1 shows.

**Table 1: Basketball incident angle and movement trajectory plane direction deviation**

Basketball incident angle(°)	90	80	70	60	50	40	33.14
Permissible deviation(cm)	10.2	9.86	8.84	7.19	4.94	2.16	0

To make basketball into hoop, it should also meet that basketball maximum offset in vertical movement trajectory plane should not above ball circle and oval internally tangent location.

Basketball circle equation:

$$(x - e)^2 + y^2 = 12.3^2 \quad (21)$$

Incident cross section oval equation:

$$\frac{x^2}{22.5^2} + \frac{y^2}{(22.5 \sin \beta)^2} = 1 \quad (22)$$

Simultaneous formula (21)、(22), it can get:

$$e^2 = \frac{(1 - \sin^2 \beta)(22.5^2 \sin^2 \beta - 12.3^2)}{\sin^2 \beta} \quad (23)$$

Solve maximum value of  $e$ , it can get:  $e_{\max} = 22.5 - 12.3 = 10.2\text{cm}$

Get incident angle  $\beta = 47^\circ 41'$  at that time, basketball has the highest field-goal percentage.

Synthesize shooting angle influences on basketball in the incident cross section long axis and short axis two directions; it can get different incident angles influence on basketball field-goal percentage.

- (1) When  $\beta < 33^\circ 8'$ , basketball field-goal percentage is zero.
- (2) When  $33^\circ 8' \leq \beta < 47^\circ 41'$ , basketball field-goal percentage is lower.
- (3) When  $\beta \geq 47^\circ 41'$ , basketball has the highest field-goal percentage.

#### Minimum speed variation angle

When basketball player shoots with angle  $\alpha$ , due to basketball initial speed variation  $\partial V_0$ , causing basketball drop point deviation  $\partial x$ . To solve minimum speed variation angle, it needs to make minimum drop point deviation that caused by basketball initial speed variation. From formula (5), it is clear that.

$$y \cos^2 \alpha = \frac{1}{2} x \sin 2\alpha - \frac{gx^2}{2V_0^2}$$

Let  $v = V_0^2$ , it can get:

$$\frac{gx^2}{v} = x \sin 2\alpha - \frac{gx^2}{v} \quad (23)$$

Convert (23) it can get:

$$v = \frac{gx^2}{x \sin 2\alpha - 2y \cos^2 \alpha} \quad (24)$$

Simultaneously solve partial derivatives of two sides on formula (24), it can get:

$$\frac{\partial v}{\partial x} = \frac{gx^2 \sin 2\alpha - 4xyg \cos^2 \alpha}{(x \sin 2\alpha - 2y \cos^2 \alpha)^2} \quad (25)$$

Solve formula (25) reciprocal, it can get by sorting:

$$\frac{\partial x}{\partial v} = \frac{(2x \sin \alpha - 2y \cos \alpha)^2}{2gx^2 \tan \alpha - 4gxy} \quad (26)$$

Convert formula (26), it can get:

$$\frac{\partial x}{\partial v} = \frac{(\sin \alpha - \frac{y}{x} \cos \alpha)^2}{\frac{g}{2} \tan \alpha - g \frac{y}{x}} \quad (27)$$

Let  $K = \frac{y}{x}$ , input it into formula (27), so can get:

$$\frac{\partial x}{\partial v} = \frac{2 \cos^2 \alpha (\tan \alpha - K)^2}{g \tan \alpha - 2K} \quad (28)$$

Let:

$$f(\alpha, K) = \frac{2 \cos^2 \alpha (\tan \alpha - K)^2}{g \tan \alpha - 2K} \quad (29)$$

It can get:

$$\frac{\partial x}{\partial v} = f(\alpha, K) \quad (30)$$

When  $K = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$ , according to formula (29), respectively draw images of  $f(\alpha, K)$ , as following Figure 2, Figure 3, Figure 4, Figure 5, Figure 6, Figure 7 shows.

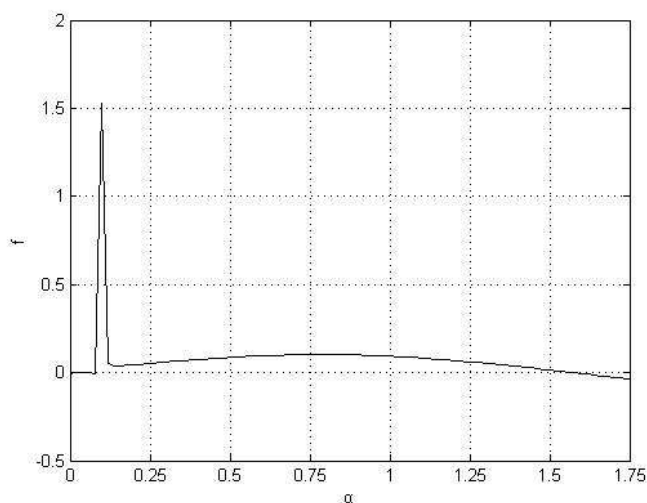


Figure 2: when  $K = 0.05$ ,  $f(\alpha, K)$  Figure

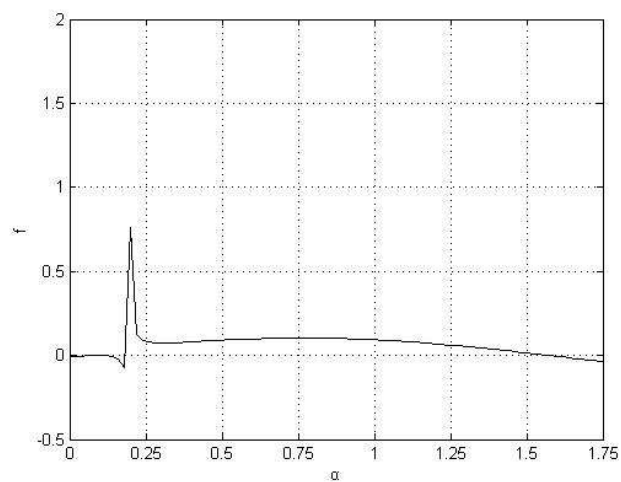


Figure 3: when  $K = 0.1$ ,  $f(\alpha, K)$  figure

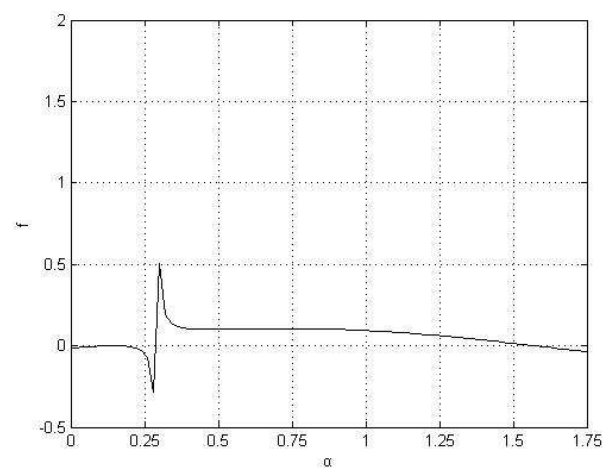


Figure 4: when  $K = 0.15$ ,  $f(\alpha, K)$  figure

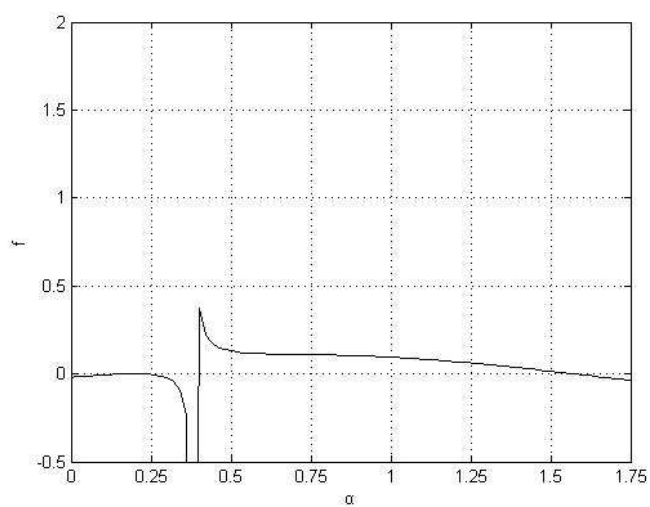


Figure 5: when  $K = 0.2$ ,  $f(\alpha, K)$  figure

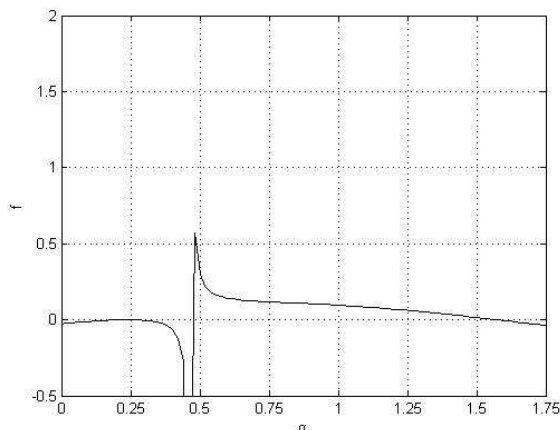


Figure 6: when  $K = 0.25$ ,  $f(\alpha, K)$  figure

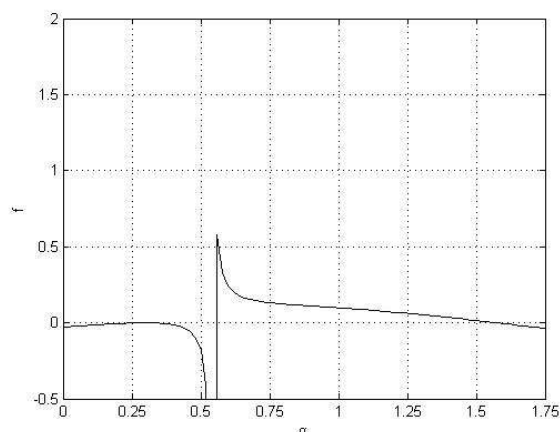


Figure 7: when  $K = 0.3$ ,  $f(\alpha, K)$  figure

Through observing above figures, it is clear that when basketball player shooting angle  $\alpha$  is above  $\frac{\pi}{4}$ ,  $\frac{\partial x}{\partial v}$  would be decreased with  $\alpha$  increasing. Therefore, when basketball player shooting angle is above  $\frac{\pi}{4}$ , player shooting angle would become bigger, shooting speed variation generated basketball incident location deviation would become smaller. From formula (10), it is clear that.

$$-\frac{\pi}{2} < -\arctan \frac{x}{y} < 0$$

$$-\frac{\pi}{4} < -\frac{1}{2} \arctan \frac{x}{y} < 0$$

It can get:

$$\frac{\pi}{4} < \frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} < \frac{\pi}{2}$$

It can further get:

Minimum angular deviation angle interval is:

$$\frac{\pi}{4} < \alpha < \frac{\pi}{2}$$

(31)

$$\tan \alpha = \frac{2H - 2h_0}{L} + \tan \beta$$

From formula (18), it is clear that:

According to model two, it is clear that when  $\beta \geq 47^\circ 41'$ , basketball has the highest field-goal percentage,



input  $h_0 = 2.5m$ ,  $L = 3m$  into formula (18), it can get:

$$\alpha \geq 55^\circ 40' \quad (32)$$

Synthesize minimum angular deviation angle, optimal incident angle, minimum speed variation angle, it is concluded that formula (30)、(31)、(32) intervals and get that highest basketball field-goal percentage shooting angle is in the following interval.

$$\frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} < \alpha < \frac{\pi}{2} \quad (33)$$

And  $\alpha$  should meet  $\alpha \geq 55^\circ 40'$ .

### CONCLUSION

Through establishing basketball movement trajectory model when player shoots, respectively considering minimum angular deviation angle, optimal incident angle, minimum speed variation angle such three aspects, analyzed player shooting basketball angle size influences on field-goal percentage. Through comparing three models, respectively got projection angles optimal intervals and made optimization of the projection angles optimal intervals, achieved

that highest basketball field-goal percentage shooting angle interval is  $\frac{\pi}{2} - \frac{1}{2} \arctan \frac{x}{y} < \alpha < \frac{\pi}{2} (\alpha \geq 55^\circ 40')$ . Under the circumstance that guarantee basketball player projection angular deviation and projection speed variation deviation have smaller influences on hoop offsets when entering into basketball, meanwhile ensured basketball get optimal incident angle. Applied this paper research results could effective improve basketball player field-goal percentage.

In addition, the purposes of attack team applying every technique and tactics in basketball playing are all to create more and better shooting opportunities and strive to hit and get scores; Defense team positive defending is to interfere opponents basketball scoring. With basketball event development, player height, physical quality and technical level improvement propel to shooting techniques continuously developing; Takeoff part changes from low to high, takeoff speed changes from slow to fast, shooting ways become more and more, field-goal percentage is continuously improving.

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