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Research Article

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Asymmetry-width probabilistic fuzzy logic system for rigid-flexible manipulator modeling

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ABSTRACT

The probabilistic fuzzy set (PFS) and probabilistic fuzzy logic system is designed for handling the uncertainties with stochastic and nonstochastic features. In this paper, an asymmetric probabilistic fuzzy set is proposed by randomly varying the width of asymmetric Gaussian membership function. And the related asymmetry PFLS is constructed to be applied to the rigid-flexible manipulator modeling. The performance discloses that the asymmetry-width probabilistic fuzzy set performs better than precious symmetric one. It is because the asymmetric probabilistic fuzzy set's variability and malleability is higher than this of the symmetric probabilistic fuzzy set.

Keywords: asymmetric probabilistic fuzzy set, probabilistic fuzzy set, probabilistic fuzzy logic system, Rigid-flexible manipulator

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INTRODUCTION

Type 1 fuzzy set [1] is able to model linguistic variable and it can deal with fuzzy information especially the uncertainties related the linguistics. It is noted that the crisp membership grade is used in this set. Yet, when the uncertainties are very complex, it may not be suitable to use a crisp membership grade in [0, 1], so type-2 fuzzy set [2] blurs the boundary of type 1 FS to directly model the uncertainty of linguistic expression. As such, the grades of membership in type 2 FS themselves are fuzzy set, thus able to capture the uncertainties in membership function. Currently, type-1 and type-2 fuzzy set have been successfully applied in many fields such as decision making [3], function approximation [4] and so on.

Unfortunately, in most of real-world applications, both stochastic and fuzzy uncertainties exist simultaneously, yet the traditional fuzzy theory and probabilistic models are only good at processing one aspect of uncertainties [5][6]. So it would be valuable to integrate the probability theory with the fuzzy theory. In recent years, several integration strategies have been proposed [7-8]. The probabilistic fuzzy set (PFS) is proposed and developed by introducing the probabilistic theory into the traditional fuzzy set described by center and width [9]. The fuzzy grades in the traditional fuzzy set become the stochastic variables described by the secondary probability density function (PDF). Then the probabilistic fuzzy set that integrates the fuzzy dimension and the probabilistic dimension is able to capture both stochastic and nonstochastic uncertainties. And based on probabilistic fuzzy set, the probability fuzzy logic system is proposed to capture stochastic and nonstochastic uncertainties in engineering process by training the data of the learning sample set. Recently, it has been applied for stochastic modeling and control [10], function approximation problem [11][12] and so on.

However, the previous probabilistic fuzzy set is constructed through randomizing the center of the symmetric fuzzy set and it is symmetric. In fact, asymmetric fuzzy set is discussed and used in fuzzy logic system to improve its modeling capability. So it would be interesting to construct asymmetric PFS and applied to system modeling.

In this paper, an asymmetric probabilistic fuzzy set is proposed by randomizing the width of asymmetric Gaussian fuzzy set. Then the related probabilistic fuzzy logic system is constructed and it is applied to the rigid-flexible manipulator modeling. The results show that the asymmetry-width probabilistic fuzzy set performs better than symmetric ones. It is because the variability and malleability of asymmetric probabilistic fuzzy set are higher than those of the symmetric probabilistic fuzzy set. This work will broaden the application capability in engineering.

This paper is organized as following: a new asymmetric probability fuzzy set and the related probabilistic fuzzy logic system are proposed in section II. In section Ⅲ, the designation of parameter is designed. In section Ⅳ, the asymmetric probability fuzzy logic system is applied to the rigid-flexible manipulator modeling problem. Finally, the conclusion is given in section V.

Ⅱ. Probabilistic Fuzzy Logic System with Asymmetric Probabilistic Fuzzy Set

The concept of probabilistic fuzzy sets have been proposed to capture uncertainties with both stochastic and fuzzy features [9] by introducing probability into the traditional fuzzy set described by center and width. In probabilistic fuzzy sets, for an input x, there no longer is a single value or values for the membership function; instead, the membership function becomes a random variable that can be described by the secondary PDF as shown in Fig.1.

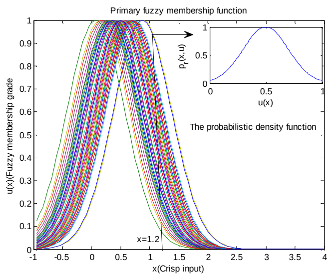

Fig.1. Fuzzy MF in the symmetric probabilistic fuzzy set for the perturbed center

2.1 Probabilistic fuzzy logic system

Similar to the ordinary fuzzy logic system, the PFLS still has operations of fuzzification, inference engine and defuzzification. Different to the ordinary fuzzy logic system, the PFLS uses the probabilistic fuzzy set that is described by a three-dimensional MF.

2.2 Construction of asymmetric probabilistic fuzzy set

In general, given an system input data x_i, and the desired output y, the inference in PFLS has the form:

$$Rule\ j: If\ \ x_1\ is\ \tilde{A}_{1,j}\ and\ ...\ and\ x_i\ is\ \tilde{A}_{i,j}\ and\ ...and\ x_n\ is\ \tilde{A}_{n,j},\ Then\ \ y\ \ is\ \ \tilde{B}_j \tag{1}$$

where $\tilde{A}_{i,j}\ (i=1,2,...,n)\ (j=1,2,...,j)$ and $\tilde{B}_j$ are probabilistic fuzzy sets. Here the probabilistic fuzzy sets $\tilde{A}_{i,j}$ and $\tilde{B}_j$ are asymmetric ones, which is constructed as follows.

An asymmetric Gaussian membership function shown in (2) is used as the primary membership function:

$$u=\begin{cases} e^{(-\frac{(x-c)^2}{2\xi_{(l)}^2})} & for\ \ x\le c \\ e^{(-\frac{(x-c)^2}{2\xi_{(r)}^2})} & for\ \ x>c \end{cases} \tag{2}$$

where c is the center, $\xi_{(l)}$ is the left width and $\xi_{(r)}$ is the right width, x is the input.

Similarly, the width $\xi_{(l)}$ and $\xi_{(r)}$ in equation (2) can be regarded as random variables following the normal distribution described as

$$\xi_{(l)} \sim N(\omega_{(l)},\lambda_{(l)}{}^2) \qquad \xi_{(r)} \sim N(\omega_{(r)},\lambda_{(r)}{}^2) \tag{3}$$

Accordingly, shown as in Fig.2, the probability distribution of fuzzy grade U can be obtained:

$$F_U(u)=\begin{cases}\int_0^{\frac{|x-c|}{\sqrt{-2\ln u}}}\frac{1}{\sqrt{2\pi}\lambda_{(l)}}e^{-\frac{(\varsigma_{(l)}-\omega_{(l)})^2}{2\lambda_{(l)}^2}}d\xi_{(l)} & 0<u<1\ and\ x<c\\ \int_0^{\frac{|x-c|}{\sqrt{-2\ln u}}}\frac{1}{\sqrt{2\pi}\lambda_{(r)}}e^{-\frac{(\varsigma_{(r)}-\omega_{(r)})^2}{2\lambda_{(r)}^2}}d\xi_{(r)} & 0<u<1\ and\ x>c\\ 0 & otherwise\end{cases} \tag{4}$$

And the secondary PDF is

$$\Pr ob_A(u)=\begin{cases}\frac{|x-c|(-2\ln u)^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda_{(l)}u}e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(l)})^2}{2\lambda_{(l)}^2}} & 0<u<1\ and\ x\le c\\ \frac{|x-c|(-2\ln u)^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda_{(r)}u}e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(r)})^2}{2\lambda_{(r)}^2}} & 0<u<1\ and\ x>c\\ 0 & otherwise\end{cases} \tag{5}$$

(1) when $x<c$

Suppose $\xi_{(l)}\sim N(\omega_{(l)},\lambda_{(l)}^2)$, then the density function is $\Phi(\xi_{(l)})=\frac{1}{\sqrt{2\pi}\lambda_{(l)}}e^{-(1/2)(\xi_{(l)}-\omega_{(l)}/\lambda_{(l)})^2}$. The random variable primary membership grade is $U=e^{-(1/2)(x-c/\xi_{(l)})^2}$ $(u\in(0,1))$. Though U is non-monotonic, it is monotonically decreasing in $(0,+\infty)$, so the distribution function of fuzzy grade u can be obtained as following:

Obviously $F_U(u)=P(U<u)=0$, when $u\le 0$

When $0<u<1$

$$F_U(u)=P(U<u)=P(\mathrm{e}^{-\frac{(x-c)^2}{2\xi_{(l)}^2}}<u)=P(\frac{(x-c)^2}{-2\ln u}>\xi_{(l)}^2) \tag{6}$$

As variance $\xi_{(l)}$ must be positive, equation (6) can be written as:

$$P(\sqrt{\frac{(x-c)^2}{-2\ln u}}>\xi_{(l)}>0))=\int_0^{\frac{|x-c|}{\sqrt{-2\ln u}}}\phi(\xi_{(l)})d\xi_{(l)} \tag{7}$$

Thus, the probabilistic distribution of U is:

$$F_U(u)=\begin{cases}\int_0^{\frac{|x-c|}{\sqrt{-2\ln u}}}\frac{1}{\sqrt{2\pi}\lambda_{(l)}}\mathrm{e}^{-\frac{(\xi_{(l)}-\omega_{(l)})^2}{2\lambda_{(l)}^2}}d\xi_{(l)} & 0<u<1\\ 0 & otherwise\end{cases} \tag{8}$$

Again, we consider the first derivative of u, the probabilistic density function can be obtained from Variable Limit Integral Derivation Formula as:

$$F_U'(u) = \frac{1}{\sqrt{2\pi}\lambda_{(l)}} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(l)})^2}{2\lambda_{(l)}{}^2}} (\frac{|x-c|}{\sqrt{-2\ln u}})' = \frac{|x-c|(-2\ln u)^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda_{(l)}u} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(l)})^2}{2\lambda_{(l)}{}^2}} \tag{9}$$

And so PDF of u can be expressed:

$$\Pr ob_A(u) = \begin{cases} \frac{|x-c|(-2\ln u)^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda_{(l)}u} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(l)})^2}{2\lambda_{(l)}{}^2}} & if \;\; 0 < u < 1 \\ 0 & otherwise \end{cases} \tag{10}$$

(2) When $x > c$

Similarly, the right side of PFS is:

$$\Pr ob_A(u) = \begin{cases} \frac{|x-c|(-2\ln u)^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda_{(r)}u} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u}}-\omega_{(r)})^2}{2\lambda_{(r)}{}^2}} & if \;\; 0 < u < 1 \\ 0 & otherwise \end{cases} \tag{11}$$

So it is followed the equation (5).

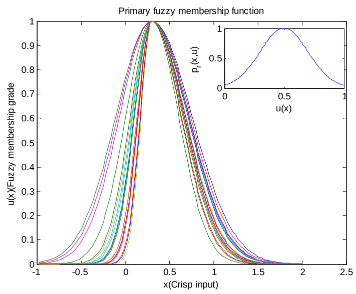

Fig.2. Fuzzy MF in the asymmetry-width probabilistic fuzzy set for the perturbed width

Ⅲ The Designation of Parameter

For the PFLS with the asymmetric probabilistic fuzzy set, the mean of width $\xi_{(l)}$ and $\xi_{(r)}$ maybe not the same and the designation of their initial parameter is given as follows:

The fuzzy c-mean variance (FCMV) algorithm is used to obtain the clustering results. Cluster centers are projected to $x1$ and $x2$ axis to obtain the Gaussian membership function of each clustering. The width $\xi_{(l)}$ and $\xi_{(r)}$ of the primary asymmetric membership function is formulated as:

$$\xi_{(l)} = \sqrt{\frac{\sum_{k=1}^{n}(\omega_{p,k}^{(l)})^m \left\|x_k - c_p\right\|_Q^2}{\sum_{k=1}^{n}(\omega_{p,k}^{(l)})^m}} \;\; for\; x_k < c_p \qquad \xi_{(r)} = \sqrt{\frac{\sum_{k=1}^{n}(\omega_{p,k}^{(r)})^m \left\|x_k - c_p\right\|_Q^2}{\sum_{k=1}^{n}(\omega_{p,k}^{(r)})^m}} \;\; for\; x_k > c_p \tag{12}$$

where

$$\omega_{p,k}^{(l)} = \frac{(\frac{1}{\left\|x_k - c_p\right\|_Q^2})^{1/m-1}}{\sum_{p=1}^{C}(\frac{1}{\left\|x_k - c_p\right\|_Q^2})^{1/m-1}} \;\; for\; x_k < c_p \qquad \omega_{p,k}^{(r)} = \frac{(\frac{1}{\left\|x_k - c_p\right\|_Q^2})^{1/m-1}}{\sum_{p=1}^{C}(\frac{1}{\left\|x_k - c_p\right\|_Q^2})^{1/m-1}} \;\; for\; x_k > c_p \tag{13}$$

where the convergent vector c_p ($p = 1, 2, ...C$) is the cluster center, C is the number of cluster partition and n is the number of the input.

Ⅳ. Rigid-flexible manipulator modeling

In this section, based on the proposed asymmetric PFS, the related asymmetric PFLS is constructed and applied to the rigid-flexible manipulator modeling problem to investigate its distinctive modeling capability in stochastic circumstance.

The Rigid-flexible manipulator system [13-15] is designed for verify the validity of the distributed parameter system as shown in Fig.3. When torque is applied on the joint of the flexible arm and the flexible arm, the strain will happen on the flexible link arms during the process of movement, then Strain gauge which is pasted in flexible link arms will appear distorted and produce strain voltage value. The strain voltage value can be used to deduce the deformation of mechanical arm to verify the validity and accuracy of the distributed parameter system. In this system, it is very important for obtaining the strain voltage value.

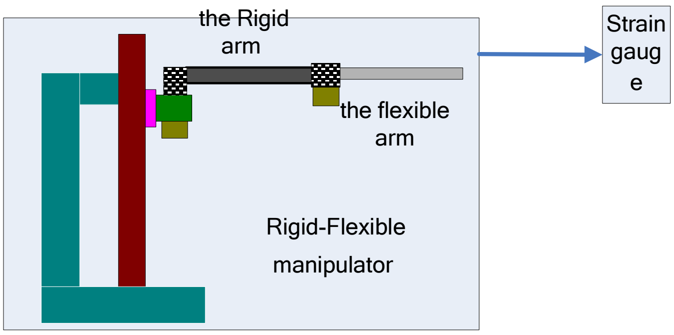

Fig.3: Rigid-Flexible manipulator system

A nonlinear modeling is used to approximate the change process of strain voltage data collected from Strain gauge

$$y(k) = f(y_1(k), y_2(k)) \tag{14}$$

where $y_1(k)$ and $y_2(k)$ is the input signal of first k moment, f is the desired Innuendo relationship, $y(k)$ is the output. Some pieces of Strain gauge is pasted in flexible link arms, the two input variables are signal sin, and the strain voltage value of Strain gauge is the output of the system.

Three pieces of Strain gauge are pasted in flexible link arms, each cycle time is set for 2 seconds and the sample time is 10 seconds. First, 100 input-output data pairs are obtained, and the FCMV algorithm is used to classify the fuzzy rule and estimate the uncertainties among these rules. The identified parameters (clustering center C, the width mean ω, the width variance λ) are obtained from 100 Monte Carlo simulations. Finally, the asymmetry-width-based PFLS is constructed to model the nonlinear system (14). The learning results of parameters obtained for asymmetry-width -based PFLS are given in Table 1, and the comparison of approximation output between symmetry Gau_{width} -based PFLS and asymmetry-width -based PFLS is shown in Fig.4.

TABLE 1 The parameter of asymmetry-width-based PFLS

Rule number	$y_1(k)(c_k, \omega_k, \lambda_k)$	$y_2(k)(c_k, \omega_k, \lambda_k)$
1	(0.6538,0.0186,0.0360)	(0.9999,0.4385,0.0795)
2	(0.1497,0.4773,0.0349)	(0.0113,0.0171,0.0520)
3	(0.2238,0.4083,0.0891)	(0.1606,0.2081,0.0231)
4	(0.4266,0.7209,0.0630)	(0.2820,0.4882,0.0420)
5	(0.0417,0.0962,0.0172)	(0.0798,0.0874,0.0033)

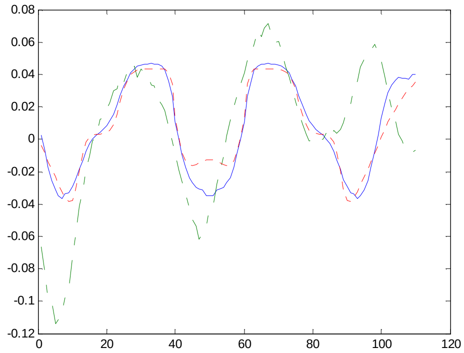

Fig.4: The comparison of approximation output for symmetry Gau_{width} -based PFLS (red solid line) and asymmetry-width -based PFLS (dotted line)

It is clearly that strong relations are found between modeling capability of PFS and the random disturb. The reason is that the asymmetry-width PFLS has the better potential ability to handle uncertainties than symmetric Gau_{width} -based PFLS under certain stochastic circumstance.

CONCLUSION

In this paper, an asymmetric probabilistic fuzzy set is proposed by randomly varying the width of Gaussian membership function. Then the related probabilistic fuzzy logic system is constructed and applied to a logistics demand forecasting problem. The results show that the asymmetry-width probabilistic fuzzy set performs better than symmetric ones. It is because the variability and malleability of asymmetric probabilistic fuzzy set are higher than those of the symmetric probabilistic fuzzy set. This will broaden the potential application of probabilistic fuzzy sets. In the future, more designation may be conducted for PFS, such as the general model of PFS. It is believed that the PFS will be very promising for many engineering application.

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