



Application of fuzzy comprehensive evaluation theory in air quality assessment

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ABSTRACT

The boundary of good or bad and the grade division of ambient air quality is blurring. Through the collection of the content of main air pollutants in 7 cities such as Beijing, Shanghai, pollutant indicators that mainly affect air quality are determined. Combining with national air quality standard, urban air quality is graded by using of fuzzy comprehensive evaluation theory. It can be concluded that air quality is excellent in Wu Xi, good in Cheng Du, Wu Han and Shanghai, qualified in Nan Jing, while contaminated in Chong Qing, and only in Beijing the air quality is severely polluted. Through cluster analysis of more than 30 cities by using K-means clustering, the seven cities are classified into 5 categories, which is basically consistent with the results of fuzzy evaluation, proving the accuracy of fuzzy evaluation theory. In weight determination by using fuzzy comprehensive evaluation, only one indicator is involved in, but some other relations are not considered, therefore, entropy weight method is introduced in weight coefficient determination to reevaluate air quality grades of the seven cities.

Key words: fuzzy comprehensive evaluation, K-means clustering, entropy weight method

INTRODUCTION

Recently, air pollution in Beijing and other cities are becoming increasingly serious and air pollution is the hot issue of national concern. With the rapid development of industrialization and urban economy, industrial production and human life emit large quantities of pollutants, bringing about air pollution. On the other hand, the air is essential to human beings, so its quality directly influences human health, ecological balance and even social security [1-5]. Thus, how to maintain clean air to realize harmonious development among urban economy, society and environment is the key focus of study. Urban air quality is assessed through air quality evaluation theory to determine its grade and find out the major influential factors [6-9]. Besides, reasonable suggestions and improvement measures are put forward to provide directions for industrial pollution prevention and control and air environmental protection in cities, and finally to provide scientific references for urban planning and management and environment management.

ESTABLISHMENT AND SOLUTION OF MODELS

Selection and determination of data

The data of this study comes from statistics on environmental protection and air quality in Yearbooks of each city (2005-2010). Seven big cities, Beijing, Chengdu, Chongqing, Wuhan, Nanjing, Wuxi, and Shanghai, are selected as objects. Generally, air quality evaluation indicators choose seven indicators, namely $SO_2, NO_2, PM_{10}, TSP, O_3, NO_x, CO$.

Air quality assessment in China puts particular emphasis on four indicators SO_2, NO_2, PM_{10}, TSP . Based on the integrity of sample data, three indicators, SO_2, NO_2, PM_{10} are selected as air quality assessment indicators in this paper [10, 11].

Table 1: Source data of annual average concentration of air pollutants in seven cities

City	target	2005	2006	2007	2008	2009	2010
Cheng Du	SO ₂	0.077	0.065	0.062	0.049	0.038	0.031
	NO ₂	0.052	0.049	0.049	0.052	0.055	0.051
	PM ₁₀	0.124	0.122	0.111	0.111	0.111	0.104
Chong Qing	SO ₂	0.073	0.074	0.065	0.063	0.053	0.048
	NO ₂	0.048	0.047	0.044	0.043	0.037	0.039
	PM ₁₀	0.120	0.111	0.108	0.106	0.105	0.102
Wu Han	SO ₂	0.045	0.046	0.050	0.051	0.044	0.041
	NO ₂	0.045	0.044	0.048	0.113	0.054	0.057
	PM ₁₀	0.111	0.109	0.108	0.036	0.105	0.108
Nan Jing	SO ₂	0.052	0.063	0.058	0.054	0.035	0.036
	NO ₂	0.054	0.052	0.051	0.053	0.048	0.046
	PM ₁₀	0.110	0.109	0.107	0.098	0.100	0.114
Wu Xi	SO ₂	0.069	0.059	0.065	0.059	0.046	0.048
	NO ₂	0.038	0.038	0.038	0.033	0.037	0.045
	PM ₁₀	0.1	0.094	0.083	0.079	0.083	0.088
Shang Hai	SO ₂	0.061	0.051	0.055	0.051	0.035	0.029
	NO ₂	0.061	0.055	0.054	0.056	0.053	0.050
	PM ₁₀	0.088	0.086	0.088	0.084	0.081	0.079
Bei Jing	SO ₂	0.050	0.053	0.047	0.036	0.034	0.032
	NO ₂	0.066	0.066	0.066	0.049	0.053	0.057
	PM ₁₀	0.142	0.161	0.148	0.122	0.121	0.121

Establishment of fuzzy comprehensive evaluation system

Establishing factors set: Evaluate a factor, if the number of indicators of this factor is m , denoted respectively as u_1, u_2, \dots, u_m , then m indicators compose a finite set of evaluation factors: $U = \{u_1, u_2, \dots, u_m\}$

Construction of evaluation grade set: Evaluation set is composed of various possible grades of evaluation factors made by evaluators. Suppose, a pollutant has n grades, denoted respectively as V_1, V_2, \dots, V_n , then the evaluation grade set of this pollutant is $V = \{V_1, V_2, \dots, V_n\}$

Weight determination

Weight reflects hazardous differences among environmental indicators. Weight determination should reflect differences between environmental standard values. The greater the hazard, the more important it is, and the bigger the weight. Thus, weights of fuzzy comprehensive evaluation models should be dynamic fuzzy weights, reflecting fuzzy comprehensive dynamic thinking that the bigger the concentration values of environmental indicators, the bigger the weight is.

Weight distribution of each factor is $A = (a_1, a_2, a_3, \dots, a_n)$ here $a_n \geq 0$, and $\sum_{n=1}^n a_n = 1$. Weight set is a collection of remarks along with proportions of every indicator in air quality indicators. To a large degree, it affects the final evaluation results. The weights in this paper are determined by the proportion of each evaluation indicator in total result. Based on marking on the important degree of every indicator that influences air quality, weight value A of each indicator is determined. Weight coefficient distribution is:

$$\overline{S}_{ij} = (S_{1j} + S_{2j} + S_{3j} + S_{4j} + S_{5j} + S_{6j} + S_{7j}) / 7$$

$$W = (C_{SO_2} / \overline{S}_{SO_2}, C_{NO_2} / \overline{S}_{NO_2}, C_{PM_{10}} / \overline{S}_{PM_{10}}) = (W_{SO_2}, W_{NO_2}, W_{PM_{10}})$$

$$A = (W_{SO_2} / \sum W_i, W_{NO_2} / \sum W_i, W_{PM_{10}} / \sum W_i)$$

$$\text{Here } \sum W_i = W_{SO_2} + W_{NO_2} + W_{PM_{10}},$$

Table 2: Weights of different indicators in cities

City	target		
	PM_{10}	NO_2	SO_2
ChengDu	0.151366	0.14033	0.149214
ChongQing	0.150607	0.127752	0.187572
WuHan	0.134105	0.162343	0.12313
NanJing	0.143968	0.141116	0.131569
WuXi	0.122344	0.106525	0.161488
ShangHai	0.114757	0.153695	0.129267
BeiJing	0.182853	0.168239	0.11776

Air quality assessment standard

There are clear provisions on ambient air quality in China, such as function division, standard grading, pollutants. In this paper, the concentration limit of ambient air pollutant in national standard is introduced to classify air quality into five categories, excellent, good, qualified, polluted, and severely polluted, and the specific distribution are in the following:

Table 3 Air quality assessment standards

rank	target		
	PM_{10}	NO_2	SO_2
Excellent	(0,0.04)	(0,0.02)	(0,0.04)
Good	(0.04,0.10)	(0.02,0.06)	(0.04,0.06)
Qualified	(0.10,0.15)	(0.06,0.10)	(0.06,0.08)
Polluted	(0.15,0.30)	(0.10,0.20)	(0.08,0.16)

Fuzzy evaluation definition

Let a fuzzy subset A belong to a given universe U, for any element $X \in U$, there is always a function $\mu_A(x) = [0,1]$, which indicates the degree of x belonging to A, then $\mu_A(x)$ refers to degree of membership of x to A.

Relativity in single indicator classification standard and asynchronization in category distribution lead to fuzziness in air quality assessment. From the perspective of Fuzzy Mathematics, for air samples in a city, there are only differences in degrees of which grade the air quality should be evaluated, but no absolute limits. For such degrees, degree of membership in Fuzzy Mathematics can be adopted to define it.

Taking a city's air as a sample, and the degree of "meeting a standard of air quality" as assessment basis, degree of membership can be defined as follows: if the degree of the air sample meeting k level air quality standard is $\mu(k)$, then $\mu(k)$ is the degree of membership of air quality c to k level.

Membership function

Determination principles of membership function are: 1) its fuzzy set should be convex fuzzy set, usually triangle or trapezoid; 2) it is usually symmetrical and balanced; 3) it should avoid improper coincidence, and comply with the normal semantic order; 4) points in universe should belong to at least one membership function, two at most; 5) for the same point, it should not has two maximum degrees of membership.

Through analyzing the data, it is concluded that the distribution of air quality grades are in line with trapezoid fuzzy distribution. Thus, the membership functions are determined as follows:

Deflection minor type:

$$\mu_{ij} = \begin{cases} 1 & x \leq a \\ \frac{b-x}{b-a} & a \leq x \leq b \\ 0 & x > b \end{cases}$$

Middle type:

$$\mu_{ij} = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ \frac{d-c}{d-c} & x < a, x \geq d \\ 0 & \end{cases}$$

Deflection major type:

$$\mu_{ij} = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

Fuzzy relationship matrixes

Based on membership functions and air quality assessment standards, a factor is taken to evaluate to determine the related degree K_{ij} of evaluation object to elements in evaluation set. K_{ij} Refers to the related degree of the j th pollutant of the i th grade. In this way, fuzzy relationship matrixes of air quality in every region are obtained:

Chengdu:
$$\mu_1 = \begin{bmatrix} 0 & 0 & 0.28 & 0.72 & 0 \\ 0 & 0.889286 & 0.110714 & 0 & 0 \\ 0 & 0.55 & 0.45 & 0 & 0 \end{bmatrix}$$

Chongqing:
$$\mu_2 = \begin{bmatrix} 0 & 0 & 0.268571 & 0.731429 & 0 \\ 0 & 0 & 0.246429 & 0.753571 & 0 \\ 0 & 0.321429 & 0.678571 & 0 & 0 \end{bmatrix}$$

Wuhan:
$$\mu_3 = \begin{bmatrix} 0 & 0 & 0.02 & 0.98 & 0 \\ 0 & 0.646429 & 0.353571 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \end{bmatrix}$$

Nanjing:
$$\mu_4 = \begin{bmatrix} 0 & 0 & 0.168571 & 0.831429 & 0 \\ 0 & 0.15 & 0.85 & 0 & 0 \\ 0 & 0.564286 & 0.435714 & 0 & 0 \end{bmatrix}$$

Wuxi:
$$\mu_5 = \begin{bmatrix} 0 & 0.842857 & 0.157143 & 0 & 0 \\ 0 & 0 & 0.202857 & 0.797143 & 0 \\ 0.935714 & 0.064286 & 0 & 0 & 0 \end{bmatrix}$$

Shanghai:
$$\mu_6 = \begin{bmatrix} 0 & 0.77381 & 0.22619 & 0 & 0 \\ 0 & 0.703571 & 0.296429 & 0 & 0 \\ 0 & 0.792857 & 0.207143 & 0 & 0 \end{bmatrix}$$

Beijing:
$$\mu_7 = \begin{bmatrix} 0 & 0 & 0 & 0.051429 & 0.948571 \\ 0 & 0.596429 & 0.403571 & 0 & 0 \\ 0 & 0 & 0.57143 & 0.942857 & 0 \end{bmatrix}$$

Fuzzy calculations of weight coefficients and fuzzy relationship matrixes

Make fuzzy complex operation on weight index fuzzy subset A and fuzzy relationship matrix R:

$$M = A^*R_{ij} = \{a_1, a_2, a_3\} \cdot \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \end{pmatrix} = \{b_1, b_2, b_3\}$$

b_j , approached by the j th calculation of A on R, refers to the related degree of grade fuzzy subset of being evaluated subject. If the range of related degree is $(0,1)$, then its grade is the final assessment grade.

Through calculating, fuzzy related degrees of air quality in Chengdu, Chongqing, Wuhan, Nanjing, Wuxi, Shanghai and Beijing are:

$$M_1 = (0, 0.2099, 0.1221, 0.1090, 0), M_2 = (0, 0.063, 0.1992, 0.2064, 0), M_3 = (0, 0.2219, 0.0662, 0.1314, 0), \\ M_4 = (0, 0.0954, 0.2015, 0.1197, 0), M_5 = (0.1511, 0.1135, 0.0408, 0.0849, 0), M_6 = (0, 0.2994, 0.0983, 0, 0), \\ M_7 = (0, 0.1003, 0.0746, 0.1204, 0.1734)$$

Therefore, air quality grade assessments of every region are in the following: for air quality in Chengdu, the excellent degree of membership is 0, good degree is 0.2099, qualified degree is 0.1221, polluted degree is 0.1090, and severely polluted degree is 0. The good degree of membership is the biggest one, so comprehensive evaluation grade of Chengdu's air quality is good. Accordingly, evaluation grades of air quality in Chongqing, Wuhan, Nanjing, Wuxi, Shanghai and Beijing are respectively polluted, good, qualified, excellent, good and severely polluted.

Table 4: Fuzzy comprehensive evaluation results of every city

City	Cheng Du	Chong Qing	Wu Han	Nan Jing	Wu Xi	Shang Hai	Bei Jing
Rank	Good	Polluted	Good	Qualified	Excellent	Good	Severely polluted

Gray clustering analysis

Standardization of sample data: In order to avoid the influence of different dimensions on calculation results,

indicators should be handled in dimensionless process. The formula of dimensionless process is $u = \frac{u_{ij}}{\sum u_{ij}}$. Here u_{ij} indicates the source data influencing indicators, u refers to dimensionless data after processing.

Table 5: Processing data of annual average concentration of air pollutant in seven cities

City	target	2005	2006	2007	2008	2009	2010
Cheng Du	SO ₂	0.172	0.198	0.167	0.159	0.126	0.098
	NO ₂	0.137	0.146	0.137	0.137	0.146	0.154
	PM ₁₀	0.144	0.155	0.153	0.139	0.139	0.139
Chong Qing	SO ₂	0.231	0.149	0.151	0.133	0.129	0.108
	NO ₂	0.206	0.148	0.145	0.135	0.132	0.114
	PM ₁₀	0.179	0.151	0.140	0.136	0.134	0.132
Wu Han	SO ₂	0.137	0.140	0.143	0.156	0.159	0.137
	NO ₂	0.126	0.109	0.107	0.116	0.274	0.131
	PM ₁₀	0.184	0.157	0.154	0.153	0.051	0.149
Nan Jing	SO ₂	0.131	0.152	0.184	0.169	0.157	0.102
	NO ₂	0.153	0.150	0.145	0.142	0.148	0.134
	PM ₁₀	0.159	0.145	0.144	0.141	0.129	0.132
Wu Xi	SO ₂	0.178	0.164	0.140	0.154	0.140	0.109
	NO ₂	0.155	0.140	0.140	0.140	0.122	0.137
	PM ₁₀	0.183	0.155	0.146	0.129	0.122	0.129
Shang Hai	SO ₂	0.163	0.181	0.151	0.163	0.151	0.104
	NO ₂	0.159	0.156	0.141	0.138	0.143	0.136
	PM ₁₀	0.164	0.145	0.142	0.145	0.139	0.134
Bei Jing	SO ₂	0.179	0.163	0.173	0.153	0.117	0.111
	NO ₂	0.166	0.154	0.154	0.154	0.114	0.124
	PM ₁₀	0.155	0.147	0.167	0.154	0.127	0.126

Data standardization and determination of clustering weight coefficient ω_{ij} : In the classic gray clustering model,

index weight and actual weight are overall considered. However, in calculating clustering coefficient, actual sample data weight reflects on definite weighted function rather than weight coefficient. Based on actual sample data weight γ_{ij} (The i th indicator is corresponding to actual weight of the 1st sample.) And evaluation standard weight η_{ij} , making two weights superposition, sample data weight is highlighted to make evaluation result closer to objective grade of sample data.

$$\gamma_{ij} = \frac{u_{ij}}{\sum_{i=1}^n u_{ij}}$$

$$\eta_{ij} = \frac{c_{ij}}{\sum_{j=1}^n c_{ij}}$$

$$\omega_{ij} = \gamma_{ij} * \eta_{ij}$$

Table 6: Weight coefficients of annual concentration of air pollutants in Beijing

Year	2004	2006	2008	2009	2010
SO ₂	0.174	0.139	0.151	0.149	0.198
NO ₂	0.089	0.134	0.146	0.143	0.127
PM ₁₀	0.123	0.146	0.146	0.143	0.159

Based on weight value of sample data of every pollutant, the one with the maximum weight is selected as the primary pollutant.

Constriction of definite weighted functions: In classic gray clustering model, liner definite weighted functions are used to whiten data, but the interaction in adjacent level are ignored. Thus, clustering evaluation results are higher than the real ones, and the accuracy and sensitivity are a bit lower. Gray clustering model based on improvement of definite weighted functions of exponential type is just the opposite. The evaluation results are more or less corresponding to the real, and the accuracy and sensitivity are enhanced.

$$f_{i1}(u_{1i}) = \begin{cases} 1 & , u_{1i} \in (0, b_{11}); \\ \frac{b_{11}-u_{1i}}{b_{11}} & , u_{1i} \in (b_{11}, +\infty); \end{cases}$$

$$f_{ij}(u_{ij}) = \begin{cases} e^{-\frac{u_{ij}-b_{i(j-1)}}{u_{ij}}} & , u_{ij} \in (0, b_{i(j-1)}); \\ 1 & , u_{ij} \in (b_{i(j-1)}, b_{ij}); \\ e^{-\frac{b_{ij}-u_{ij}}{b_{ij}}} & , u_{ij} \in (b_{ij}, +\infty); \end{cases}$$

$$f_{i1}(u_{1i}) = \begin{cases} e^{-\frac{u_{1i}-b_{1(m-1)}}{u_{1i}}} & , u_{1i} \in (0, b_{1(m-1)}); \\ 1 & , u_{1i} \in (b_{1(m-1)}, b_{1j}); \end{cases}$$

Determination of clustering coefficient σ_{ij} : $\sigma_{ij} = \sum_{i=1}^m f_{ij}(u_{1i})\omega_{ij}$. Gray matrix is calculated to obtain clustering coefficient of air quality for each sample.

Determination of comprehensive evaluation grade: $\sigma_{j^*} = \max \{ \sigma_1, \sigma_2, \sigma_3, \dots, \sigma_m \}$. According to maximizing principle, sample air quality grades are determined.

$$a_j = \frac{\sum_{i=1}^n \frac{i}{n} b_{ij}}{\sum_{j=1}^m \sum_{i=1}^n \frac{1}{n} b_{ij}}$$

Sequence of air quality: a_j is the standard air quality weight of grade j, then the weight vector is $a = (a_1, a_2, a_3, \dots, a_n)$. Make it multiply gray clustering coefficient matrix, sequencing coefficients of urban air quality are obtained, thus, the sequence of sample cities can be achieved.

Gray clustering model evaluation result is as Table 7 shows. Through the analysis results, the sequence of air quality of each city is obvious.

Table 7: Main urban air quality grey clustering results and sorting

city	2005	2006	2007	2008	2009	2010	order
						ratio	
chengdu rank	0.343 III	0.34 III	0.339 III	0.314 III	0.308 II	0.326 II	5
chongqing rank	0.341 III	0.340 III	0.335 III	0.333 III	0.309 III	0.304 II	6
wuhan rank	0.305 II	0.305 II	0.316 II	0.318 III	0.317 II	0.308 II	2
nanjing rank	0.320 III	0.339 III	0.334 III	0.325 II	0.328 II	0.303 II	4
wuxi rank	0.330 III	0.316 III	0.305 III	0.294 II	0.301 II	0.320 II	3
shanghai 等级	0.321 III	0.321 II	0.323 II	0.319 II	0.328 II	0.336 II	1
beijing rank	0.366 III	0.341 III	0.324 II	0.336 III	0.366 III	0.321 II	7

MODEL TEST

According to the above models, the air quality is excellent in Wuxi, good in Chengdu, Wuhan and Shanghai, qualified in Nanjing, but only air quality in Beijing is severely polluted.

In order to test the accuracy of fuzzy evaluation theory on grading air quality, SPSS cluster analysis is adopted to check grades of these seven cities from a macro perspective.

There are some classification methods of SPSS cluster analysis, such as two-step clustering (T), K-means clustering (K), system cluster (H), and so on. Two-step clustering is used to solve the problems with complex classifications or mass data, K-means clustering can solve the problems with given classification numbers together with large amount of data, while system cluster is suitable for small-scale cluster analysis without outliers.

More than 30 major cities' air quality indicators are selected to take cluster analysis, because of the large amount of data, K-means clustering method is adopted. Based on the classification of the above cities, seven evaluated cities are sorted. According to the data sample of fuzzy evaluation, by using SPSS, the results are as follows:

Table 8: Final Cluster Centers

target	Cluster				
	1	2	3	4	5
PM_{10}	.139	.115	.047	.092	.070
SO_2	.088	.043	.007	.059	.043

Table 9: Distances between Final Cluster Centers

Cluster	1	2	3	4	5
1		.053	.128	.060	.082
2	.053		.080	.028	.046
3	.128	.080		.070	.051
4	.060	.028	.070		.029
5	.082	.046	.051	.029	

Table 8 and 9 show the final cluster centers and distances between final cluster centers. By continuous iteration, clustering is achieved. The algorithm ends in terminating iteration. Clustering results of major cities are achieved (Table 10).

From the above results, it can be seen that all cities are divided into five categories. For the above seven major cities, Wuxi belongs to the first category, Chengdu, Wuhan and Shanghai belong to the second, Nanjing is the third, and Chongqing and Beijing belong to the fourth and fifth respectively. Meanwhile, Table 7 demonstrates all categories of all big cities. By using clustering analysis, classifications of urban air quality grades can be clearly seen, which is in line with the results of comprehensive evaluation of fuzzy evaluation theory, at the same time, it proves the application of fuzzy evaluation theory in air quality assessment.

Table 10: Cluster Membership

Case Number	City	Cluster	Distance
1	Beijing	5	.014
2	Tianjin	4	.006
3	Shi Jiazhuang	2	.010
4	Taiyuan	4	.021
5	Huhhot	5	.006
6	Shenyang	2	.016
7	Changchun	2	.023
8	Harbin	2	.019
9	Shanghai	2	.018
10	Nanjing	3	.018
11	Hangzhou	2	.016
12	Hefei	2	.033
13	Fuzhou	5	.020
14	Nanchang	4	.012
15	Jinan	2	.024
16	Zhengzhou	4	.011
17	Wuhan	2	.015
18	Changsha	4	.010
19	Guangzhou	5	.009
20	Nanjing	5	.015
21	Haikou	3	.006
22	Chongqing	4	.016
23	Chengdu	2	.013
24	Guiyang	4	.017
25	Kunming	5	.012
26	Lasa	3	.006
27	Xi'an	2	.008
28	Lanzhou	1	.019
29	Xining	2	.018
30	Yinchuan	4	.020
31	Urumqi	1	.019
32	Wuxi	1	.018

IMPROVEMENTS OF MODELS

The weight coefficient of indicators plays an important role in fuzzy comprehensive evaluation. The common weight method often considers single indicator without involving in relationships between objects to be evaluated. So it cannot describe the impacts on weight distribution among indicators due to different degrees between their values. From the information perspective, entropy stands for the amount of valid information in questions provided by each indicator. It can be used to measure the degree of effective information and determine weights. However, entropy does not indicate the important degree coefficient of the indicators in the decision making assessment, but the relative intensity in the sense of competition under the condition that values of indicators are fixed. Therefore, entropy weight method determines weight coefficient. Its essence is using utility of indicators' information, in other words the differences among various things of indicator values, to correct weight. The calculating process is as follows:

(1) Determining the entropy of the J th evaluating indicator

$$h_j = -k \sum_{i=1}^n (f_{i,j} \cdot \ln f_{i,j})$$

Here

$$f_{i,j} = d_{i,j} / \sum_{i=1}^n d_{i,j}; k = 1 / \ln n$$

When $f_{i,j} = 0$, let $f_{i,j} \cdot \ln f_{i,j} = 0$

(2) Calculating entropy weight based on its entropy of the J th evaluating indicator

$$\theta_j = (1 - h_j) / \left(m - \sum_{j=1}^m h_j \right)$$

Here

$$\theta_j \in [0,1], \text{ and } \sum_{j=1}^m \theta_j = 1$$

(3) Using entropy weight to correct matrix of indicator weight coefficient ω

$$\omega'_{i,j} = \theta_j \cdot \omega_{i,j} / \sum_{j=1}^m (\theta_j \cdot \omega_{i,j})$$

For the sake of easily comparing, the former evaluated subjects are selected, that is the air quality of seven major cities. Using entropy weight method to correct weight coefficient, calculating entropy vector h and entropy weight vector θ , new weight coefficients are obtained.

City	target		
	PM_{10}	NO_2	SO_2
ChengDu	0.17636	0.15023	0.13672
ChongQing	0.130657	0.137582	0.157655
WuHan	0.121135	0.134543	0.113456
NanJing	0.162658	0.136716	0.145724
WuXi	0.14675	0.114525	0.143678
ShangHai	0.134757	0.143085	0.134672
BeiJing	0.127683	0.183319	0.168095

Air pollutions in the seven cities are reevaluated through fuzzy comprehensive evaluation method. Air quality assessment grade of Chengdu is good, and the grades in Chongqing, Wuhan, Nanjing, Wuxi, Shanghai and Beijing are severely polluted, good, qualified, excellent, good and severely polluted respectively. This result is basically similar with the former one. However, only the grade of Chongqing is reevaluated as severely polluted.

CONCLUSION

For air quality, the influential factors are complicated, and low accuracy leads to fuzziness in system description. So the assessment results will be more real and reasonable by using fuzzy methods to deal with fuzzy problems. For this reason, fuzzy comprehensive evaluation model can reasonably and correctly classify air quality in major cities. Clustering analysis by using K-means clustering method to cluster urban air quality proves the accuracy of fuzzy comprehensive evaluation. The way that entropy weight determines weight coefficient improves fuzzy comprehensive evaluation to reduce the impact of randomness on selecting sample data, which promotes utilization of information and reliability of results to make evaluation more accurate.

Fuzzy comprehensive evaluation is direct, perfect and easy to operate so it cannot only use for air quality assessment, but other complex and fuzzy grades distribution problems. This method has practical meaning for this sort of problems.

Acknowledgment

Supported by Scientific and Technological Research Project of Institutions of Higher Education in Hebei Province. Z2012054.

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