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Analytical Solutions of SIRS-SI Malaria Disease Model Using HPM

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ABSTRACT

In this paper, we discussed the transmission of malaria disease in the work of an SIRS-SI model with treatments are given to humans and mosquitoes. Furthermore, we investigated the Homotopy Perturbation Method (HPM) to construct the approximate analytical solution of the simultaneous non linear ordinary differential equations arise in this model. A stability analysis was then performed and MATLAB numerical simulation was provided to clarify the result. These analytical solutions represent a significant simplification of the system's description allowing easy curve fitting to experiment. Analytical results are compared with simulation results and satisfactory agreement is noted.

Key words: HPM, Malaria Disease, SIRS-SI model.

INTRODUCTION

The general epidemic model with some effective features is described in [1, 2]. The epidemiological impact of immunity to malaria has been investigated in [3, 4]. The effect of vaccines for malaria has been investigated in [5]. The disease modification idea has been discussed in [5]. However the intensity of malaria transmission changes over the evolution of drug resistance is explained in [6]. In [7], the different fluctuation of the mosquito density is investigated. In [8], the authors considered the treatment and vaccination strategies on the transmission of malaria. A mathematical model for malaria treatment and drug resistance is described in [9]. A model interaction with constant immigration in human population and infective immigrants has been depicted in [10].

The dynamics of a malaria transmission model in environment is dealt in [11]. The Modelling of malaria transmission by considering the human population is exhibited in [12], where it is assumed that the individuals recovered from malaria can act as infectives for susceptibles mosquitoes. The idea of reservoir class is also incorporated in [13]. This method of control is cheap and is being used in many part of the area. But the disease is endemic. The larvivorous fish to control malaria by decreasing the larvae population is developed in [14]. The optimal control approach is used to minimize the infectives rate of mosquito. The paper [15] discussed on the study of optimal control on vaccination program. In this paper, we formulated a non-linear differential equations model by malaria transmission. We found the HPM and analyzed the stability of this equilibrium [16-22].

In this current work, we discuss the effectiveness of the use of drugs in a malaria transmission model. Modification of the model is done by considering the assumption that humans belong to recovered class have possibility to be susceptible, i.e., we consider a SIRS-SI model. Moreover, we also consider the application of vaccine and spraying as introduced in malaria transmission. Stability analysis is then performed to reveal the effects of treatments on population dynamics. Additionally we also propose the use of the HPM in providing an approximate solution of the model.

NOMENCLATURE

S_h	Susceptible human
I_h	Infected human
R_h	Recovered human,
S_m	Susceptible mosquito
I_m	Infected mosquito
λ_h	rate of constant
$a\beta_1$	rate of blood transfusion
$b\beta_2$	rate of infected mosquito bite
$c\beta_3$	rate of humans
θ	rate of vaccination
μ_h, μ_m	rate of death
η	rate of congenital
$k\gamma$	rate of anti-malarial drugs
α	rate of death due to malaria
λ_m	rate of susceptible class with a constant
ρ	rate of spraying

EXPERIMENTAL SECTION

Malaria is a common and serious disease. It is reported that the incidence of malaria in the world may be in the order of 300 million clinical cases each year. Malaria mortality is estimated at almost 2 million deaths worldwide per year. The vast numbers of malaria deaths occur among young children in Africa, especially in remote rural areas. In addition, an estimated over 2 billion people are at risk of infection, no vaccines are available for the disease. Malaria is transmitted to humans through the bite of an infected female Anopheles mosquito, following the successful sporozoite inoculation; plasmodium falciparum is usually first detected 7-11 days. This is followed after few days of the bites, by clinical symptoms such as sweats, shills, pains, and fever. Mosquitoes on the other hand acquire infection from infected human after a blood meal. Although malaria is life-threatening it is still preventable and curable if the infected individual seek treatment early. Prevention is usually by the use of insecticide treated bed nets and spraying of insecticide but according to the World Health Organization position statement on insecticide treated mosquito nets, the insecticide treated bed nets, long-lasting insecticide nets, indoor residual spraying, and the other main method of malaria vector control, may not be sufficiently effective alone to achieve and maintain interruption of transmission of malaria, particularly in holo-endemic areas of Africa.

Mathematical Modeling

In constructing the model we employ the following assumptions. Compartmental diagram of the model is illustrated in Fig.1 and its dynamical equations are formulated in Aron, J.L. [3].

Human population model:

$$\frac{dS_h}{dt} = \lambda_h + \sigma R_h - (a\beta_1 I_h + b\beta_2 I_m)S_h - (\theta + \mu_h)S_h \quad (1)$$

$$\frac{dI_h}{dt} = \eta I_h + (a\beta_1 I_h + b\beta_2 I_m)S_h - (\mu_h + \alpha + k\gamma)I_h \quad (2)$$

$$\frac{dR_h}{dt} = k\gamma I_h - (\mu_h + \sigma)R_h + \theta S_h \quad (3)$$

Mosquito population model:

$$\frac{dS_m}{dt} = \lambda_m - (c\beta_3 I_h + \mu_m + \rho)S_m \quad (4)$$

$$\frac{dI_m}{dt} = c\beta_3 I_h S_m - (\mu_m + \rho) I_m \tag{5}$$

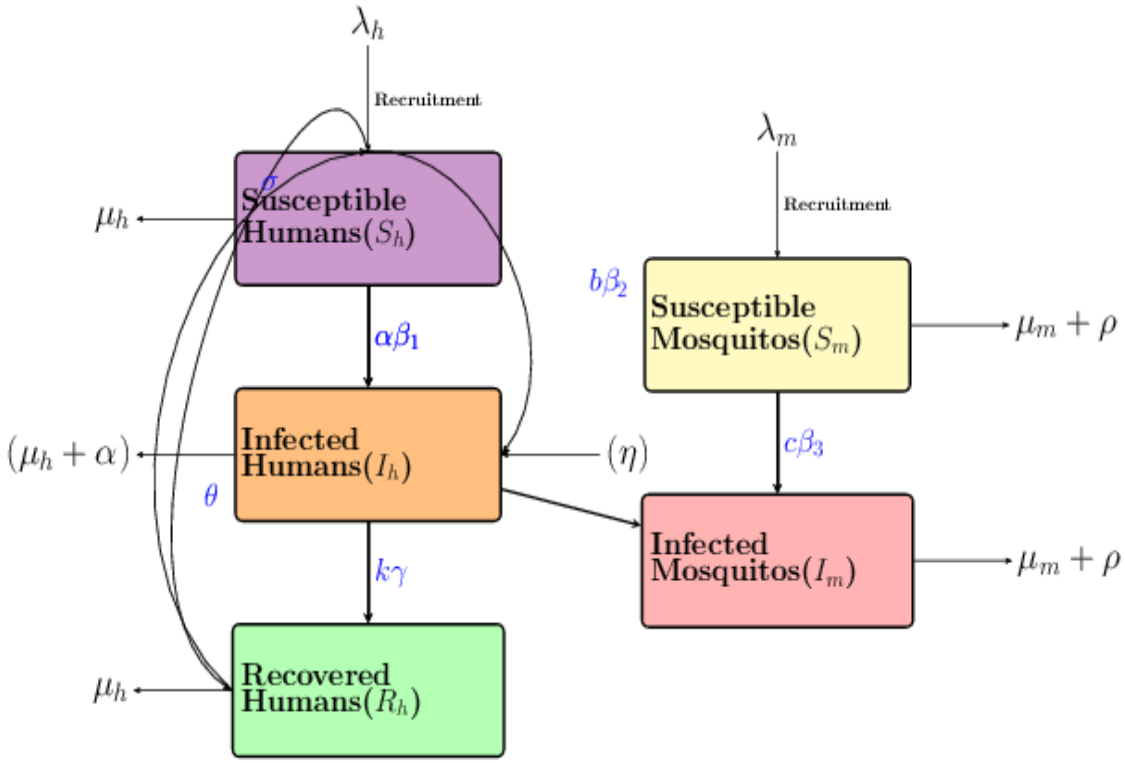

Fig. 1: Compartmental diagram of malaria disease

Homotopy Perturbation Method

Recently, many authors have applied the homotopy perturbation method (HPM) to solve the non-linear problems in physics and engineering sciences [16-18]. Recently this method is also used to solve some of the non-linear problem in physical sciences [18-20]. This method is a combination of homotopy in topology and classic perturbation techniques. He used to solve the Lighthill equation, the Diffusion equation, and the Blasius equation. The HPM is unique in its applicability, accuracy and efficiency [20-22]. The HPM uses the imbedding parameter p as a small parameter, and only a few iterations are needed to search for an asymptotic solution. Using this method, we can obtain the following solution to (1) - (5) (see Appendices B):

$$S_h = \frac{\lambda_h}{\theta_3} + \left(40 - \frac{\lambda_h}{\theta_3}\right) e^{-\theta_3 t} + \left(\frac{A}{\eta - \theta_4} + \frac{B}{\theta_1 + \theta_3} + \frac{C}{\theta_1}\right) e^{-\theta_3 t} - \frac{A}{\eta - \theta_4} e^{(\eta - \theta_4 - \theta_3)t} - \frac{B}{\theta_1 + \theta_3} e^{\theta_1 t} - \frac{C}{\theta_1} e^{(\theta_1 - \theta_3)t} \tag{6}$$

$$I_h = 2e^{(\eta - \theta_4)t} - \left[\frac{D_3}{D_1 - \eta - \theta_4 - \theta_3} + \frac{D_4}{D_1 + \theta_1} + \frac{D_5}{D_1 + \theta_1 - \theta_3}\right] e^{-D_1 t} + tD_2 e^{-D_1 t} + \frac{D_3}{D_1 - \eta - \theta_4 - \theta_3} e^{(-\eta - \theta_4 - \theta_3)t} + \frac{D_4}{D_1 + \theta_1} e^{\theta_1 t} + \frac{D_5}{D_1 + \theta_1 - \theta_3} e^{(\theta_1 - \theta_3)t} \tag{7}$$

$$R_h = \left(-\frac{2k\gamma}{\eta - \theta_4 + \theta_2} + \frac{\theta\lambda_h}{\theta_2\theta_3} + \frac{\theta\left(40 - \frac{\lambda_h}{\theta_3}\right)}{\theta_2 - \theta_3} \right) e^{-\theta_2 t} + \frac{2k\gamma}{\eta - \theta_4 + \theta_2} e^{(\eta - \theta_4)t} - \frac{\theta\lambda_h}{\theta_2\theta_3} - \frac{\theta\left(40 - \frac{\lambda_h}{\theta_3}\right)}{\theta_2 - \theta_3} e^{-\theta_3 t} \quad (8)$$

$$S_m = \frac{\lambda_m}{\theta_1} + \left(500 - \frac{\lambda_m}{\theta_1}\right) e^{-\theta_1 t} + \left(\frac{2c\beta_2\lambda_m}{\theta_1(\eta - \theta_4 - \theta_1)} + \frac{2c\beta_2\left(500 - \frac{\lambda_m}{\theta_1}\right)}{\eta - \theta_4 - 2\theta_1} \right) e^{\theta_1 t} - \frac{2c\beta_2\lambda_m}{\theta_1(\eta - \theta_4 - \theta_1)} e^{(\eta - \theta_4)t} - \frac{2c\beta_2\left(500 - \frac{\lambda_m}{\theta_1}\right)}{\eta - \theta_4 - 2\theta_1} e^{(\eta - \theta_4 - \theta_1)t} \quad (9)$$

$$I_m = 10e^{\theta_1 t} - \left(\frac{2c\beta_2\lambda_m}{\theta_1(\eta - \theta_4 - \theta_1)} + \frac{2c\beta_2\left(500 - \frac{\lambda_m}{\theta_1}\right)}{\eta - \theta_4 - 2\theta_1} \right) e^{\theta_1 t} + \frac{2c\beta_2\left(\frac{\lambda_m}{\theta_1}\right)}{\eta - \theta_4 - \theta_1} e^{(\eta - \theta_4)t} + \frac{2c\beta_2\left(500 - \frac{\lambda_m}{\theta_1}\right)}{\eta - \theta_4 - 2\theta_1} e^{(\eta - \theta_4 - \theta_1)t} \quad (10)$$

RESULTS AND DISCUSSION

In this simulation, the population dynamics are observed in conditions such that $R_0 < 1$. Thus we would like to show the effect of vaccination, anti-malarial drug, and spraying in a situation where the disease doesn't spread. Simulation is performed to demonstrate the effectiveness of the use of anti-malaria drugs on human populations and mosquito populations. In this case, it will be shown that an increase or decrease in the value of the parameter γ can alter the basic reproduction number R_0.

In the human susceptible population, as shown in Fig 2 to Fig 4, the solution of S_h using HPM for fixed values of $\gamma = \rho = \theta = 0.05, 0.1 \text{ and } 0.15$ when the rate of vaccination value and S_h value also increased. If the effectiveness of the use of anti-malarial drugs is increased, then it increases the number of susceptible humans as well as the number of recovered humans, but it decreases the number of infected humans. The use of anti-malarial drugs given to human has also an impact on the mosquito population, as shown in Fig 5 to Fig 7, the solution of I_h using HPM for fixed values of $\gamma = \rho = \theta = 0.05, 0.1 \text{ and } 0.15$ when the rate of spraying increased and I_h decreased in Human susceptible and Mosquito susceptible of anti-malarial drugs, vaccines and spraying.

The same treatment on γ causes a decrease in the number of infected mosquitoes but an increase in the number of susceptible mosquitoes. Increasing or decreasing in the number of humans and mosquitoes in each class tend to be equal to any increase in the effectiveness of the use of anti-malarial drugs. The maximum number of infected humans and mosquitoes occurs at t=25 days. At this point, the effectiveness level of 20% can reduce the number of infected humans up to 23.81% of the total human population and can reduce the number of infected mosquitoes up to 5.88%. It is assumed that infected humans treated with anti-malarial drugs by 10%.

In this case, it will be shown the effect of parameters θ and the basic reproduction number R_0 to the population dynamic. In the human population, as shown in Fig 8 to Fig 10, the solution of R_h using HPM for fixed values of

$\gamma = \rho = \theta = 0.05, 0.1$ and 0.15 when the rate of spraying increased and R_h decreased in Human Infected, Mosquito Infected of anti-malarial drugs, if the effectiveness of the vaccine is improved and the other parameters are fixed, then the number of humans in susceptible and recovered classes is increased. It indirectly causes a decrease in the number of human in infected class. While, in the mosquito population as shown in Fig 11 to Fig 13, the solution of S_m using HPM for fixed values of $\gamma = \rho = \theta = 0.05, 0.1$ and 0.15 when the rate of spraying value and S_m value also increased in Human Infected, Mosquito Infected of vaccines, improvement on vaccine effectiveness indirectly decreases the number of mosquito in infected class.

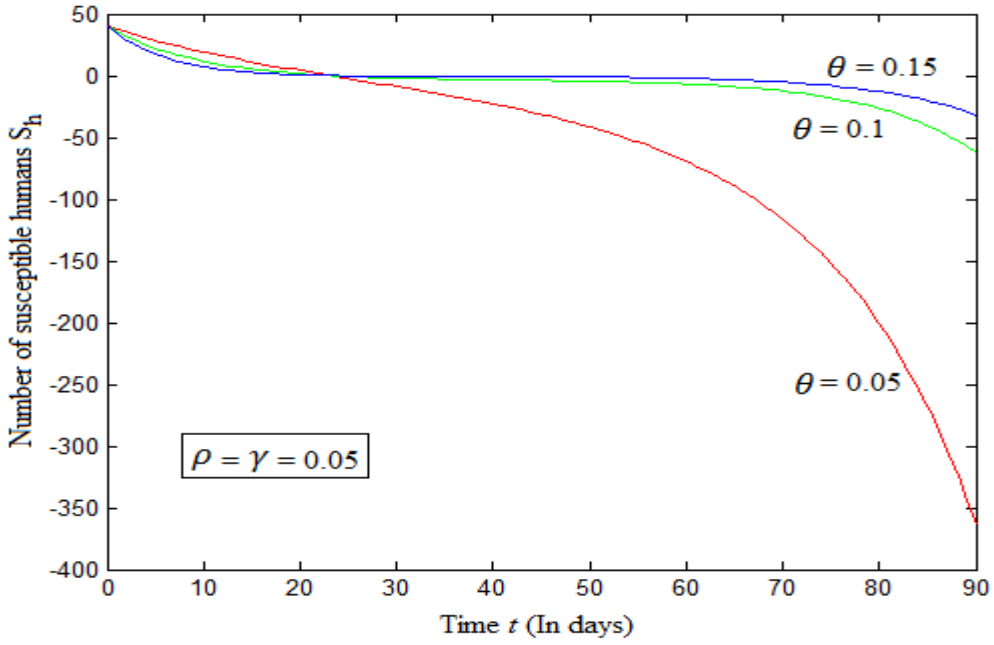

Fig. 2: The solution of S_h using HPM for fixed values of $\gamma = \rho = 0.05$ and $\theta = 0.05, 0.1$ and 0.15

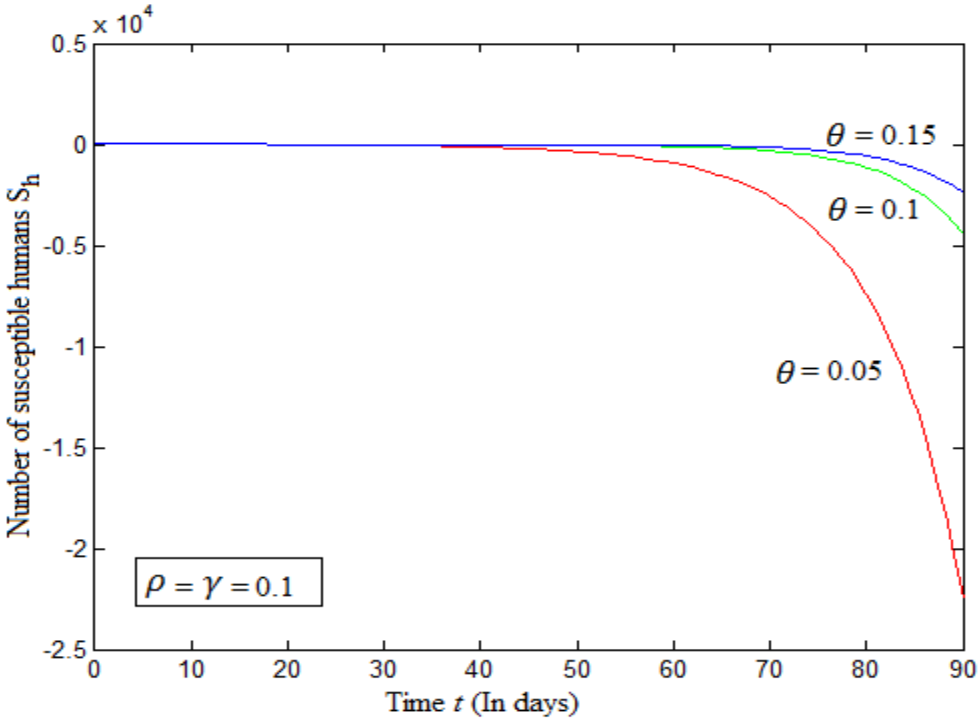

Fig. 3: The solution of S_h using HPM for fixed values of $\gamma = \rho = 0.1$ and $\theta = 0.05, 0.1$ and 0.15

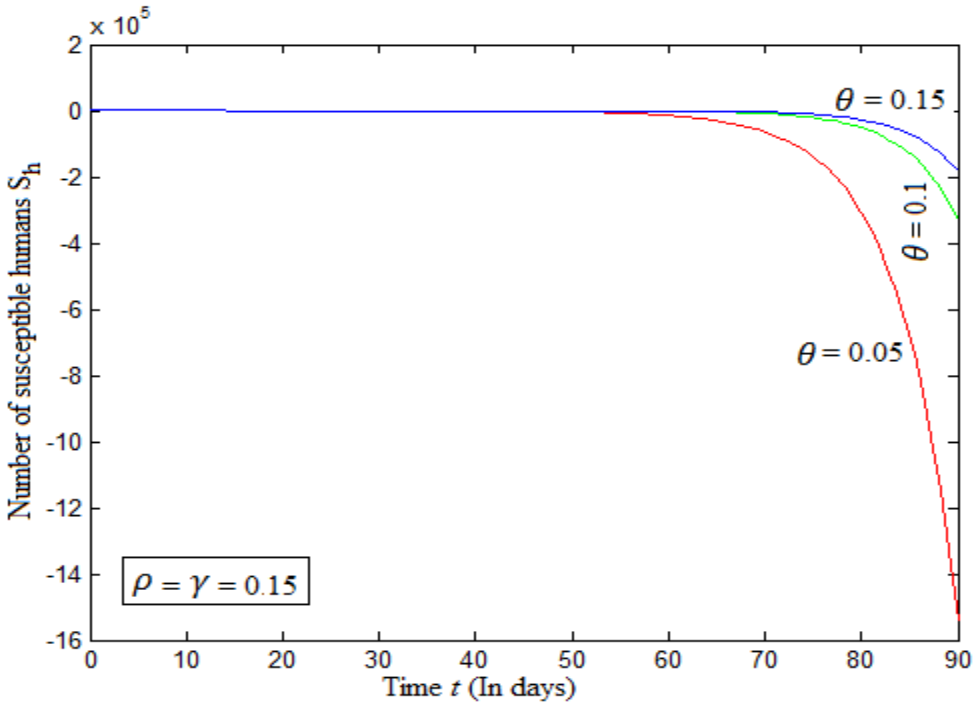

Fig. 4: The solution of S_h using HPM for fixed values of $\gamma = \rho = 0.15$ and $\theta = 0.05, 0.1$ and 0.15

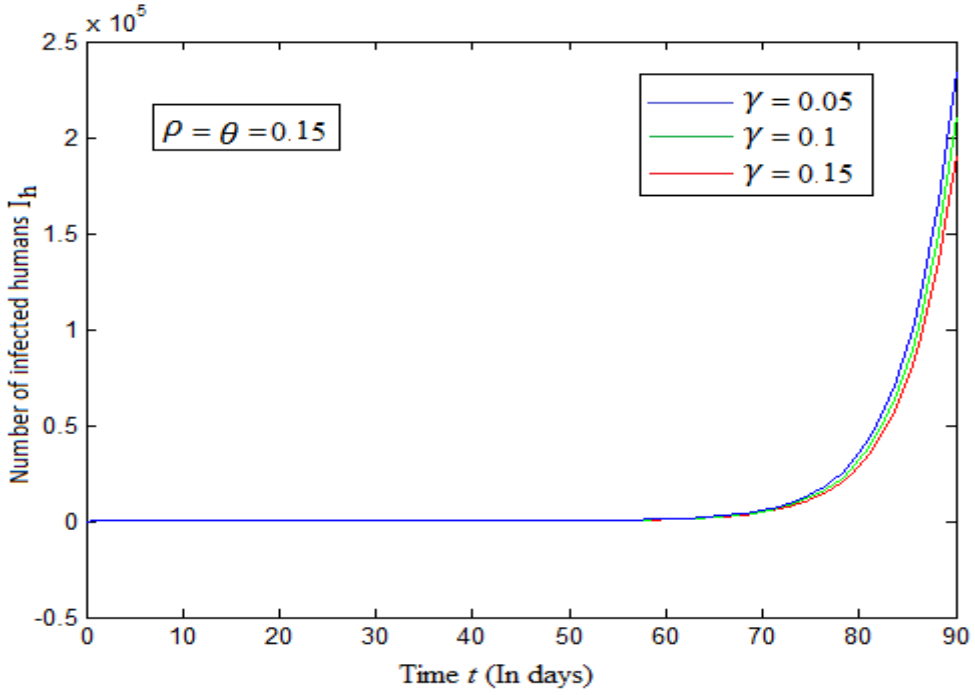

Fig. 5: The solution of I_h using HPM for fixed values of $\theta = \rho = 0.15$ and $\gamma = 0.05, 0.1$ and 0.15

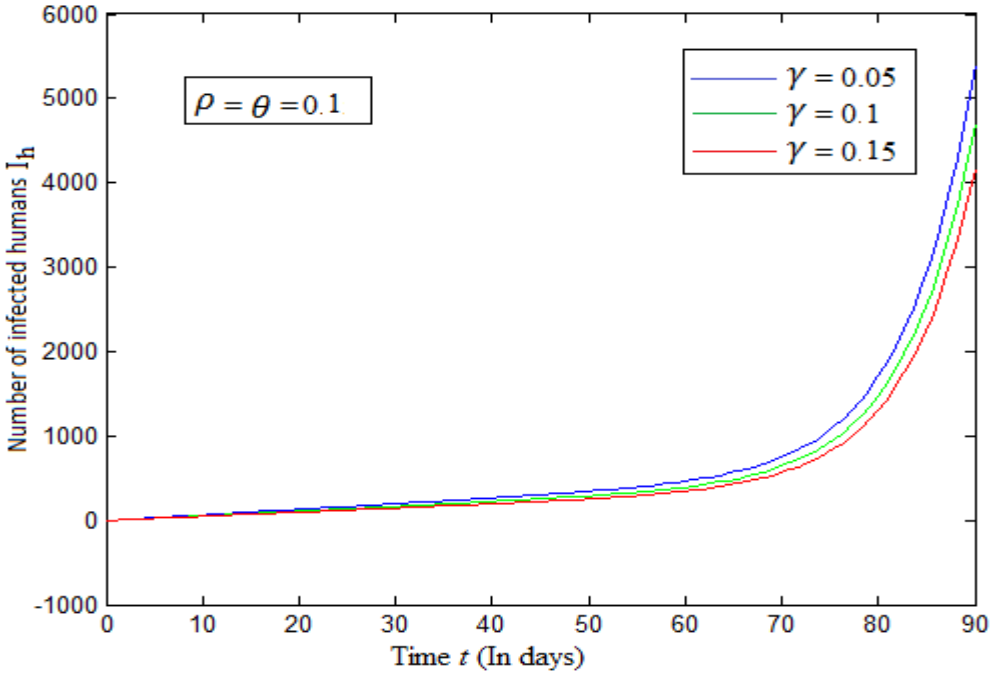

Fig. 6: The solution of I_h using HPM for fixed values of $\theta = \rho = 0.1$ and $\gamma = 0.05, 0.1$ and 0.15

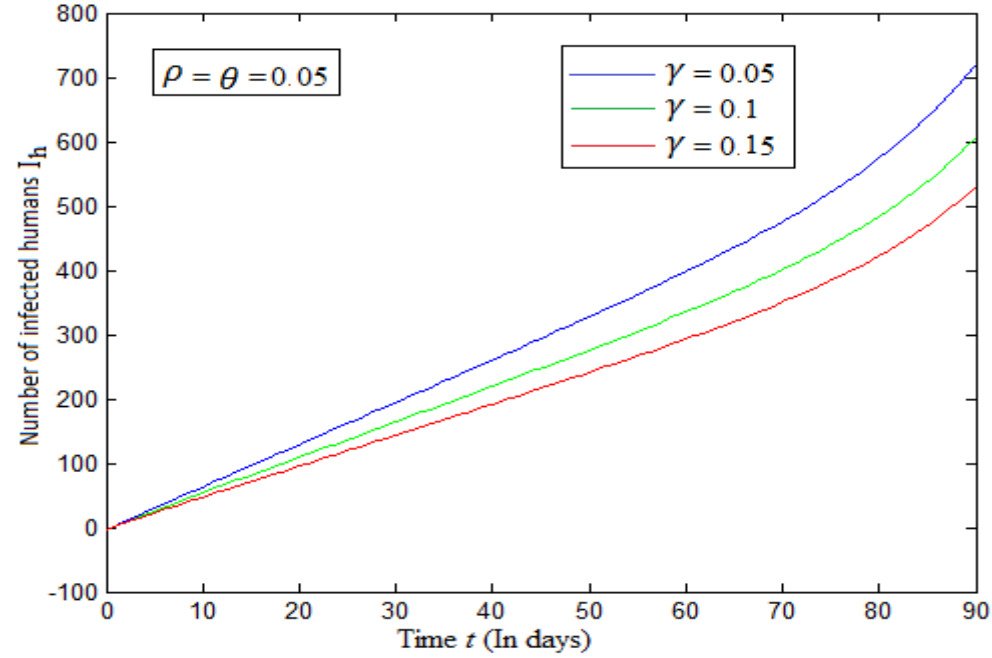

Fig. 7: The solution of I_h using HPM for fixed values of $\theta = \rho = 0.05$ and $\gamma = 0.05, 0.1$ and 0.15

Now we demonstrate the effectiveness of the use of spraying on human populations and mosquito populations. In this case we still assumed that infected humans consume anti-malarial drugs by 10% and we show the effect of parameter ρ and R_0 on population dynamics. In Fig. 5 Human Infected, Mosquito Infected of spraying, we can see that an improvement on the effectiveness of spraying increases the number of susceptible humans, but decreases the numbers of infected mosquitoes and recovered humans. In Fig 14 to Fig 16, the solution of I_m using HPM for fixed values of $\gamma = \rho = \theta = 0.05, 0.1$ and 0.15 when the rate of spraying value and I_m value also increased in Human Recovered of anti-malarial drugs, vaccines and sprayin meanwhile, toward the vector population, the same treatment decreases the population in both classes as spraying is aimed to mosquitoes.

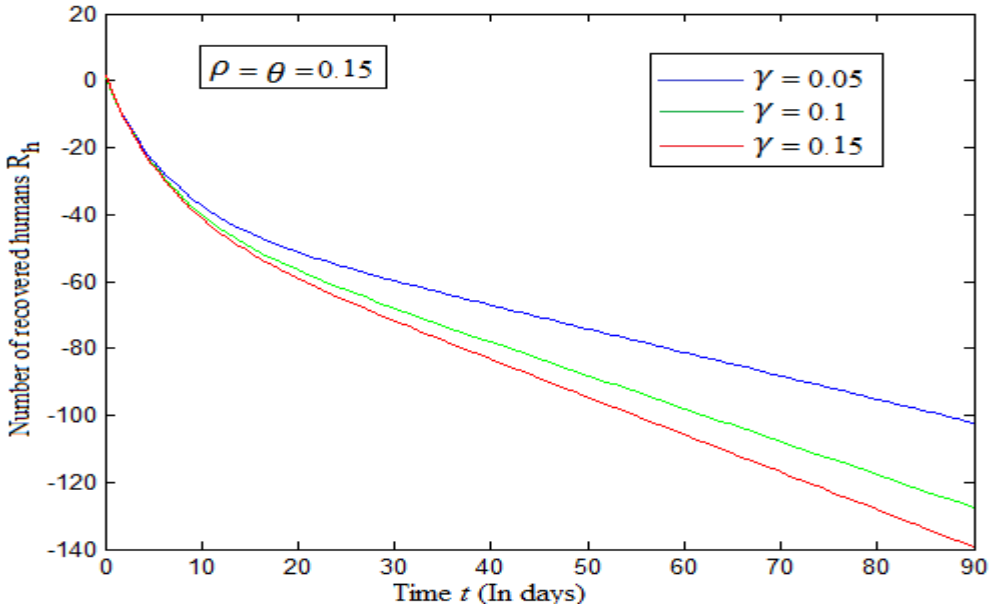

Fig. 8: The solution of R_h using HPM for fixed values of $\theta = \rho = 0.15$ and $\gamma = 0.05, 0.1$ and 0.15

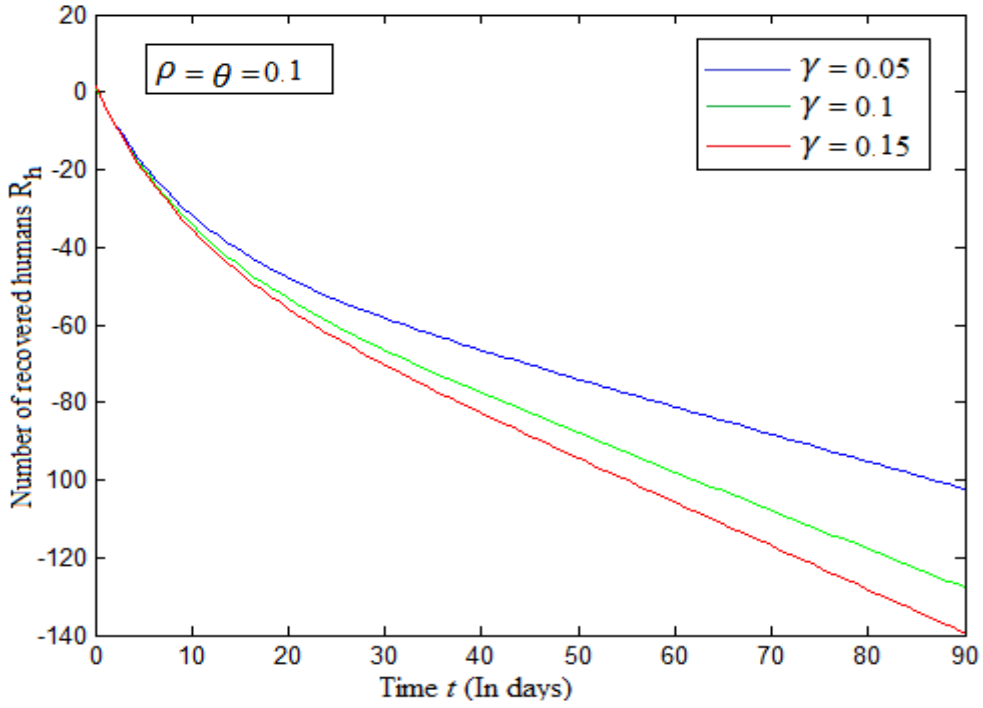

Fig. 9: The solution of R_h using HPM for fixed values of $\theta = \rho = 0.1$ and $\gamma = 0.05, 0.1$ and 0.15

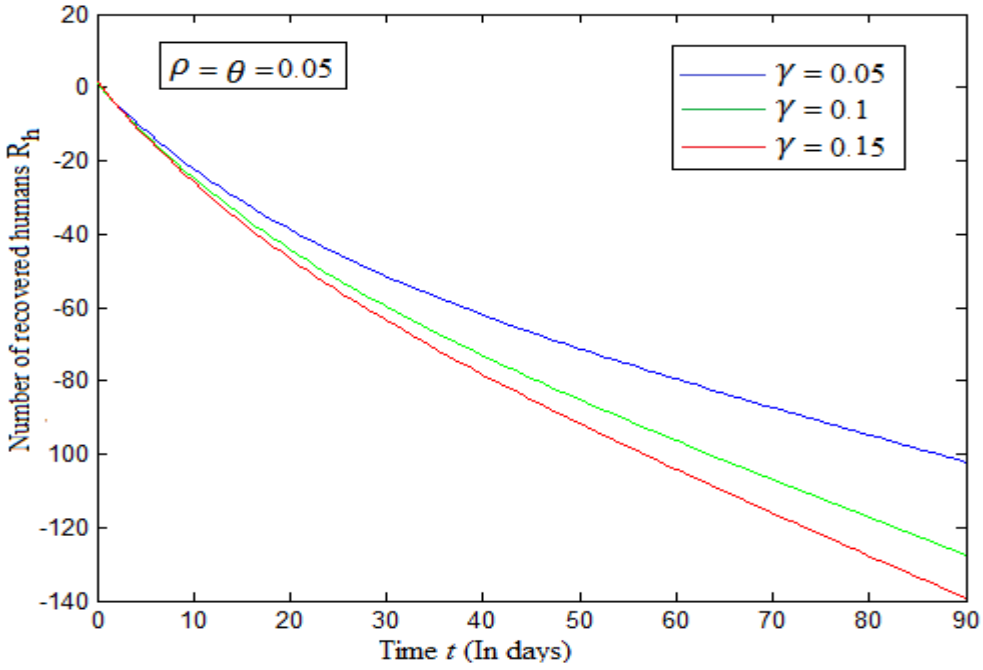

Fig.10: The solution of R_h using HPM for fixed values of $\theta = \rho = 0.05$ and $\gamma = 0.05, 0.1$ and 0.15

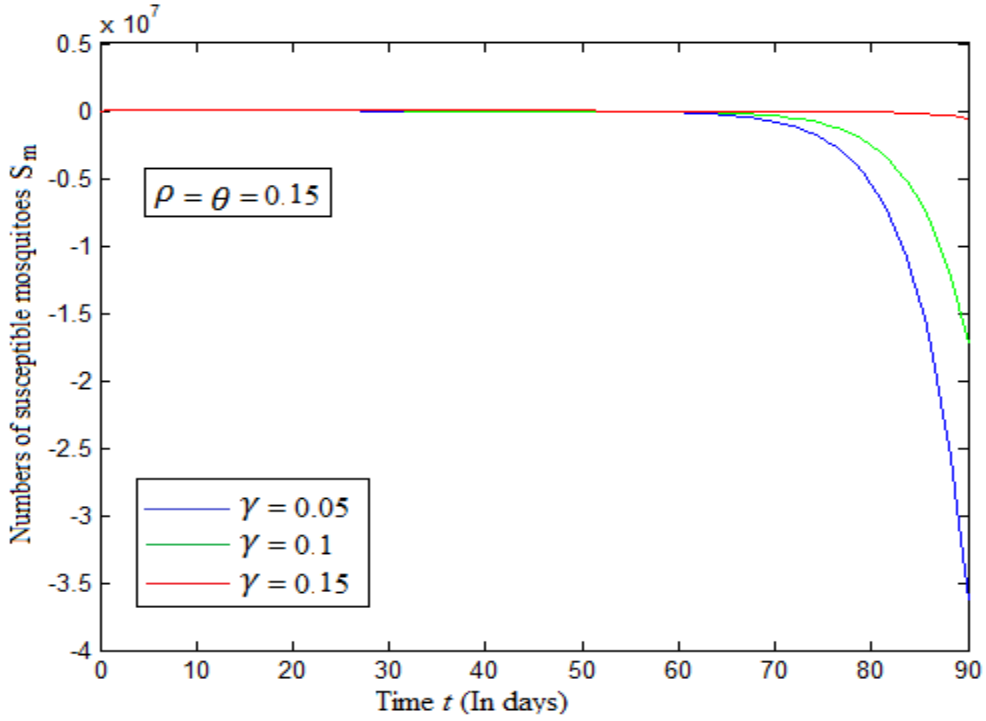

Fig. 11: The solution of S_m using HPM for fixed values of $\theta = \rho = 0.15$ and $\gamma = 0.05, 0.1$ and 0.15

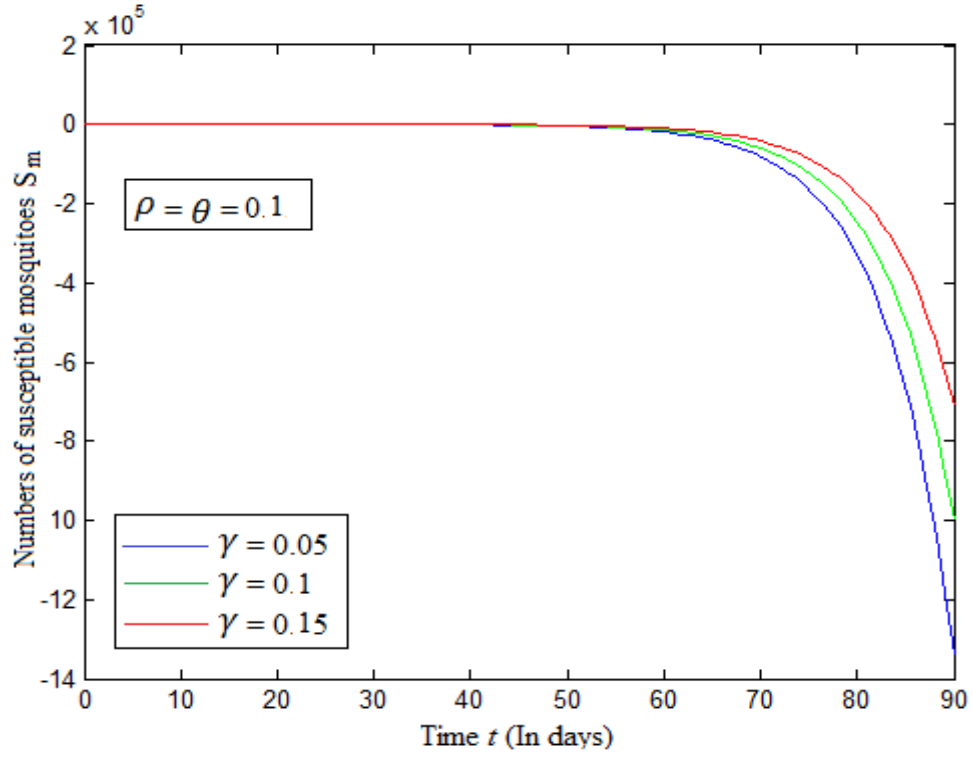

Fig. 12: The solution of S_m using HPM for fixed values of $\theta = \rho = 0.15$ and $\gamma = 0.05, 0.1$ and 0.15

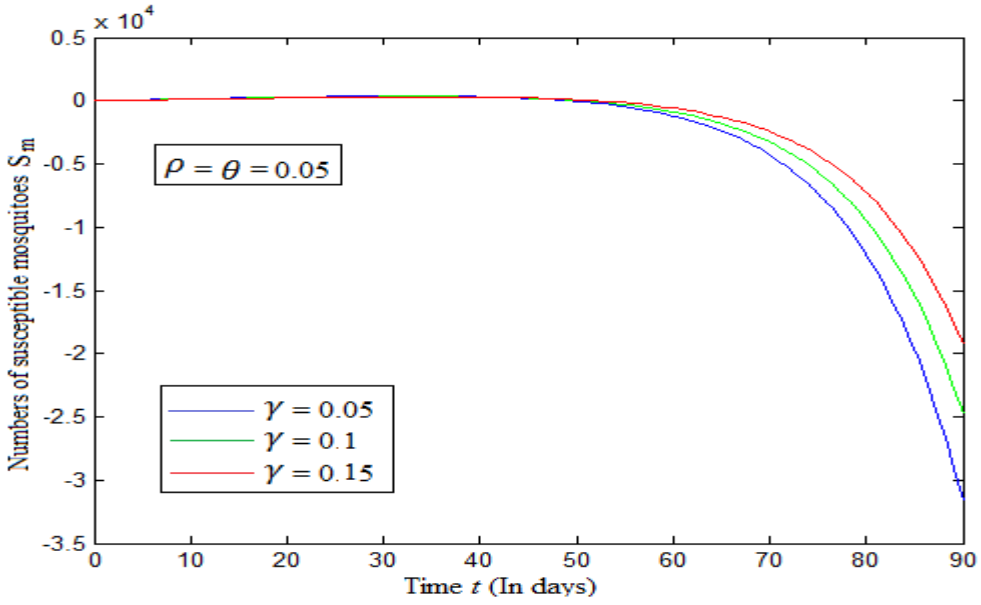

Fig. 13: The solution of S_m using HPM for fixed values of $\theta = \rho = 0.05$ and $\gamma = 0.05, 0.1$ and 0.15

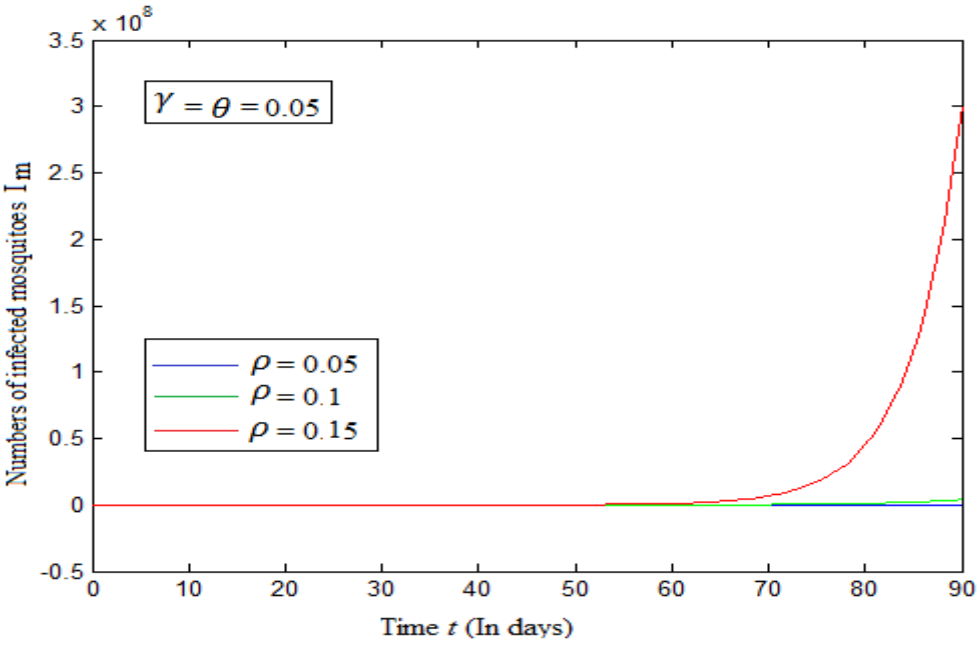

Fig. 14: The solution of I_m using HPM for fixed values of $\theta = \gamma = 0.05$ and $\rho = 0.05, 0.1$ and 0.15

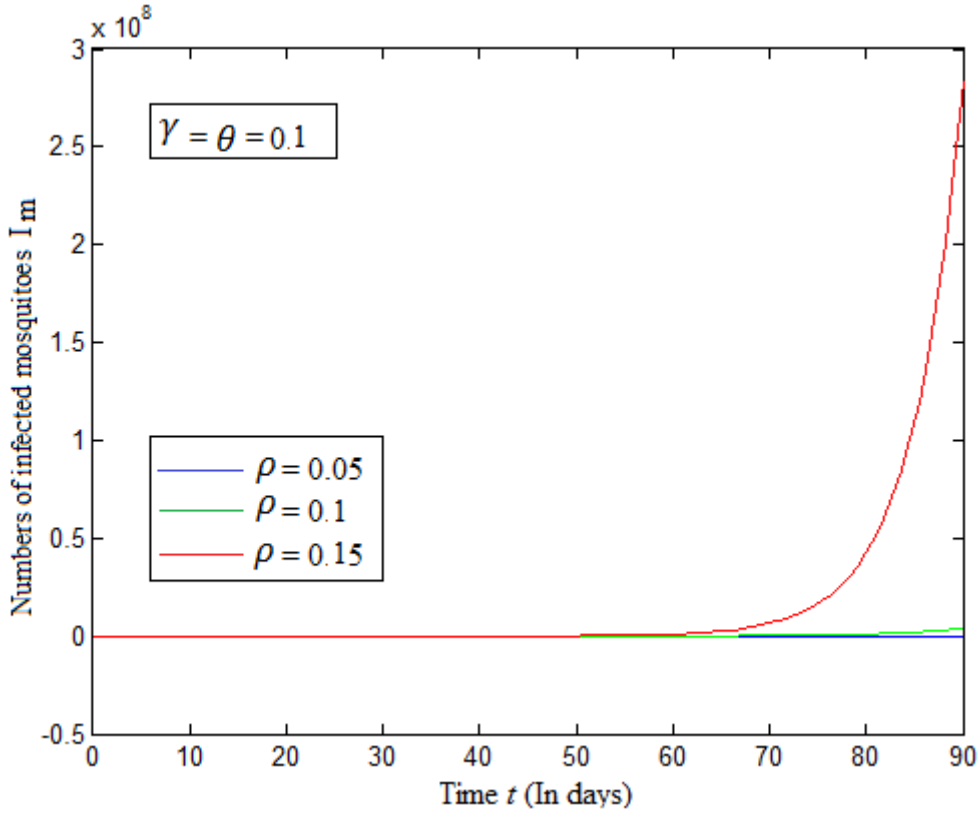

Fig. 15: The solution of I_m using HPM for fixed values of $\theta = \gamma = 0.1$ and $\rho = 0.05, 0.1$ and 0.15

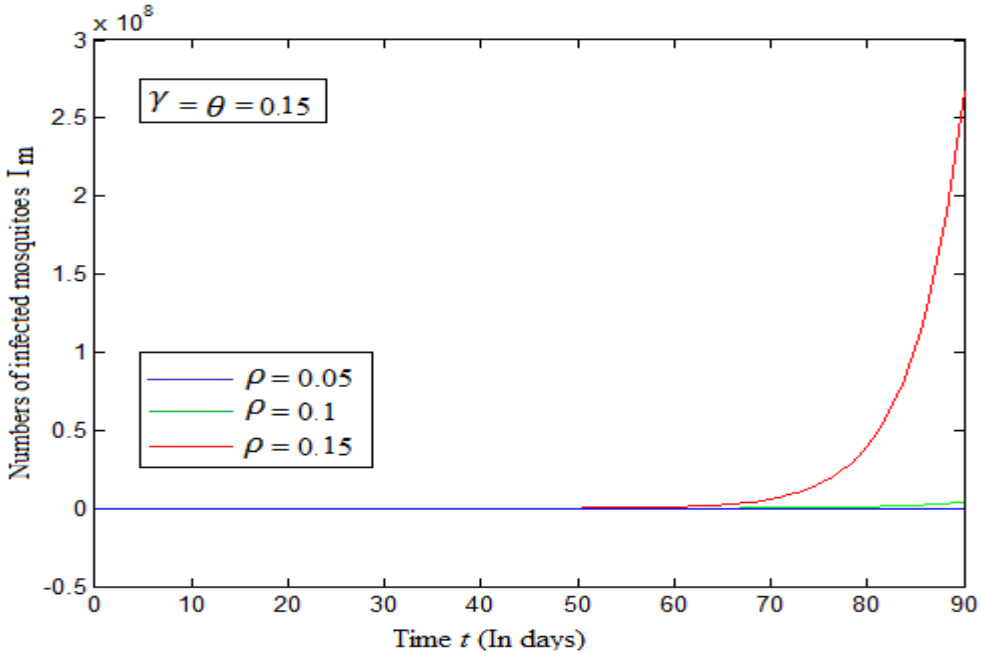

Fig. 16: The solution of I_m using HPM for fixed values of $\theta = \gamma = 0.15$ and $\rho = 0.05, 0.1$ and 0.15

CONCLUSION

In this paper, we have presented an SIRS-SI model of malaria disease. The malaria model non-linear differential equations have been solved analytically and numerically. Simple and approximate dimensionless concentrations are derived by using the HPM for all values of dimensionless parameters. The HPM is an extremely simple method and it is also a promising method to solve other nonlinear equations. This method can be easily extended to find the solution of all other non-linear equations. This present method is quick and efficient and is able to significantly the amount of computations in simulations of this model.

APPENDIX A

Basic Concepts of the Homotopy Perturbation Method

The HPM method has overcome the limitations of traditional perturbation methods. It can take full advantage of the traditional perturbation techniques, so a considerable deal of research has been conducted to apply the homotopy technique to solve various strong non-linear equations. To explain this method, let us consider the following function:

$$D_o(u) - f(r) = 0, \quad r \in \Omega \tag{A1}$$

With the boundary conditions of

$$B_o(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma \tag{A2}$$

Where, D_o is a general differential operator, B_o is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. Generally speaking, the operator D_o can be divided into a linear part L and a nonlinear part N. Eq. (A1) can therefore be written as

$$L(u) + N(u) - f(r) = 0 \tag{A3}$$

By the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0,1] \to \Re$ that satisfies

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[D_o(v) - f(r)] = 0 \tag{A4}$$

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad \text{(A5)}$$

Where $p \in [0,1]$ is an embedding parameter , and u_0 is an initial approximation of Eq. (A1) that satisfies the boundary conditions. From Eqs. (A4) and (A5), we have

$$H(v,0) = L(v) - L(u_0) = 0 \quad \text{(A6)}$$

$$H(v,1) = D_o(v) - f(r) = 0. \quad \text{(A7)}$$

When p=0, Eq. (A4) and Eq. (A5) become linear equations. When p =1, they become non-linear equations. The process of changing p from zero to unity is that of $L(v) - L(u_0) = 0$ to $D_o(v) - f(r) = 0$. We first use the embedding parameter p as a "small parameter" and assume that the solutions of Eqs. (A4) and (A5) can be written as a power series in p :

$$v = v_0 + pv_1 + p^2 v_2 + ... \quad \text{(A8)}$$

Setting $p = 1$ results in the approximate solution of Eq. (A1):

$$u = \lim_{p \to 1} v = \mathrm{v}_0 + v_1 + v_2 + ... \quad \text{(A9)}$$

This is the basic idea of the HPM.

APPENDIX B

Solution of Nonlinear Differential Equations (1) - (5) Using HPM

In this appendix, we indicate how (6) – (10) in this paper are derived. To find the solution of (1) – (5), they can be simplified to

$$\frac{dS_h}{dt} - \lambda_h - \sigma R_h + a\beta_1 S_h I_h + b\beta_2 S_h I_m + \theta_3 S_h = 0 \quad \text{(B1)}$$

$$\frac{dI_h}{dt} - \eta I_h - a\beta_1 S_h I_h - b\beta_2 S_h I_m + \theta_4 I_h = 0 \quad \text{(B2)}$$

$$\frac{dR_h}{dt} - k\gamma I_h + \theta_2 R_h - \theta S_h = 0 \quad \text{(B3)}$$

$$\frac{dS_m}{dt} - \lambda_m + c\beta_3 I_h S_m + \theta_1 S_m = 0 \quad \text{(B4)}$$

$$\frac{dI_m}{dt} - c\beta_3 I_h S_m - \theta_1 I_m = 0 \quad \text{(B5)}$$

where

$$\theta_1 = \mu_m + \rho,\ \theta_2 = \mu_h + \sigma,\ \theta_3 = \mu_h + \theta,\ \theta_4 = \mu_h + \alpha + \gamma k \quad \text{(B6)}$$

In order to obtain the solution of (B1) – (B5), we first construct a homotopy as follows:

$$(1-p)\left[\frac{dS_h}{dt}-\lambda_h+\theta_3 S_h\right]+p\left[\frac{dS_h}{dt}-\lambda_h-\sigma R_h+a\beta_1 S_h I_h+b\beta_2 S_h I_m+\theta_3 S_h\right]=0$$
$$(1-p)\left[\frac{dI_h}{dt}-\eta I_h+\theta_4 I_h\right]+p\left[\frac{dI_h}{dt}-\eta I_h-a\beta_1 S_h I_h-b\beta_2 S_h I_m+\theta_4 I_h\right]=0$$
$$(1-p)\left[\frac{dR_h}{dt}+\theta_2 R_h\right]+p\left[\frac{dR_h}{dt}-k\gamma I_h+\theta_2 R_h-\theta S_h\right]=0 \quad \text{(B7)}$$
$$(1-p)\left[\frac{dS_m}{dt}-\lambda_m+\theta_1 S_m\right]+p\left[\frac{dS_m}{dt}-\lambda_m+c\beta_3 I_h S_m+\theta_1 S_m\right]=0$$
$$(1-p)\left[\frac{dI_m}{dt}-\theta_1 I_m\right]+p\left[\frac{dI_m}{dt}-c\beta_3 I_h S_m-\theta_1 I_m\right]=0$$

The analytical solutions of the system (B1)-(B5) are

$$S_h = S_{h0}+pS_{h1}+p^2S_{h2}+\ldots$$
$$I_h = I_{h0}+pI_{h1}+p^2I_{h2}+\ldots$$
$$R_h = R_{h0}+pR_{h1}+p^2R_{h2}+\ldots \quad \text{(B8)}$$
$$S_m = S_{m0}+pS_{m1}+p^2S_{m2}+\ldots$$
$$I_m = I_{m0}+pI_{m1}+p^2I_{m2}+\ldots$$

Substituting (B8) into (B7) and comparing the coefficient of like powers of p in (B7), we get

$$p^0:\frac{dS_{h0}}{dt}-\lambda_h+\theta_3 S_{h0}=0$$
$$p^0:\frac{dI_{h0}}{dt}-\eta I_{h0}+\theta_4 I_{h0}=0$$
$$p^0:\frac{dR_{h0}}{dt}+\theta_2 R_{h0}=0$$
$$p^0:\frac{dS_{m0}}{dt}-\lambda_m+\theta_1 S_{m0}=0$$
$$p^0:\frac{dI_{m0}}{dt}-\theta_1 I_{m0}=0 \quad \text{(B9)}$$

$$p^1:\frac{dS_{h1}}{dt}-\sigma R_{h0}+a\beta_1 S_{h0}R_{h1}+b\beta_2 S_{h0}I_{m0}+\theta_3 S_{h1}=0$$
$$p^1:\frac{dI_{h1}}{dt}-\eta R_{h1}-a\beta_1 S_{h0}R_{h0}-b\beta_2 S_{h0}I_{m0}+\theta_4 R_{h1}=0$$
$$p^1:\frac{dR_{h1}}{dt}-k\gamma R_{h0}+\theta_2 R_{h1}-\theta S_{h0}=0 \quad \text{(B10)}$$
$$p^1:\frac{dS_{m1}}{dt}+c\beta_3 I_{h0}S_{m0}-\theta_1 S_{m1}=0$$
$$p^1:\frac{dI_{m1}}{dt}-c\beta_3 I_{h0}S_{m0}-\theta_1 I_{m1}=0$$

The initial approximations are as follow

$$S_h(0)=40,\ I_h(0)=2,\ R_h(0)=0,\ S_m(0)=500,\ I_m(0)=10$$
$$S_h(i)=I_h(i)=R_h(i)=S_m(i)=I_m(i)=0, i=1,2,...,\infty \tag{B11}$$

Solving (B9) and using the boundary conditions (B11), we obtain the following results.

$$S_{h0}=\frac{\lambda_h}{\theta_3}+\left(40-\frac{\lambda_h}{\theta_3}\right)e^{-\theta_3 t} \tag{B12}$$

$$I_{h0}=2e^{(\eta-\theta_4)t} \tag{B13}$$

$$R_{h0}=0 \tag{B14}$$

$$S_{m0}=\frac{\lambda_m}{\theta_1}+\left(500-\frac{\lambda_m}{\theta_1}\right)e^{-\theta_1 t} \tag{B15}$$

$$I_{m0}=10e^{\theta_1 t} \tag{B16}$$

Solving (B10) and using the boundary conditions (B11), we obtain the following results.

$$S_{h1}=\left(\frac{A}{\eta-\theta_4}+\frac{B}{\theta_1+\theta_3}+\frac{C}{\theta_1}\right)e^{-\theta_3 t}-\frac{A}{\eta-\theta_4}e^{(\eta-\theta_4-\theta_3)t}-\frac{B}{\theta_1+\theta_3}e^{\theta_1 t}-\frac{C}{\theta_1}e^{(\theta_1-\theta_3)t} \tag{B17}$$

where $A=40a\beta_1,\ B=10b\beta_2\left(\frac{\lambda_h}{\theta_3}\right),\ C=10b\beta_2\left(40-\frac{\lambda_h}{\theta_3}\right)$

$$I_{h1}=-\left[\frac{D_3}{D_1-\eta-\theta_4-\theta_3}+\frac{D_4}{D_1+\theta_1}+\frac{D_5}{D_1+\theta_1-\theta_3}\right]e^{-D_1 t}+tD_2e^{-D_1 t}+\frac{D_3}{D_1-\eta-\theta_4-\theta_3}e^{(-\eta-\theta_4-\theta_3)t}$$
$$+\frac{D_4}{D_1+\theta_1}e^{\theta_1 t}+\frac{D_5}{D_1+\theta_1-\theta_3}e^{(\theta_1-\theta_3)t} \tag{B18}$$

where

$$D_1=\theta_4-\eta,\ D_2=2a\beta_1\left(\frac{\lambda_h}{\theta_3}\right),\ D_3=2a\beta_1\left(40-\frac{\lambda_h}{\theta_3}\right),\ D_4=10b\beta_2\left(\frac{\lambda_h}{\theta_3}\right),\ D_5=10b\beta_2\left(40-\frac{\lambda_h}{\theta_3}\right)$$

$$R_{h1}=\left[-\frac{2k\gamma}{\eta-\theta_4+\theta_2}+\frac{\theta\lambda_h}{\theta_2\theta_3}+\frac{\theta\left(40-\frac{\lambda_h}{\theta_3}\right)}{\theta_2-\theta_3}\right]e^{-\theta_2 t}+\frac{2k\gamma}{\eta-\theta_4+\theta_2}e^{(\eta-\theta_4)t}-\frac{\theta\lambda_h}{\theta_2\theta_3}-\frac{\theta\left(40-\frac{\lambda_h}{\theta_3}\right)}{\theta_2-\theta_3}e^{-\theta_3 t} \tag{B19}$$

$$S_{m1}=\left[\frac{2c\beta_2\lambda_m}{\theta_1(\eta-\theta_4-\theta_1)}+\frac{2c\beta_2\left(500-\frac{\lambda_m}{\theta_1}\right)}{\eta-\theta_4-2\theta_1}\right]e^{\theta_1 t}-\frac{2c\beta_2\lambda_m}{\theta_1(\eta-\theta_4-\theta_1)}e^{(\eta-\theta_4)t}-\frac{2c\beta_2\left(500-\frac{\lambda_m}{\theta_1}\right)}{\eta-\theta_4-2\theta_1}e^{(\eta-\theta_4-\theta_1)t} \tag{B20}$$

$$I_{m1}=\left[-\frac{2c\beta_2\lambda_m}{\theta_1(\eta-\theta_4-\theta_1)}-\frac{2c\beta_2\left(500-\frac{\lambda_m}{\theta_1}\right)}{\eta-\theta_4-2\theta_1}\right]e^{\theta_1 t}+\frac{2c\beta_2\left(\frac{\lambda_m}{\theta_1}\right)}{\eta-\theta_4-\theta_1}e^{(\eta-\theta_4)t}+\frac{2c\beta_2\left(500-\frac{\lambda_m}{\theta_1}\right)}{\eta-\theta_4-2\theta_1}e^{(\eta-\theta_4-\theta_1)t} \tag{B21}$$

According to the HPM, we can conclude that

$$
\begin{aligned}
S_h &= \lim\ p \to 1 S_h(t) = S_{h0} + S_{h1} + S_{h2} + \ldots \\
I_h &= \lim\ p \to 1 I_h(t) = I_{h0} + I_{h1} + I_{h2} + \ldots \\
R_h &= \lim\ p \to 1 R_h(t) = R_{h0} + R_{h1} + R_{h2} + \ldots \\
S_m &= \lim\ p \to 1 S_m(t) = S_{m0} + S_{m1} + S_{m2} + \ldots \\
I_m &= \lim\ p \to 1 I_m(t) = I_{m0} + I_{m1} + I_{m2} + \ldots
\end{aligned}
\qquad \text{(B22)}
$$

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