Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(6):1716-1736

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Analytical solutions of an asymmetrical dynamic crack design for bridging fiber pull-out in composite materials

Lü Nian-chun[1,4], Li Xin-gang[2], Cheng Yun-hong[3] and Cheng Jin[3]

[1]*School of Material Science and Engineering, Shenyang Ligong University, Shenyang, P R China*
[2]*School of Civil Engineering, Harbin Engineering University, Harbin, P R China*
[3]*Department of Civil Engineering, Northeastern University, Shenyang, P R China*
[4]*Department of Astronautics and Mechanics, Harbin Institute of Technology, Harbin, P R China*

ABSTRACT

In order to examine the distributions stress and displacement with the internal dissymmetrical crack, an asymmetrical dynamic crack design is presented for bridging fiber pull-out in unidirectional composite materials. The crack extension should also appear in the format of self-similarity because fiber failure is ascertained by the maximum tensile stress. The formulation involves the development of a Riemann-Hilbert problem. Analytical solutions under the conditions of an increasing motive force Pt^3/x^3, Px^3/t^2 *are closed for an asymmetrical dynamic crack with bridging fiber pull-out, respectively.*

Key words: asymmetrical dynamic design; bridging fiber pull-out; analytical solutions; crack; composite materials

---

INTRODUCTION

It is well known that the matrix crack, as well as fracture process of the bridging fibers, is one of the critical mechanisms of the crack expansion in fiber-reinforced composite materials, e.g. the unidirectional fiber-reinforced brittle matrix composites [1-2]. It is essential to deal with the mechanical analysis of matrix cracking with bridging fibers, in order to evaluate the distribution of the axial traction force in each fiber. Literature [3] proposed a measure to estimate the distribution of the traction force for a crack with bridging fibers in an infinite orthotropic elastic plane under a uniform remote tensile stress and also presented a design of bridging fiber pull-out by the same process. Most of researchers [4-13] have investigated the bridging fibrous problem of the crack by means of boundary collocation method (BCM), but all of them studied static problems concerning composite materials with numerical solutions. Researches in [14-18] et al. obtained a closed solution to the elastodynamic crack problem in an orthotropic medium. However, each active crack problem mentioned above was not concerned with fracture process of the bridging fiber pull-out of composite materials. Many researchers [19-22] have studied dynamic problems of the bridging fiber pull-out of composite materials and concluded analytical solutions, but they did not address the asymmetrical dynamic problem of bridging fiber pull-out. Because of the complexity and difficulty, dynamic fracture problems of composite materials researched are not enough thoroughly [23-26]. When a crack occurs in composite materials, bridging fiber pull-out often exists in the front of the crack tip, and this is an inevitable phenomenon. Composite materials are often regarded as orthotropic anisotropic body in terms of their fibrous directions, therefore, investigating dynamic fracture problems concerning the bridging fiber pull-out is an extremely significant aspect on the mechanics of composite materials because many engineering structures will destruct in dynamic conditions as time goes on. So far, the analytical solution to asymmetrical dynamic crack extension of bridging fiber pull-out has not been found out very less[27], the authors try to approach and examine the problem from a new perspective because the new dynamic design is obvious different from the old one in [27].

In this paper, the problem of an asymmetrical crack with bridging fiber pull-out of unidirectional composite material is analyzed under the dynamic conditions by means of Keldysh-Sedov mixed boundary value method, and that analytical solutions for unidirectionally reinforced material with fibers parallel to the free surface are presented. In the finite orthotropic plate, the distribution of displacement and stress can be calculated by the stress functions under the conditions of a non-uniform tensile stress and the traction forces of bridging fibers on crack edges. In order to simulate the state of the bridging fibers, the asymmetrical design of bridging fiber pull-out is introduced. By utilizing this design, the relation between the traction forces on the bridging fiber and the crack opening displacement is ascertained. The solution of a sole dislocation in an elastic half-plane is derived by complex variable analysis. The crack is then described as a consecutive distribution of dislocation. This solution in conjunction with a bridging fiber pull-out force gives rise to a system of self-similar functions with dislocation density as unknown units. The self-similar functions are then resolved analytically by means of Keldysh-Sedov approach. In order to settle efficaciously fracture dynamics problems of bridging fiber pull-out of composite materials, it is indispensable to build a dynamic design of bridging fiber pull-out.

The problem under consideration is that of a crack, running in one plane, assumed to nucleate from an infinitesimally small micro-crack with the unlike velocity from the start. This asymmetrical crack is moving with constant velocity V_1 and V_2 at subsonic velocity in both the positive and negative directions of the x-axis, respectively [28] which was not concerned with bridging fiber pull-out of composite materials. It is entirely different from the symmetrical crack moving in both directions of the x-axis with constant velocity V in literatures [16-22, 25-26, 29]. All of them considered motion in materials, which were supposed to be homogeneous and isotropic, with respect to stress-strain relationships and fracturing character. If the fiber failure is governed by maximum tensile stress, which occurs at the crack plane, the fiber breaks and hence the crack extension should also appear in a self-similar modality. The fiber breaks lie along a transverse line and therefore, present a notch [3, 30-31]. When a crack expands at higher speed, bridging fiber pull-out of composite materials still exists in the dynamic circumstance, which is more significant than those in the static case. Since bridging fibers can lead to an arresting purpose on crack extension along the original notch plane, dynamic fracture effect on bridging fiber pull-out of composite materials will be expressed, at the same time, stresses and displacements as well as stress intensity factors are deducted properly. In this paper, a dynamic design of bridging fiber pull-out is developed to study the asymmetric propagation of the finite length crack in unidirectional composite materials.

[1] **The correlative expressions of self-similar functions**

In order to resolve efficiently fracture dynamics problems on composite materials, solutions will be obtained under the action of mutative loads for a mode Ⅰ asymmetric propagation crack. In terms of the theorem of generalized functions, the dissimilar boundary condition problems considered will be transformed into Keldysh-Sedov mixed boundary value problem utilizing self-similar functions, then correlative solutions will be attained.

Assume that there are any number of loaded segments and displacement segments along the x-axis, the ends of these sections are running with different constant velocity. At the initial moment $t = 0$, the half-plane is stillness. In these segments the loads and displacements are discretionary linear combination of the following functions [18-22,26-27,32-34]:

$$\frac{d^k f_{k_1}(x)}{dx^k} \cdot \frac{d^s f_{s_1}(t)}{dt^s} \tag{1}$$

Where $$f_i(\xi) = \begin{cases} 0 & \xi < 0 \\ \xi^i & \xi > 0 \end{cases} \tag{2}$$

Here k 、 k_1 and s 、 s_1 are arbitrary integer positive numbers.

A discretional sequential function of two variables x and t may be represented as a linear superposition of Eq. (1); thus it has significance in principle to seek the loadings or the displacements satisfying the modality of Eq. (2). Let us introduce the following linear differential operator as well as integral operator:

$$L = \frac{\partial^{m+n}}{\partial x^m \partial t^n} \quad , \qquad \text{inverse:} \quad L^{-} = \frac{\partial^{-m-n}}{\partial x^{-m} \partial t^{-n}} \tag{3}$$

Here $+m+n$, $-m-n$ and 0 represent the $(m+n)$th order derivative, the $(m+n)$th order integral and function's self. It is easy to prove that there were exist constants m and n. When putting Eq.(3) into Eqs. (2), (1); we will attain functions that are homogeneous functions of x and t of zeroth dimension (homogeneous), the couple m, n will be called an index of self-similarity [26-27,32-34].

Utilizing corresponding expressions of elastodynamics equations of motion for an orthotropic anisotropic body [18-22, 26-27,32-34]:

for the case when functions Lu and Lv are homogeneous

$$u^0 = Lu\,,\qquad v^0 = Lv\,,\qquad \tau_{xy}^0 = L\tau_{xy}\,,\qquad \sigma_y^0 = L\sigma_y \tag{4}$$

for the case when functions $L\sigma_y$ and $L\tau_{xy}$ are homogeneous

$$u^0 = \frac{\partial}{\partial t}Lu\,,\ \ v^0 = \frac{\partial}{\partial t}Lv\,,\quad \sigma_y{}^0 = \frac{\partial}{\partial t}L\sigma_y\,,\quad \tau_{xy}{}^0 = \frac{\partial}{\partial t}L\tau_{xy} \tag{5}$$

The relative self-similar functions are as follows [18-23,26-27]:

$$\sigma_y^0 = (1/t)\operatorname{Re}F(\tau)\,,\qquad v^0 = \operatorname{Re}W(\tau)\,, \tag{6}$$

$$W'(\tau) = [D_1(\tau)/D(\tau)]F(\tau) \tag{7}$$

where: $\tau = x/t$, $F(\tau)$ 、$W(\tau)$ are self-similar functions. The values of $D_1(\tau)/D(\tau)$ can be ascertained form Appendix 1 of literatures [21,18, 27]. Indicated here are only $D_1(\tau)/D(\tau)$ in the subsonic speed range with purely imaginary values. Thus, elastodynamics problems for an orthotropic anisotropic body can be reduced to seeking the sole unknown function for which $F(\tau)$ and $W(\tau)$ must satisfy the boundary-value conditions. This case is Riemann-Hilbert problem in the theory of complex functions while for the simplest case, there is results from the Keldysh-Sedov or Dirichlet problem. Refer to literatures detailedly [35-37].

Fracture dynamics problems will be researched for an infinite orthotropic anisotropic body. Assuming at the initial moment $t = 0$ a crack appears at the origin of coordinates and begins to spread asymmetrically at constant velocity V_1 、V_2 at subsonic speed in both the positive and negative directions of the $x-$axis, respectively; and at, $t < 0$, the half-plane was at rest. The surfaces of the crack are subjected to the unlike types of loads under the plane strain states according to the presumption.

[2] **Foundational modality of the solution to dynamic asymmetrical propagation problem concerning mode I crack**

At the initial moment $t = 0$, a micro-crack suddenly is postulated to occur in an orthotropic anisotropic body. Let the Cartesian co-ordinate axes accord with the axes of elastic symmetry of the body. The problem is restricted to motion in the $x - y -$ plane. The crack is running asymmetrically with constant velocity V_1 and V_2 respectively in the positive and negative directions of x-axis such that. $V_1 > V_2 > 0$. Consider the translation of the following boundary condition as

$$\begin{aligned} &\sigma_y(x,0,t) = f_1(x,t)\,, && -V_2 t < x < V_1 t \\ &v(x,0,t) = 0, && x < -V_2 t \quad \text{or} \quad x > V_1 t \end{aligned} \tag{8}$$

Let's introduce the variable $\tau = x/t$. By application of the above corresponding expressions and $t\delta(x) = \delta(x/t)$ in the theory of generalized functions [37-39], the boundary conditions can be transformed as:

$$\begin{aligned} &\operatorname{Re}F(\tau) = f_2[\tau, \delta(\tau)]\,, && -V_2 < \tau < V_1 \\ &\operatorname{Re}W'(\tau) = 0\,, && \tau < -V_2 \quad \text{or} \quad \tau > V_1 \end{aligned} \tag{9}$$

In the light of the relationship $F(\tau)$ and $W'(\tau)$ in Eq.(7) and the previous conditions, the format of sole unknown function $W'(\tau)$ can be confirmed:

$$W'(\tau) = f_3[(\tau, \xi(\tau)] \tag{10}$$

Then the considered problem can be easily decreased to Keldysh-Sedov problem:

$$\operatorname{Re}\xi(\tau) = 0\,,\qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1$$

$$\operatorname{Im}\xi(\tau) = 0\,, \qquad -V_2 < \tau < V_1 \tag{11}$$

Synthetically considering asymmetry and the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the stress at the crack tip [40-41], the solution of the above problems can be deducted by literatures [27-28,42] as follows:

$$\xi(\tau) = T[(V_1 - \tau),\ (V_2 + \tau)] \tag{12}$$

Then using Eqs. (6) or (7), people will facilely gain the stress, the displacement and the stress intensity factor for the problems concerning asymmetrical crack propagation.

4 An asymmetrical dynamic crack design for bridging fiber pull-out in unidirectional composite materials

The crack is postulated to nucleate from an infinitesimally small micro-crack located along the $x-$axis in self-similar modality, and to move asymmetrically with constant velocity V_1 and V_2 in the positive and negative *x* directions such that. $V_1 > V_2 > 0$. Bridging fiber pull-out of unidirectional composite materials considered is designed on a two-dimensional region, having a single row of parallel, identical, equally spaced fibers, segregated by matrix. The initial damage is taken to consist of an arbitrary number of broken fibers such that all breaks lie along the $x-$axis forming a curved notch. In addition to this notch, a discretional number of self-similar (off-axis) fiber breaks, i. e. fiber pull-out, with asymmetry with respect to *x* and along a transverse line are also considered. The sketch of an asymmetrical dynamic crack design of bridging fiber pull-out configuration is displayed in Fig.1. The contour in Fig.1 is symmetry both in geometry and loading with respect to the $x-$axis, but it is asymmetry with respect to the $y-$axis on account of crack asymmetrical expansion. The fibers and the matrix are taken to be linearly elastic. It is further assumed that the fibers have a much higher elastic modulus in the axial direction than the matrix and therefore, the fibers are taken as supporting all of the axial loads in composite materials [43]. Load is transferred between adjacent fibers through the matrix by a simple shear mechanism. The shear stresses are independent of transverse displacements and the equilibrium equation in the fiber direction decreases to an equation in the longitudinal displacements alone, as is a typical of shear-lag theory [21-23, 27, 30-31]. The approach and designing procedure developed by [21-23, 30] will be used. The major difference among [21-23, 30] and the present work is in the mode of fiber break. Unlike in [30] where static problems are considered, the fiber fractures in turn occur along two single planes, i.e. the fiber fracture was self-similar fiber (off-axis) break and inevitably had relation to both time *t* and displacement *x*. In view of this, the geometry of the damage will not be the same about the $x-$axis, i.e. break lie is also dissymmetry about $y-$axis on account of crack asymmetrical extension. That is, the fiber fracture could be self-similar fiber fracture and therefore. also present a notch. The fibrous fracture speeds presumed are α_1 and α_2 such that. $V_1 > V_2 > \alpha_1 > \alpha_2 > 0$, as displayed in Fig. 1. The crack or notch is at. $y = 0$, $-V_2 t < x < V_1 t$ in the matrix, and the bridging fiber pull-out location is ahead of the crack tips, i. e. $x > V_1 t$ or $x < -V_2 t$. The fibrous area is located at the interval of. $-V_2 t < x < -\alpha_2 t$ and $\alpha_1 t < x < V_1 t$, whereas the broken fibers are located in the domain of. $-\alpha_2 t < x < \alpha_1 t$.

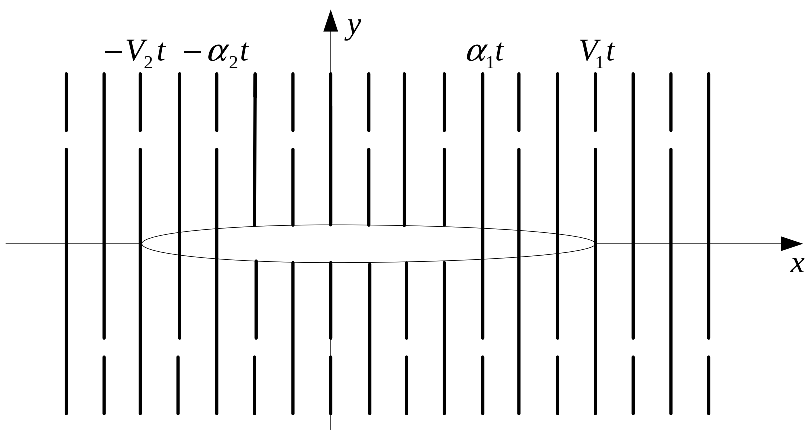

Fig.1. Sketch of an asymmetrical dynamic crack design of bridging fiber pull-out in unidirectional composite materials

Obviously, the asymmetrical dynamic design of crack expansion problem with bridging fiber pull-out in Fig.1 is shown by the mechanical design of a dynamic crack of bridging fibers in Fig.2. The bridging fibers are symmetrical state with respect to $x-$ axis. Each bridging fiber is replaced by a pair of vertical traction forces that act at the points with the same *x*-coordinate on the upper and lower track surfaces, but in the opposite direction. Each bridging fiber is assumed to be balanced with the fracture load of a fiber from the matrix. In order to analyze the design conveniently break lie presumed has no effects on the crack. The present design has the symmetry of geometrical and mechanical conditions with respect to $x-$ axis, but it has no such a character with respect to y axis because of the asymmetrical crack extension. Closure forces act on the segment of. $y=0$, $-V_2t < x < -\alpha_2 t$ and $\alpha_1 t < x < V_1 t$, which represent tension forces of bridging fibers. Fibers of the composite materials are usually arranged tightly, separated by matrix, therefore bridging fiber traction forces are postulated to be distributed continuously. It is obvious that traction forces are larger near the points of. $\alpha_1 t, \alpha_2 t$, and they are smaller close to the points of. $V_1 t, V_2 t$ [27]. This situation is similar to that of [19-23] except that the crack asymmetrically runs. When a crack moves at high speed, its dimension is related to the parameters. x and t, and the edges of the crack subjected to loads must also be related to. x and t. The fibers in the matrix are supposed to be distributed homogenously. Each fiber has the same power. When a fracture occurs, both the fiber and the matrix are in the same plane of crack expansion [4, 19-20]. Certainly, this is an assumed mechanical design which maybe not accord with real cases, and it needs more improvements in future.

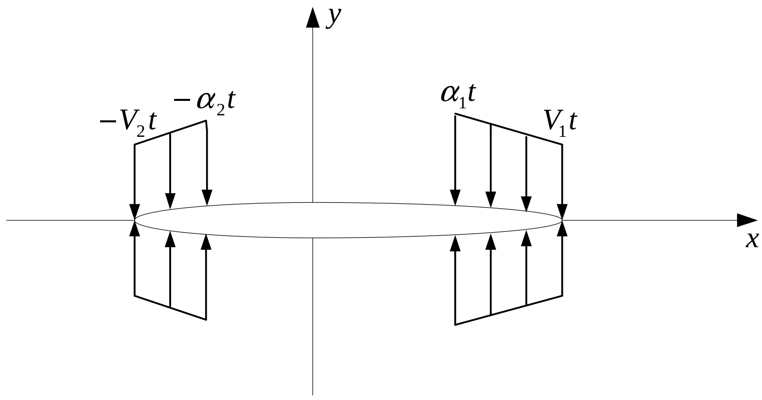

Fig.2. The mechanical design of an asymmetrical dynamic crack in bridging fiber pull-out

5 The solutions of idiographic problems

In order to solve efficaciously asymmetrical dynamics problems with bridging fiber pull-out in unidirectional composite materials, solutions will be found under the conditions of different loads for mode I moving crack. In the light of the theorem of generalized functions, the unlike boundary condition problems studied will be readily changed into Keldysh-Sedov mixed boundary value problem by the measures of self-similar functions, and the corresponding solutions will be acquired. The problems researched are under the plane strain states.

1) Presume at the initial moment. $t=0$, a crack suddenly occurs and begins to propagate asymmetrically with constant velocity V_1 and V_2 in the positive and negative directions of x-axis respectively; such that. $V_1 > V_2 > 0$. The surfaces of the crack are subjected to standard point force. Pt^3/x^3, moving at a constant velocity β along the positive direction of *x*-axis, where $\beta < V_1$; at $t<0$ the half-plane was at rest. On the half-plane at $y=0$, the boundary conditions will be as follows:

$$\sigma_y(x,0,t) = -Pt^3/x^3 \cdot \delta(x-\beta t), \qquad -V_2 t < x < V_1 t$$
$$v(x,0,t) = 0, \qquad x < -V_2 t \quad \text{or} \quad x > V_1 t \tag{13}$$

In this case, the displacement will apparently be homogeneous functions, in which. $L=1$. Using $\tau = x/t$ and the theory of generalized functions [37-39] as well as Eqs. (4) and (6), the first representation of Eq. (13) can be written as:

$$\operatorname{Re} F(\tau) = -Pt^3/x^3 \cdot t\delta(x-\beta t) = -P\tau^{-3}\delta(\tau-\beta) \qquad -V_2 < \tau < V_1$$
$$\operatorname{Re} W'(\tau) = 0, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1 \tag{14}$$

In the light of Eq. (7) , boundary conditions (14) will be further rewritten as follows:

$$\mathrm{Re}[\frac{D(\tau)}{D_1(\tau)}\cdot W'(\tau)] = -P\tau^{-3}\delta(\tau-\beta)\,, \qquad -V_2 < \tau < V_1$$
$$\mathrm{Re}W'(\tau) = 0\,, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1 \tag{15}$$

Deducting from the above formulas, the sole solution of $W'(\tau)$ must have the format:

$$W'(\tau) = \xi(\tau)/[\tau^3(\tau-\beta)] \tag{16}$$

In the formula $\xi(\tau)$ has no singularity in the realm of. $-V_2 < \tau < V_1$,while $D(\tau)/D_1(\tau)$ is purely imaginary for the subsonic speeds, therefore $\xi(\tau)$ must be purely real in the neighborhood of $-V_2 < \tau < V_1$. Thus, Eq. (15) becomes as:

$$\mathrm{Re}\,\xi(\tau) = 0\,, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1$$
$$\mathrm{Im}\,\xi(\tau) = 0\,, \qquad -V_2 < \tau < V_1 \tag{17}$$

According to asymmetry and the conditions of the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the crack tip [40-41], the unique solution of the Keldysh-Sedov problem (17) must have the following shape:

$$\xi(\tau) = \frac{A}{\sqrt{(V_1-\tau)(V_2+\tau)}} \tag{18}$$

Where A is an unknown constant.

Substituting Eq. (18) into Eqs. (16) and (7), there results:

$$W'(\tau) = \frac{A}{\tau^3(\tau-\beta)\sqrt{(V_1-\tau)(V_2+\tau)}} \tag{19}$$

$$F(\tau) = \frac{A\cdot D(\tau)/D_1(\tau)}{\tau^3(\tau-\beta)\sqrt{(V_1-\tau)(V_2+\tau)}} \tag{20}$$

Then putting Eq. (20) into the first of Eq. (14), at $\tau \to \beta$, constant A can be ensured:

$$A = -\frac{P\sqrt{(V_1-\beta)(V_2+\beta)}}{\pi\,\mathrm{Im}[D(\beta)/D_1(\beta)]} \tag{21}$$

Inserting Eqs. (20) into (6) and (4), at the surface $y = 0$, the stress τ_{yz} and dynamic stress intensity factor $K_1(t)$ are gained, respectively:

$$\sigma_y = \frac{1}{t}\mathrm{Re}\,F(\tau) = \frac{1}{t}\cdot\frac{A\cdot\mathrm{Im}[D(\tau)/D_1(\tau)]}{\tau^3(\tau-\beta)\sqrt{(\tau-V_1)(\tau+V_2)}}$$
$$= \mathrm{Im}[\frac{D(\tau)}{D_1(\tau)}]\cdot\frac{A}{\tau^3(\tau-\beta)\sqrt{(x-V_1t)(x+V_2t)}}\,, \qquad x < -V_2t \quad \text{or} \quad x > V_1t \tag{22}$$

Known from the above, at $x \to -V_2t$ or $x \to V_1t$ the stress of the crack tip tends to infinity and has apparent singularity, hence its result is right.

$$K_{\mathrm{I}}^{(1)}(t) = \lim_{x\to V_1t}\sqrt{2\pi(x-V_1t)}\cdot[\frac{1}{t}\cdot\frac{A\cdot\mathrm{Im}[D(\tau)/D_1(\tau)]}{\tau^3(\tau-\beta)\sqrt{(\tau-V_1)(\tau+V_2)}}]$$

$$= A \cdot \frac{\sqrt{2\pi}\,\mathrm{Im}[D(V_1)/D_1(V_1)]}{V_1^3 (V_1-\beta)\sqrt{(V_1+V_2)t}} \tag{23}$$

$$K_{\mathrm{I}}^{(2)}(t) = \lim_{x\to -V_2 t} \sqrt{-2\pi(x+V_2 t)} \cdot [\frac{1}{t} \cdot \frac{A\cdot \mathrm{Im}[D(\tau)/D_1(\tau)]}{\tau^3(\tau-\beta)\sqrt{(\tau-V_1)(\tau+V_2)}}]$$

$$= A \cdot \frac{\sqrt{2\pi}\,\mathrm{Im}[D(-V_2)/D_1(-V_2)]}{V_2^{\,3}(V_2+\beta)\sqrt{(V_1+V_2)t}} \tag{24}$$

The superscripts of Eqs. (23) and (24) show the stress intensity factor at. $x \to V_1 t$ and. $x \to -V_2 t$, respectively. In order to represent expediently, one postulates:

$$X = (V_1-\tau)(V_2+\tau) = V_1V_2 + (V_1-V_2)\tau - \tau^2 \tag{25}$$

The correlative constants of the above can show only [44] as follows: $a = V_1V_2$, $b = V_1 - V_2$, $c = -1$, $K = 4ac - b^2 = -(V_1+V_2)^2$

Putting Eq. (19) Into Eqs. (4), (6), after integrating with respective to τ we can attain v :

$$v = v^0 = \mathrm{Re}\int_\infty^{x/t} \frac{A}{\tau^3(\tau-\beta)\sqrt{(V_1-\tau)(V_2+\tau)}} d\tau$$

$$= A\mathrm{Re}\int_\infty^{x/t} \frac{1}{\beta}[-\frac{1}{\tau\beta^2} - \frac{1}{\beta\tau^2} - \frac{1}{\tau^3} + \frac{1}{(\tau-\beta)\beta^2}]\frac{d\tau}{\sqrt{X}} = \frac{A}{\beta}\mathrm{Re}\{\int_\infty^{x/t} -\frac{1}{\beta^2\tau\sqrt{X}}d\tau$$

$$-\int_\infty^{x/t}\frac{1}{\beta\tau^2\sqrt{X}}d\tau - \int_\infty^{x/t}\frac{1}{\tau^3\sqrt{X}}d\tau + \frac{1}{\beta^2}\int_\infty^{x/t}\frac{1}{(\tau-\beta)\sqrt{X}}d\tau\} \tag{26}$$

Utilizing correlative integral formulas [44] to yield:

$$\int \frac{d\tau}{\tau^2\sqrt{X}} = -\frac{\sqrt{X}}{a\tau} + \frac{b}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau} + \frac{b}{2\sqrt{a}}\right| \tag{27}$$

$$\int \frac{d\tau}{\tau\sqrt{X}} = -\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau} + \frac{b}{2\sqrt{a}}\right| \tag{28}$$

$$\int \frac{d\tau}{\tau^3\sqrt{X}} = (-\frac{1}{2a\tau^2} + \frac{3b}{4a^2\tau})\sqrt{X} + (\frac{3b^2}{8a^2} - \frac{c}{2a})\int\frac{d\tau}{\tau\sqrt{X}} = (-\frac{1}{2a\tau^2} + \frac{3b}{4a^2\tau})\sqrt{X}$$

$$-(\frac{3b^2}{8a^2} - \frac{c}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau} + \frac{b}{2\sqrt{a}}\right| \tag{29}$$

$$\int \frac{1}{(\tau-\beta)\sqrt{X}}d\tau = -\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta} + \frac{b_1}{2\sqrt{a_1}}\right| \tag{30}$$

where: $a_1 = V_1V_2 + (V_1-V_2)\beta - \beta^2$, $b_1 = V_1 - V_2 - 2\beta$.

Putting Eqs. (27), (28), (29) , (30) into (26), we can easily gain v as follows:

$$v = \frac{A}{\beta}\mathrm{Re}\{+\frac{1}{\beta^2\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau} + \frac{b}{2\sqrt{a}}\right| - \frac{1}{\beta}[-\frac{\sqrt{X}}{a\tau} + \frac{b}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau} + \frac{b}{2\sqrt{a}}\right|]$$

$$-(-\frac{1}{2a\tau^2}+\frac{3b}{4a^2\tau})\sqrt{X}\ +(\frac{3b^2}{8a^2}-\frac{c}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{1}{\beta^2}\int_\infty^{x/t}\frac{1}{(\tau-\beta)\sqrt{X}}d\tau\}$$

$$=\frac{A}{\beta}\mathrm{Re}\{+\frac{1}{\beta^2\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{\sqrt{X}}{\beta a\tau}-\frac{b}{2a^{3/2}\beta}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|]$$

$$+(\frac{1}{2a\tau^2}-\frac{3b}{4a^2\tau})\sqrt{X}\ +(\frac{3b^2}{8a^2}+\frac{1}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|$$

$$-\frac{1}{\beta^2\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|\}\Bigg|_\infty^{x/t}=\frac{A}{\beta}\mathrm{Re}\{-\frac{1}{\beta^2\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$+(\frac{1}{\beta^2}-\frac{b}{2a\beta}+\frac{3b^2}{8a^2}+\frac{1}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+(\frac{1}{\beta}+\frac{1}{2\tau}-\frac{3b}{4a})\frac{\sqrt{X}}{a\tau}\}\}\Bigg|_\infty^{x/t}$$

$$=\frac{A}{\beta}\{-\frac{1}{\beta^2\sqrt{a_1}}\ln\left|\frac{t\sqrt{a_1}+\sqrt{(V_1t-x)(V_2t+x)}}{x-\beta t}+\frac{b_1}{2\sqrt{a_1}}\right|+(\frac{1}{\beta}+\frac{t}{2x}-\frac{3b}{4a})\frac{\sqrt{(V_1t-x)(V_2t+x)}}{a\,x}$$

$$+(\frac{1}{\beta^2}-\frac{b}{2a\beta}+\frac{3b^2}{8a^2}+\frac{1}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|\},$$

$$-V_2t<x<V_1t \qquad (31)$$

where: $a=V_1V_2$, $b=V_1-V_2$, $K=4ac-b^2=-(V_1+V_2)^2$, $a_1=V_1V_2+(V_1-V_2)\beta-\beta^2$, $b_1=V_1-V_2-2\beta$. Its result is obtained by relative integral formulas in literature [38].

By application of the solution of Eq. (31), the bridging fibrous fracture speeds α_1 and α_2 of composite materials can be attained.

$$\Delta_1=\frac{A}{\beta}\{-\frac{1}{\beta^2\sqrt{a_1}}\ln\left|\frac{\sqrt{a_1}+\sqrt{(V_1-\alpha_1)(V_2+\alpha_1)}}{\alpha_1-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$+(\frac{1}{\beta^2}-\frac{b}{2a\beta}+\frac{3b^2}{8a^2}+\frac{1}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{(V_1-\alpha_1)(V_2+\alpha_1)}+\sqrt{a}}{\alpha_1}+\frac{b}{2\sqrt{a}}\right|$$

$$+(\frac{1}{\beta}+\frac{1}{2\alpha_1}-\frac{3b}{4a})\frac{\sqrt{(V_1-\alpha_1)(V_2+\alpha_1)}}{a\cdot\alpha_1}\},\qquad x=\alpha_1t \qquad (32)$$

$$\Delta_2=\frac{A}{\beta}\{-\frac{1}{\beta^2\sqrt{a_1}}\ln\left|\frac{\sqrt{a_1}+\sqrt{(V_1+\alpha_2)(V_2-\alpha_2)}}{-\alpha_2-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$+(\frac{1}{\beta^2}-\frac{b}{2a\beta}+\frac{3b^2}{8a^2}+\frac{1}{2a})\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{(V_1+\alpha_2)(V_2-\alpha_2)}+\sqrt{a}}{\alpha_2}+\frac{b}{2\sqrt{a}}\right|$$

$$-(\frac{1}{\beta}+\frac{1}{2\alpha_1}-\frac{3b}{4a})\frac{\sqrt{(V_1+\alpha_2)(V_2-\alpha_2)}}{a\cdot\alpha_2}\},\qquad x=-\alpha_2t \qquad (33)$$

Each fibre has the same power [16-18], therefore the fibrous fracture power must be equal. That is to say: $\Delta_1=\Delta_2=\Delta$, while, Δ can be determined by a sole axial tensile test with V_1 , V_2 and β referred to as known constants. According to this measure, the fibrous fracture speeds α_1 and α_2 can be only attained numerical solution, respectively, because α_1 and α_2 will not be represented in the format of explicit functions.

2) With all conditions holding the same as those in the previous example, the applied loads become. Px^3/t^2. The boundary conditions will be as follows:

$$\sigma_y(x,0,t) = -Px^3/t^2 \cdot \delta(x-\beta t), \qquad -V_2 t < x < V_1 t$$
$$v(x,0,t) = 0, \qquad x < -V_2 t \quad \text{or} \quad x > V_1 t \tag{34}$$

In this case, the stress will obviously be homogeneous functions, in which. $L=1$. According to Eqs. (5), (6) and the theory of generalized functions [37-39], boundary conditions (34) can be rewritten as follows:

$$\operatorname{Re} F(\tau) = -[-2Px^3/t^3 \cdot t\delta(x-\beta t)] = 2P\tau^3\delta(\tau-\beta), \qquad -V_2 < \tau < V_1$$
$$\operatorname{Re} W'(\tau) = 0, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1 \tag{35}$$

Owing to the derivative of Dirac's function equaling zero at. $x \neq \beta t$, the above expression is deduced [37-39].

In terms of Eq. (7), boundary conditions (35) will be further rewritten as:

$$\operatorname{Re}[\frac{D(\tau)}{D_1(\tau)} \cdot W'(\tau)] = 2P\tau^3\delta(\tau-\beta), \qquad -V_2 < \tau < V_1$$
$$\operatorname{Re} W'(\tau) = 0, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1 \tag{36}$$

From the above formulas, the unique solution of $W'(\tau)$ can be easily deduced as follows:

$$W'(\tau) = \xi(\tau) \cdot \tau^3/(\tau-\beta) \tag{37}$$

In the formula, $\xi(\tau)$ has no singularity in the scope of. $-V_2 < \tau < V_1$,while, $D(\tau)/D_1(\tau)$ is purely imaginary for the subsonic speeds, therefore, $\xi(\tau)$ must be purely real at the interval of. $-V_2 < \tau < V_1$. So, question (36) takes:

$$\operatorname{Re}\xi(\tau) = 0, \qquad \tau < -V_2 \quad \text{or} \quad \tau > V_1$$
$$\operatorname{Im}\xi(\tau) = 0, \qquad -V_2 < \tau < V_1 \tag{38}$$

In terms of asymmetry and the conditions of the infinite point of the plane corresponding to the origin of coordinates of the physical plane as well as singularities of the crack tip [40-41], the sole solution of the Keldysh-Sedov problem (38) can be attained:

$$\xi(\tau) = \frac{A}{[(V_1-\tau)(V_2+\tau)]^{3/2}} \tag{39}$$

Where A is an unknown constant.

Then putting Eq. (39) Into Eqs. (37) and (7), there results:

$$W'(\tau) = \frac{A\tau^3}{(\tau-\beta)[(V_1-\tau)(V_2+\tau)]^{3/2}} \tag{40}$$

$$F(\tau) = \frac{D(\tau)}{D_1(\tau)} \cdot \frac{A\tau^3}{(\tau-\beta)[(V_1-\tau)(V_2+\tau)]^{3/2}} \tag{41}$$

Substituting Eq. (41) Into the first of Eq. (36), at $\tau \to \beta$, constant *A* can be ascertained:

$$A = \frac{2P[(V_1-\beta)(V_2+\beta)]^{3/2}}{\pi \operatorname{Im}[D(\beta)/D_1(\beta)]} \tag{42}$$

In an orthotropic isotropic body, the disturbance range of elastic wave can be depicted by the circular area of radius $c_1 t$ and. $c_2 t$. Here c_1 and c_2 are the velocities of longitudinal and transverse waves ($c_1 > c_2$) of elastic body，respectively. In an orthotropic anisotropic body, the disturbance range of elastic wave is not the circular area and can not surpass the threshold value $C_d = \sqrt{C_{11} / \rho}$ of elastic body, Where. C_{11} is an elastic constant of materials. At. $|x| > C_d t$, with this expression: $\mathrm{Im}[D_1(\tau)/D(\tau)] = 0$, thus the stresses and the displacements are zero which coincide with the initial boundary conditions; and this illuminates that at $y = 0$, disturbance of elastic wave can not exceed. $C_d t$.

Now inserting Eq. (41) into (6) and (5), at the surface $y = 0$, the stresses and the dynamic stress intensity factor are gained, respectively:

$$\sigma_y = \int_{\infty}^{x} \frac{1}{t} \mathrm{Re} \frac{A\tau^3 \cdot \mathrm{Im}[D(\tau)/D_1(\tau)]}{(\tau-\beta)[(V_1-\tau)(V_2+\tau)]^{3/2}} dt$$
$$= -\mathrm{Re} \int_{C_d}^{\frac{x}{t}} \frac{A\tau^2 \cdot \mathrm{Im}[D(\tau)/D_1(\tau)]}{(\tau-\beta)[(\tau-V_1)(\tau+V_2)]^{3/2}} d\tau \quad , \quad x < -V_2 t \ \text{ or } \ x > V_1 t \tag{43}$$

$$K_{\mathrm{I}}^{(1)}(t) = \lim_{x \to V_1 t} \sqrt{2\pi(x - V_1 t)} \cdot \mathrm{Re} \int_{C_d}^{\frac{x}{t}} - \frac{A\tau^2 \cdot \mathrm{Im}[D(\tau)/D_1(\tau)]}{(\tau-\beta)[(\tau-V_1)(\tau+V_2)]^{3/2}} d\tau$$
$$= 2\sqrt{2\pi t} \cdot \frac{AV_1^2 \cdot \mathrm{Im}[D(V_1)/D_1(V_1)]}{(V_1-\beta)(V_1+V_2)^{3/2}} \tag{44}$$

$$K_{\mathrm{I}}^{(2)}(t) = \lim_{x \to -V_2 t} \sqrt{-2\pi(x + V_2 t)} \cdot \mathrm{Re} \int_{C_d}^{\frac{x}{t}} - \frac{A\tau^2 \cdot \mathrm{Im}[D(\tau)/D_1(\tau)]}{(\tau-\beta)[(\tau-V_1)(\tau+V_2)]^{3/2}} d\tau$$
$$= 2\sqrt{2\pi t} \cdot \frac{AV_2^2 \cdot \mathrm{Im}[D(-V_2)/D_1(-V_2)]}{(V_2+\beta)(V_1+V_2)^{3/2}} \tag{45}$$

The limit of Eqs.(44) and (45) remains with the shape $0 \cdot \infty$ which should be translated into the modality of ∞ / ∞, then the aftermath of the two formulae can be calculated by the approaches of L'Hospital theorem [45]. The superscripts of the two formulas also represent the stress intensity factor at. $x \to V_1 t$ and. $x \to -V_2 t$, respectively.

Simplified, we postulate again:

$$X = (V_1 - \tau)(V_2 + \tau) = V_1 V_2 + (V_1 - V_2)\tau - \tau^2 \tag{46}$$

The correlative constants of the above can show only [44] as follows: $a = V_1 V_2$, $b = V_1 - V_2$, $c = -1$, $K = 4ac - b^2 = -(V_1 + V_2)^2$

Integrating Eq.(40) in terms of relevant formulae in literature [44], we will attain $W(\tau)$

$$W(\tau) = \int W'(\tau) d\tau = \frac{A\tau^3}{(\tau-\beta)\sqrt{(V_1-\tau)(V_2+\tau)}} d\tau$$
$$= A\int [\tau^2 + \tau\beta + \beta^2 + \frac{\beta^3}{(\tau-\beta)}] \frac{d\tau}{X^{3/2}}, \tag{47}$$

Integrating Eq.(47), one will achieve v^0. But it has four terms, separate denotation is more expedient, then essential formulae can be utilized in literature [38], now presuming:

$$\int \frac{\tau}{X^{3/2}} d\tau = [\frac{1}{\sqrt{X}} + \frac{b}{2}\int \frac{d\tau}{X^{3/2}}] = [\frac{1}{\sqrt{X}} + \frac{b}{2} \cdot \frac{2b-4\tau}{K\sqrt{X}}] = [\frac{(K+b^2)}{K\sqrt{X}} - \frac{2b\tau}{K\sqrt{X}}] \quad (48)$$

$$\int \frac{1}{X^{3/2}} d\tau = \frac{2b-4\tau}{K\sqrt{X}} \quad (49)$$

$$\int \frac{\tau^2}{X^{3/2}} d\tau = \frac{(K-b^2)\tau - 2ab}{K\sqrt{X}} - \int \frac{d\tau}{\sqrt{X}} = \frac{(K-b^2)\tau - 2ab}{K\sqrt{X}} - \arcsin\frac{2\tau - b}{\sqrt{-K}} \quad (50)$$

Putting Eqs. (48), (49), (50) into (47), now presuming:

$$W_1(\tau) = A\beta \int \frac{\tau}{X^{3/2}} d\tau = A\beta[\frac{(K+b^2)}{K\sqrt{X}} - \frac{2b\tau}{K\sqrt{X}}] + C \quad (51)$$

$$W_2(\tau) = A \cdot \int \frac{\beta^2}{X^{3/2}} d\tau = A\beta^2 \cdot \frac{2b-4\tau}{K\sqrt{X}} + C \quad (52)$$

$$W_3(\tau) = A \cdot \int \frac{\tau^2}{X^{3/2}} d\tau = A \cdot \frac{(K-b^2)\tau - 2ab}{K\sqrt{X}} - A \cdot \arcsin\frac{2\tau - b}{\sqrt{-K}} + C \quad (53)$$

Denominator of the fourth term in Eq. (47) contains this term $(\tau - \beta)X^{3/2}$, so the calculation is incapable of applying integral formulas directly, thus integral format must change into performable integral[38].

By variable replacement: $\tau_1 = \tau - \beta$, now, putting it into Eq. (46), one can acquire:

$$X = (V_1 - \tau)(V_2 + \tau) = V_1V_2 + (V_1 - V_2)\beta - \beta^2 + (V_1 - V_2 - 2\beta)\tau_1 - \tau_1^2 \quad (54)$$

The relative constants of Eq. (54) can denote as follows: $a_1 = V_1V_2 + (V_1 - V_2)\beta - \beta^2$, $b_1 = V_1 - V_2 - 2\beta$, $c = -1$, $K_1 = 4a_1c - b_1^2 = -(V_1 + V_2)^2 = K$

Substituting Eq. (54) into the fourth term of Eq. (47), after integrating with respective to τ we can obtain $W_4(\tau)$ in terms of literature [38]:

$$W_4(\tau) = A\beta^3 \int \frac{d\tau}{(\tau - \beta)X^{3/2}} = A\beta^3 \int \frac{d\tau_1}{\tau_1 X^{3/2}} = \frac{A\beta^3}{a_1}[\frac{1}{\sqrt{X}} + \int \frac{d\tau_1}{\tau_1\sqrt{X}} - \frac{b_1}{2}\int \frac{d\tau_1}{X^{3/2}}]$$

$$= \frac{A\beta^3}{a_1}[\frac{1}{\sqrt{X}} - \frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X} + \sqrt{a_1}}{\tau_1} + \frac{b_1}{2\sqrt{a_1}}\right| - \frac{b_1}{2} \cdot \frac{-4\tau_1 + 2b_1}{K\sqrt{X}}]$$

$$= \frac{A\beta^3}{a_1}[\frac{K - b_1^2}{K\sqrt{X}} - \frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X} + \sqrt{a_1}}{\tau_1} + \frac{b_1}{2\sqrt{a_1}}\right| + \frac{2b_1\tau_1}{K\sqrt{X}}]$$

$$= \frac{A\beta^3}{a_1}[\frac{K - b_1^2 - 2b_1\beta}{K\sqrt{X}} - \frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X} + \sqrt{a_1}}{\tau - \beta} + \frac{b_1}{2\sqrt{a_1}}\right| + \frac{2b_1\tau}{K\sqrt{X}}] + C \quad (55)$$

Known from Eq.(47): $W(\tau) = W_1(\tau) + W_2(\tau) + W_3(\tau) + W_4(\tau)$. The crack spreads along the *x*-axis, therefore $W(\tau)$ including Eqs. (51), (52), (53) and (55) can be performed in a definite integral operation, one takes constant. $C = 0$.

Now replacing Eq.(51) into Eqs. (6), (5), then applying Eqs. (27) and (28) to yield the divisional displacement v_1:

$$v_1 = \int_0^t \mathrm{Re} W_1(\tau)dt = \mathrm{Re}\int_\infty^{x/t} -\frac{x}{\tau^2}W_1(\tau)d\tau = -A\beta x \cdot \mathrm{Re}\int_\infty^{x/t}\frac{1}{\tau^2}[\frac{K+b^2}{K} - \frac{2b\tau}{K}]\frac{d\tau}{\sqrt{X}}$$

$$=-A\beta x\cdot \mathrm{Re}[(-\frac{\sqrt{X}}{a\tau}+\frac{b}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)\frac{K+b^2}{K}$$

$$-\frac{2b}{K}(-\frac{1}{a^{1/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)]\Bigg|_{\infty}^{x/t}=-\frac{A\beta}{K}\mathrm{Re}[(-\frac{K+b^2}{a}\cdot t\sqrt{X}$$

$$+\frac{(K+b^2)b+4ba}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|]\Bigg|_{\infty}^{x/t}=\frac{-4A\beta}{K}\sqrt{(V_1t-x)(V_2t+x)}$$

$$-V_2t<x<V_1t \tag{56}$$

Then substituting Eq. (52) into Eqs. (6), (5) after utilizing Eqs. (27) and (28), the divisional displacement v_2 can be acquired as:

$$v_2=\int_0^t \mathrm{Re}W_2(\tau)dt=\mathrm{Re}\int_{\infty}^{x/t}-\frac{x}{\tau^2}W_2(\tau)d\tau=-A\beta^2 x\cdot\mathrm{Re}\int_{\infty}^{x/t}\frac{1}{\tau^2}[\frac{2b}{K\sqrt{X}}-\frac{4\tau}{K\sqrt{X}}]d\tau$$

$$=-A\beta^2x\cdot\mathrm{Re}\{\frac{2b}{K}[-\frac{\sqrt{X}}{a\cdot\tau}-\frac{b}{2a}\int_{\infty}^{x/t}\frac{d\tau}{\tau\sqrt{X}}]-\frac{4}{K}\cdot(-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)\}$$

$$=-A\beta^2x\cdot\mathrm{Re}\{\frac{2b}{K}[\frac{-\sqrt{X}}{a\cdot\tau}-\frac{b}{2a}(-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)]+\frac{4}{K\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)\}$$

$$=-A\beta^2x\cdot\mathrm{Re}\{-\frac{2b}{K}\cdot\frac{\sqrt{X}}{a\cdot\tau}+\frac{b^2}{Ka^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{4a}{Ka^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|\}$$

$$=-\frac{A\beta^2x}{a}\mathrm{Re}\{-\frac{2b}{K}\cdot\frac{\sqrt{X}}{\tau}+\frac{4a+b^2}{Ka^{1/2}}\cdot\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|\}\Bigg|_{C_d}^{x/t}$$

$$=-\frac{A\beta^2}{a}\{-\frac{2b}{K}\cdot\sqrt{(V_1t-x)(V_2t+x)}+\frac{b^2+4a}{a^{1/2}K}x\cdot\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|\}$$

$$=-\frac{A\beta^2}{a}\{-\frac{2b}{K}\cdot\sqrt{(V_1t-x)(V_2t+x)}-\frac{x}{a^{1/2}}\cdot\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|\}$$

$$-V_2t<x<V_1t \tag{57}$$

Putting Eq. (53) into Eqs. (6), (5) by using Eqs. (27) and (28), the divisional displacement v_3 can be attained as:

$$v_3=\int_0^t \mathrm{Re}W_3(\tau)dt=\mathrm{Re}\int_{\infty}^{x/t}-\frac{x}{\tau^2}W_3(\tau)d\tau=-Ax\cdot\mathrm{Re}\int_{\infty}^{x/t}\frac{1}{\tau^2}[\frac{(K-b^2)\tau}{K\sqrt{X}}-\frac{2ab}{K\sqrt{X}}$$

$$-\arcsin\frac{2\tau-b}{\sqrt{-K}}]d\tau=-Ax\cdot\mathrm{Re}[-\frac{K-b^2}{K}\cdot\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|$$

$$-\frac{2ab}{K}(-\frac{\sqrt{X}}{a\tau}+\frac{b}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|)-\int_{\infty}^{x/t}\frac{1}{\tau^2}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}}\cdot d\tau]$$

$$=-Ax\cdot\mathrm{Re}[-\frac{K-b^2}{K}\cdot\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|$$

$$+\frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau}-\frac{b^2}{K}\cdot\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\int_{\infty}^{x/t}\arcsin\frac{2\tau-b}{\sqrt{-K}}\cdot d(\frac{1}{\tau})$$

$$= -Ax \cdot \mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right| + \frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau}$$

$$+\frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}} - \int_{\infty}^{x/t}\frac{1}{\tau}\cdot d\arcsin\frac{2\tau-b}{\sqrt{-K}}$$

$$= -Ax \cdot \mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right| + \frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau} + \frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}}$$

$$-\int_{\infty}^{x/t}\frac{1}{\tau}\cdot\frac{1}{\sqrt{1-(\frac{2\tau-b}{\sqrt{-K}})^2}}\cdot\frac{2}{\sqrt{-K}}d\tau = -Ax\cdot\mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|$$

$$+\frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau}+\frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}} - \int_{\infty}^{x/t}\frac{1}{\tau}\cdot\frac{2}{\sqrt{-K-(2\tau-b)^2}}\cdot d\tau$$

$$= -Ax \cdot \mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right| + \frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau} + \frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}}$$

$$-\int_{\infty}^{x/t}\frac{1}{\tau}\cdot\frac{2}{\sqrt{-K-(4\tau^2-4b\tau+b^2)}}\cdot d\tau = -Ax\cdot\mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|$$

$$+\frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau}+\frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}} - \int_{\infty}^{x/t}\frac{1}{\tau}\cdot\frac{2}{\sqrt{4a-4\tau^2+4b\tau}}\cdot d\tau$$

$$= -Ax \cdot \mathrm{Re}[-\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right| + \frac{2b}{K}\cdot\frac{\sqrt{X}}{a\tau} + \frac{1}{\tau}\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}} - \int_{\infty}^{x/t}\frac{1}{\tau}\cdot\frac{1}{\sqrt{X}}\cdot d\tau$$

$$+\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|]\Bigg|_{\infty}^{x/t} = -A\cdot\mathrm{Re}[+\frac{2b}{K}\cdot\frac{t\sqrt{X}}{a}+t\cdot\arcsin\frac{2\tau-b}{\sqrt{-K}}]\Bigg|_{\infty}^{x/t}$$

$$= -A\cdot[+\frac{2b}{Ka}\cdot\sqrt{(V_1t-x)(V_2t+x)}+t\cdot\arcsin\frac{2x/t-b}{\sqrt{-K}}], \qquad -V_2t<x<V_1t \tag{58}$$

Then putting Eq. (55) into Eqs. (6), (5), the displacement v_4 can be acquired as:

$$v_4 = \int_0^t \mathrm{Re}W_4(\tau)dt = \mathrm{Re}\int_{\infty}^{x/t} -\frac{x}{\tau^2}W_4(\tau)d\tau = -\frac{A\beta^3 x}{a_1}\cdot\mathrm{Re}\int_{\infty}^{x/t}\frac{1}{\tau^2}[\frac{K-b_1^2-2b_1\beta}{K\sqrt{X}}$$

$$-\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right| + \frac{2b_1\tau}{K\sqrt{X}}]d\tau \tag{59}$$

The second item of the above (without coefficient term) can be expressed as follows:

$$\int -\frac{1}{\tau^2\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|d\tau = \frac{1}{\sqrt{a_1}}\int \ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|d(\frac{1}{\tau})$$

$$= \frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right| - \frac{1}{\sqrt{a_1}}\int\frac{1}{\tau}\cdot\frac{1}{\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}}$$

$$\times[\frac{\frac{b-2\tau}{2\sqrt{X}}\cdot(\tau-\beta)-(\sqrt{X}+\sqrt{a_1})}{(\tau-\beta)^2}]d\tau=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$-\frac{1}{\sqrt{a_1}}\int\frac{1}{\tau}\cdot\frac{2\sqrt{a_1}(\tau-\beta)}{2\sqrt{a_1X}+2a_1+b_1(\tau-\beta)}\cdot\frac{(b-2\tau)(\tau-\beta)-2X-2\sqrt{a_1X}}{2\sqrt{X}\cdot(\tau-\beta)^2}\cdot d\tau$$

$$=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$-\int\frac{1}{\tau}\cdot\frac{-b\beta-b\tau-2a+2\tau\beta-2\sqrt{a_1X}}{2\sqrt{a_1X}+2a_1+b_1(\tau-\beta)}\cdot\frac{1}{\sqrt{X}\cdot(\tau-\beta)}d\tau$$

$$=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$-\int\frac{1}{\tau}\cdot\frac{-b\beta-b\tau-2a+2\tau\beta-2\sqrt{a_1X}}{2\sqrt{a_1X}+b\beta+b\tau+2a-2\tau\beta}\cdot\frac{1}{\sqrt{X}\cdot(\tau-\beta)}d\tau$$

$$=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|+\int\frac{1}{\tau}\cdot\frac{d\tau}{(\tau-\beta)\sqrt{X}}=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$-\frac{1}{\beta}\int(\frac{1}{\tau}-\frac{1}{\tau-\beta})\cdot\frac{d\tau}{\sqrt{X}}=\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$+\frac{1}{\beta\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{1}{\beta\sqrt{a_1}}\ln\left|\frac{\sqrt{a_1}+\sqrt{X}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$=(\frac{1}{\tau}+\frac{1}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|+\frac{1}{\beta\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right| \quad (60)$$

Substituting Eqs. (60), (26), (27) into Eq. (59), the displacement v_4 can be gained:

$$v_4=\frac{-A\beta^3x}{a_1}\mathrm{Re}\{\frac{K-b_1^2-2b_1\beta}{K}[-\frac{\sqrt{X}}{a\tau}+\frac{b}{2a^{3/2}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|]$$

$$-\frac{2b_1}{K}\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$+\frac{1}{\beta\sqrt{a}}\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{1}{\beta\sqrt{a_1}}\ln\left|\frac{\sqrt{a_1}+\sqrt{X}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|\}\Bigg|_{\infty}^{x/t}$$

$$=\frac{-A\beta^3x}{a_1}\mathrm{Re}\{-\frac{K-b_1^2-2b_1\beta}{K}\cdot\frac{\sqrt{X}}{a\tau}+\frac{1}{\tau\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|+\frac{1}{\sqrt{a}}[-\frac{2b_1}{K}$$

$$+\frac{(K-b_1^2-2b_1\beta)b}{2aK}+\frac{1}{\beta}]\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|+\frac{1}{\beta\sqrt{a_1}}\ln\left|\frac{\sqrt{a_1}+\sqrt{X}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|\}\Bigg|_{\infty}^{x/t}$$

$$=\frac{A\beta^3}{a_1}\mathrm{Re}\{\frac{K-b_1^2-2b_1\beta}{K}\cdot\frac{t\sqrt{X}}{a}-(t+\frac{x}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$

$$-x\cdot\frac{(K-b_1^2-2b_1\beta)b\beta-4ab_1\beta+2aK}{2a^{3/2}K\beta}\times\ln\left|\frac{\sqrt{X}+\sqrt{a}}{\tau}+\frac{b}{2\sqrt{a}}\right|\}\Bigg|_{\infty}^{x/t} \tag{61}$$

Then a, b, K, a_1 and b_1 can be replaced with relevant constants by application of Eqs. (46), (54), one presumes:

$$E_1 = K-b_1^2-2b_1\beta = K-b_1(b_1+2\beta)=K-bb_1 \tag{62}$$

$$F_1=(K-b_1^2-2b_1\beta)b\beta-4a\beta\, b_1+2aK=(K-b_1b)b\beta-4a\beta\, b_1+2aK=Kb\beta$$
$$-b_1\beta(b^2+4a)+2aK=K(b\beta+b_1\beta+2a)=K(2b\beta-2\beta^2+2a)=2a_1K \tag{63}$$

Putting Eqs. (54), (55) into Eq. (53), the displacement v_4 can be obtained:

$$v_4=\frac{A\beta^3}{a_1}\{\frac{E_1}{aK}\sqrt{(V_1t-x)(V_2t+x)}-(t+\frac{x}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{X}+\sqrt{a_1}}{\tau-\beta}+\frac{b_1}{2\sqrt{a_1}}\right|$$
$$-\frac{xF_1}{2a^{3/2}K\beta}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|\},\qquad -V_2t<x<V_1t \tag{64}$$

The displacement v is the sum of subdistrict displacement**:** $v=v_1+v_2+v_3+v_4$. Afterward the addition of Eqs. (56), (50) , (50) and (64), the displacement v is gained:

$$v=\frac{-4A\beta}{K}\cdot\sqrt{(V_1t-x)(V_2t+x)}+\frac{2Ab\beta^2}{aK}\cdot\sqrt{(V_1t-x)(V_2t+x)}$$
$$+\frac{A\beta^2\cdot x}{a^{3/2}}\cdot\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|$$
$$-\frac{2Ab}{aK}\cdot\sqrt{(V_1t-x)(V_2t+x)}-At\cdot\arcsin\frac{2x/t-b}{\sqrt{-K}}+\frac{A\beta^3}{a_1}\cdot\frac{E_1}{aK}\sqrt{(V_1t-x)(V_2t+x)}$$
$$-\frac{A\beta^3}{a_1}(t+\frac{x}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a_1}}{x-\beta t}+\frac{b_1}{2\sqrt{a_1}}\right|$$
$$-\frac{A\beta^2x}{a^{3/2}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|$$
$$=\frac{A}{K}[-4\beta+\frac{2b\beta^2}{a}-\frac{2b}{a}+\frac{E_1\beta^3}{a_1a}]\sqrt{(V_1t-x)(V_2t+x)}-At\cdot\arcsin\frac{2x/t-b}{\sqrt{-K}}$$
$$+(\frac{A\beta^2\cdot x}{a}-\frac{A\beta^2\cdot x}{a})\cdot\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a}}{x}+\frac{b}{2\sqrt{a}}\right|$$
$$-\frac{A\beta^3}{a_1}(t+\frac{x}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a_1}}{x-\beta t}+\frac{b_1}{2\sqrt{a_1}}\right|$$
$$=\frac{A}{K}[-4\beta+\frac{2b\beta^2}{a}-\frac{2b}{a}+\frac{E_1\beta^3}{a_1a}]\sqrt{(V_1t-x)(V_2t+x)}-At\cdot\arcsin\frac{2x/t-b}{\sqrt{-K}}$$
$$-\frac{A\beta^3}{a_1}(t+\frac{x}{\beta})\frac{1}{\sqrt{a_1}}\ln\left|\frac{\sqrt{(V_1t-x)(V_2t+x)}+t\sqrt{a_1}}{x-\beta t}+\frac{b_1}{2\sqrt{a_1}}\right|,\qquad -V_2t<x<V_1t \tag{65}$$

where: $a=V_1V_2$, $b=V_1-V_2$; $a_1=V_1V_2+(V_1-V_2)\beta-\beta^2$, $b_1=V_1-V_2-2\beta$, $c=-1$,

$$K_1 = 4a_1c - b_1^2 = -(V_1+V_2)^2 = K$$

Applying the same approach as that for resolving Eqs. (32) and, (33), substitute $x = \alpha_1 t$, $x = -\alpha_2 t$ into Eq. (65) While V_1, V_2 and t were also referred to as known constants, the fibrous fracture speeds α_1 and α_2 can be facilely attained numerical solutions on account of similar reasons.

6 Description of dynamic stress intensity factor

Analytical solutions need translating into numerical solutions in terms of real case of concrete problems, therefore variable rule of dynamic stress intensity factors can be depicted validly. When relevant parameters are put into Eqs. (23), (24), (44), (45) to easily plot $K_1^{(1)}(t)$ and $K_1^{(2)}(t)$ as a function of time t, respectively, and the numerical solutions of them are facilely obtained. The following constants [40-41, 36] are presumed:

$C_{11} = 19.24Gpa$; $C_{12} = 1.25Gpa$; $C_{22} = 17.83Gpa$; $C_{66} = 1.00Gpa$
$V_1 = 300m\cdot s^{-1}$; $V_2 = 250m\cdot s^{-1}$; $\beta = 200m\cdot s^{-1}$; $P = 200N$;
$\rho = 0.5\times 9.8\times 10^3 N\cdot m^{-3}$;

Known from Eqs. (23) and (24) , after simulative software Matlab 6.5, dynamic stress intensity factors $K_1^{(1)}(t)$ and $K_1^{(2)}(t)$ decline gradually to slow and trend to constants finally and also have apparent singularity with the increase of time because only variable t locates in the denominator of the two expressions, moreover, the rest quantities are referred to as real constants. Such currents are detailedly illustrated by the curves in Fig.3. This kind of the alterable current is related to the result obtained by relative numerical calculation in literatures [47-48], therefore the outcomes obtained are proved to be right. The correlative numerical value relationships are represented in Table.1.

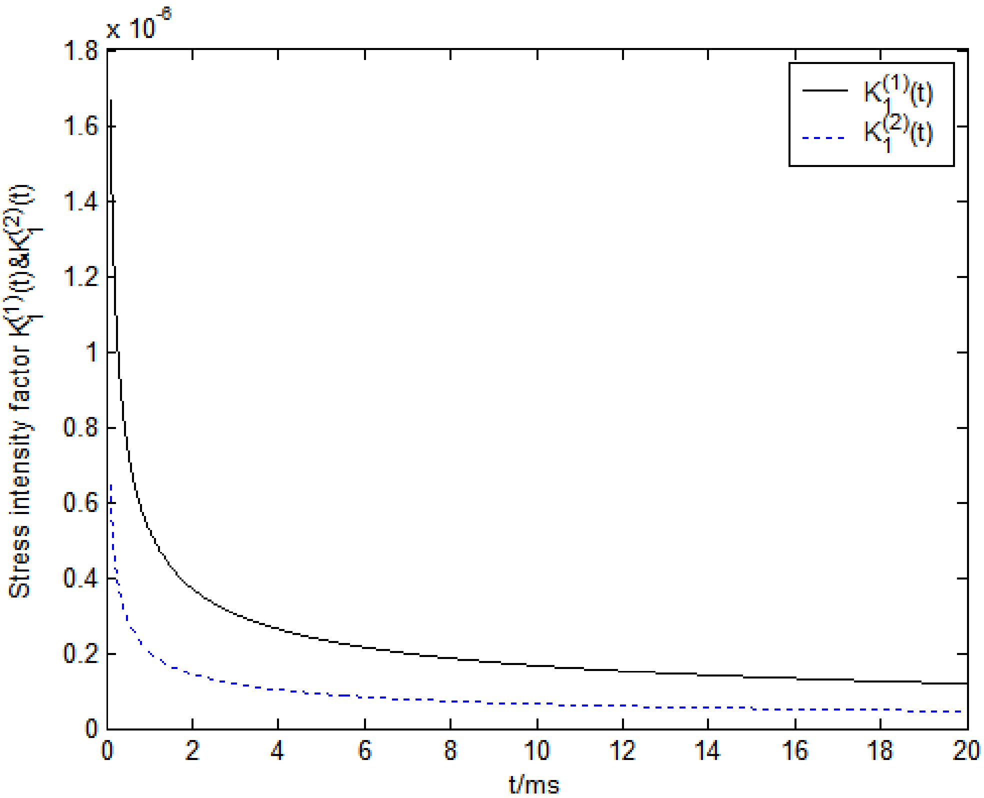

Fig.3. Dynamic stress intensity factors $K_1^{(1)}(t)$ and $K_1^{(2)}(t)$ versus time. t

Table.1 Relative numerical values between dynamic stress intensity factors $K_1^{(1)}(t)$, $K_1^{(2)}(t)$ **and time** t

$t \times 10^{-4}$ / sec	2	4	6	8	10
$K_1^{(1)}(t) \times 10^{-7} / N \cdot m^{-3/2}$	3.7271	2.6354	2.1518	1.8635	1.6668
$K_1^{(2)}(t) \times 10^{-7} / N \cdot m^{-3/2}$	1.4427	1.0202	0.8330	0.7214	0.6452
$t \times 10^{-4}$ / sec	12	14	16	18	20
$K_1^{(1)}(t) \times 10^{-7} / N \cdot m^{-3/2}$	1.5215	1.4087	1.3177	1.2423	1.1786
$K_1^{(2)}(t) \times 10^{-8} / N \cdot m^{-3/2}$	5.8899	5.4529	5.1008	4.8092	4.5623

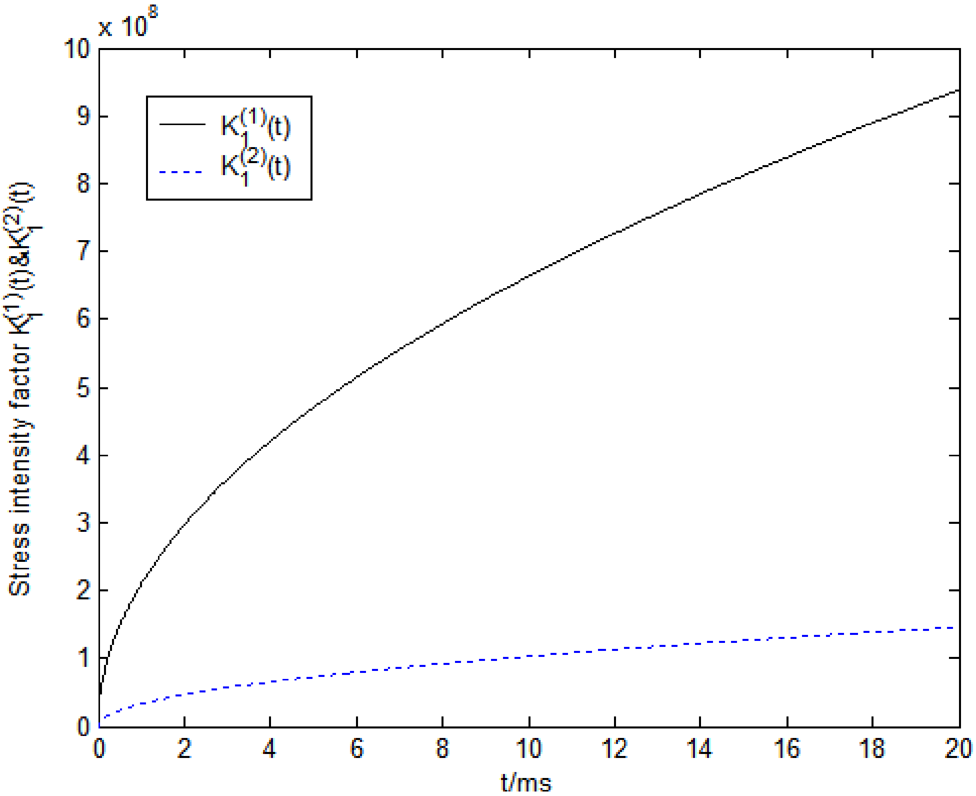

Fig.4. Dynamic stress intensity factors $K_1^{(1)}(t)$ **and** $K_1^{(2)}(t)$ **versus time.** t

Table.2 Relative numerical values between dynamic stress intensity factors $K_1^{(1)}(t)$, $K_1^{(2)}(t)$ **and time** t

$t \times 10^{-4}$ / sec	2	4	6	8	10
$K_1^{(1)}(t) \times 10^{8} / N \cdot m^{-3/2}$	2.9643	1.2199	5.1338	5.9281	1.4581
$K_1^{(2)}(t) \times 10^{7} / N \cdot m^{-3/2}$	4.6109	6.5209	7.9864	9.2219	10. 3121
$t \times 10^{-4}$ / sec	12	14	16	18	20
$K_1^{(1)}(t) \times 10^{8} / N \cdot m^{-3/2}$	7.2603	7.8421	8.3834	8.8923	9.3729
$K_1^{(2)}(t) \times 10^{8} / N \cdot m^{-3/2}$	1.1294	1.2199	1.3042	1.3833	1.4581

In terms of Eqs. (44), (45), after simulative software Matlab 6.5, $K_1^{(1)}(t)$ and $K_1^{(2)}(t)$ gradually aggrandize from zero, but their trends are slow and eventually reach or exceed fracture toughness of this material with the enhance of time, these results will conduct the structural instability because sole variable t locates in their numerator, and that the rest quantities are also regarded as real constants, therefore structural destruction will occur, as shown in Fig.4. Such trends are homogeneous to the outcomes also attained by means of correlative numerical calculation in

Literatures [49-52]， accordingly it is correct. The relative numerical value relations are illuminated in Table.2.

CONCLUSION

By means of the relevant representation: $f(x, y, t) = t^n f(x/t, y/t)$, where n is an integral number, and the problem considered can be translated into homogeneous function of *x* and *t* of zeroth dimension, namely self-similar functions. All satisfy the relationship of this function, thus the analytical solutions can be attained by Eqs. (4), (5), (6) and (7) with homogeneous function of variable. τ . This measure can utilize not only in elastodynamics, but also in elastostatics and even in other situations. Refer to literatures [53-55].

Analytical solutions of the dynamic asymmetrical crack design for bridging fiber pull-out of unidirectional composite materials were found by way of the theoretical use of a complex variable function. The approach developed in this paper based on the methods of the self-similar functions makes it conceivable to attain the idiographic solution to this design of bridging fiber pull-out of composite materials and bridging fibrous fracture speeds α_1 and. α_2. The rudimental solution of asymmetrical dynamic crack extension problems is derived based on the self-similar functions. In terms of the real boundary conditions, self-similar function $W'(\tau)$ can be easily deduced by means of corresponding to variable τ, accordingly analytical solutions of stresses, displacements and stress intensity factors will be readily computed. This case is referred to as the analogous class of dynamic problem of the elasticity theory. However, the present solution occurs to be the most straightforward and intuitive of all alternative methods appeared up to now. Indeed, relative researchers have succeeded in a mixed Keldysh-Sedov boundary value problem on a half-plane. The problem is of sufficient real interest, since all of the members of structures in which fractures may expand are of finite dimensions and are frequently in the modality of long strips. The method of solution is based exclusively on techniques of analytical-function theory and is simple and compendious. By making some observations regarding the solution of the mixed boundary value problem we have reasonably decreased the amount of the calculative work needed to solve such a crack propagation problem. The approaches of self-similar functions are still applicable in researches of mode I semi-infinite crack [56], mode Ⅲ crack [52-54] and mode Ⅲ interface crack [57-61] and mode Ⅲ interface crack [62-68] as well as axially crack [32-33, 69].

Analytic solutions of the asymmetrical dynamic design for bridging pull-out of unidirectional composite materials were found by way of the theoretical application of a complex variable function. The approach developed in this paper based on the measures of the self-similar functions makes it conceivable to acquire the concrete solution to this design and bridging fiber fracture speeds α_1 and α_2. The fundamental solution of asymmetrical dynamic crack extension problems is derived based on the self-similar functions. In the light of the concrete boundary conditions, self-similar function $W'(\tau)$ can be easily deduced by means of corresponding to variable τ, therefore analytical solutions of stresses, displacements and stress intensity factors will be readily worked out. This is regarded as the analogous class of dynamic problem of the elasticity theory. However, the present solution appears to be the simplest and intuitive of all alternative approaches appeared up to now. Indeed, we have succeeded in a mixed Keldysh-Sedov boundary value problem on a half-plane. The problem is of adequate factual interest, since all of the members of structures in which fractures may extend are of finite dimensions and are frequently in the format of long strips. The method of solution is based exclusively on techniques of analytical-function theory and is straightforward and compendious. By making some observations regarding the solution of the mixed boundary value problem we have reasonably decreased the amount of the calculative work needed to resolve such a crack propagation problem. The methods of self-similar functions are still relevant in studies of mode I semi-infinite crack [56], mode Ⅲ crack [57-61] and mode Ⅲ interface crack [62-68] as well as axially crack [32-33, 69].

Acknowledgments
This work was supported by the postdoctoral foundation of China (No.2005038199), HeiLongJiang provincial nature science foundation of China (ZJG04-08)

REFERENCES

[1] B F Sorensen, T K Jacobsen (**1998**) *Composites Part* A 29A: 1443-1451.
[2] B. Budiansky, J.W. Hutchinson, A.G. Evens, J. *Mech. and. Phy. Sol*. **34,** 167—189. (**1986**)
[3] Ji. M., and Ishikawa. H, *Int. J. Engng Sci*. **35**, 549-560. (**1997**)
[4] D. B. Marshall, B. N*Acta Metall*, **1985**, (33)**:** 2013-2021.
[5] D. B. Marshall, B. N. Cox. *Acta Metall,* **1987**, (35): 2607-2619.
[6] C. W. WOO, Y. H. WANG, *Int. J. Frac*. **62,**203—208.(**1993**)

[7] J. C. LEE, *Engng. Frac. Mech*, **37,**209—219.(**1990**)
[8] W. T. Tsai, I. R. Dharani, *Engng. Frac. Mech*, **44,**43—49.(**1993**)
[9] Liu W N. *Int. J. Frac*, **1994**,**70**(4): L31—L35..
[10] K. Liao，K. Reifsnider. *Int. J. Frac*.**2000**, (106): 95-115.
[11] V.Tamuzs, S. Tarasovs, U. Vilks. *Eng. Frac. Mech*. **2001**, 68 (5): 513-525.
[12] G Bao, Z Suo. *Appl Mech Rev* 45(8): 355-366. (**1992**)
[13] T K Jacobsen, B F Sorensen. *Composites Part* A 32: 1-11. (**2001**)
[14] A. Piva, E. Viola, *Engng. Frac. Mech*, **29,**535-547.(**1988**)
[15] J. De, B. Patra, *Engng. Frac. Mech*, **41,** 895—909. (**1998**)
[16] J. CHENG, *J. Harbin. Inst. Tech*. （Supplement of Engineering Mechanics）, **1985** ：8-21. (*in Chinese*)
[17] N. C Lü, J. Cheng. *Engng. Mech*, **2000**,**17(6),** 117—120. (*in Chinese*)
[18] N. C Lü, J. Cheng. *Chinese. Quart. Mech*. **2002**,23 (4): 504-508 (*in Chinese*)
[19] B N Cox, N Sridhar, IJ Beyerlein, *Acta Materialia* 49: 3863-3877. (**2001**)
[20] Lü N C, Cheng J, Cheng Y. H**.** *Mech. Res. Commun*. **2005**, 32 (1): 1-14
[21] Lü Nian-chun, Cheng Yun-hong, Xu Hong-min, Cheng Jin, Tang Li-qiang. *App. Math. Mech*. **2004**, 25(10): 1194－1202
[22] N. C, Lü, Y. H. Cheng. X. G. Li, J. Cheng**,** *Meccanica***. 2012,** 47(5): 1247-1260,
[23] G. C. Sih. Some elastodynamics problems of cracks. Int. J. Frac. **1968**, 1(1):51-68.
[24] E Sharon, SP Gross, J Fineberg, *Physical Review Letters* 76(12): 2117-2120. (**1996**)
[25] K. B. Broberg, *Arkve. fur Fysik*. **18,** 159-192. (**1960**)
[26] C. Atkinson, *Int. J. Engng Sci*. **3,** 77-91(**1965**)
[27] Nian-Chun Lü, Yun-Hong Cheng，Jin Cheng. *International Journal for Computational Methods in Engineering* Science and Mechanics, **2008**, 9(3):171-179
[28] N. C, Lü, Y. H Cheng. J .Cheng, *Theo. App. Frac. Mech*. **2006**, 46(3):262 –275.
[29] Y. W. CRAGGS, The Growth of A Disk-Shaped Crack, *Int. J. Engng Sci*. **4,** 113-124. (**1966**)
[30] J. G. GOREE, R. S. GROSS, *Engng. Frac. Mech,* **33,** 563—578. (**1979**)
[31] J. G. GOREE, L. R. DHARNT, W. F. JONES, Crack growth and fracture of continuous fiber metal matrix composites: analysis and experiments. *Metal Matrix Composites: Testing, Analysis, and Failure Modes,ASTM STP* **1032,** 251—269.(**1989**)
[32] G. P. Charepanov, E. F. Afanasov. Some dynamic problems of the theory of elasticity——A review. Int. J. Eng. Sci. **1970**,(**12):** 665-690.
[33] G. P. Charepanov, Mechanics of brittle fracture, Nauka Moscow, 732-792. (**1973**)
[34] A.C. Erigen, E.S.Suhubi, Elastodynymics Vol.2. Linear Theory. Academic Press, New York San Francisco London **1975**
[35] N. I. Muskhlishvili, Singular Integral Equations. Nauka, Moscow, (**1968**)
[36] N. I. Muskhlishvili, Some fundamental problems in the mathematical theory of elasticity. Nauka, Moscow, (**1966**)
[37] F. D. GAKHOV. Boundary-Value Problems. Fitzmatigiz, Moscow, (**1963**)
[38] R. F. Hoskins, Generalized Functions, Ellis Horwood,**1979**
[39] X. S Wang. Singular functions and their applications in mechanics. Beijing: Scientific Press, **1993**, 3-45 (In Chinese)
[40] G. C. Sih, *Mechanics of Fracture 4. Elastodynamics Crack Problems*[M], Noordhoff, Leyden. **1977**, 213—247
[41] R. P .Kanwal, D. L. Sharma, *J. Elasticit*y, **1976**, 6(4):405-418.
[42] XU Hong-min. Study of different kinds of problems on fracture mechanics in orthotropic medium. Ph. D of Harbin Institute of Technology, **2004**，20-60. (in Chinese)
[43] Guan-lin Shen. Mechanics of composite materials．Beijing: Tsinghua University Press,**1996**.(*in Chinese*)
[44] Editorial group of mathematics handbook. *Mathematics handbook*. Beijing: Advanced Education Press, **2002:** 244－300. (*in Chinese*)
[45] Teaching office of mathematics of Tongji University. *Advanced mathematics* (Vol.1). Beijing: Advanced Education Press, **1994**. 167-172. (*in Chinese*)
[46] Wu K C， *Int. J. Frac*.**2000**，106 (1): 1-12.
[47] Kalthof J F，Beinert J，Winkler S．Measurements of dynamic stress intensity factors for fastrunning and arresting cracks in double-cantilever-beam specimens[M1．In：Fast Fracture and crack Arrest．ASTM．STP 627，Philadelphia：Pa Publisher **1977** 161-176
[48] Kobayashi A S．Dynam ic fracture analysis by dynam ic finite element method=．generation and prediction analyses[M]．In：Nonlinear and Dynarnic Fracture Mechanics．AMD．ASME,New York: NY Publisher，**1979**，35：19—36

[49] Ravi-Chandar K．Knauss W G． *Internat J Fracture*，**1984**，25(41：247-262
[50] Ravi-Chandar K，Knauss W G． *Internat J Fracture*，**1984**，26(11)：65-80
[51] Ravi—Chandar K，Knauss W G． *Internat J Fracture*，**1984**，26(2)：141-152
[52] Ravi-Chandar K，Knauss W G． *Internat J Fracture*，**1984**，26(3)：189-200．
[53] *N. I. Sneddon.* Fourier Transform[M]. New York: McGraw-Hill, **1951**
[54] N. I. Muskhelishvili. *Some Basic Problems from the Mathematical Theory of Elasticity*[M]. Groningen-Holland: P. Noordoff, **1953**
[55] L. A. GALIN, Contact problems in elasticity theory[M]. GITTL, Moscow, (**1953**)
[56] N. C, Lü, Y. H Cheng. X. G. Li, J. Cheng, *Fati. Frac. Eng. Mater. Struct*. **2010**, 33,(3): 141–148.
[57] Y H Wang, Y K Cheung, C W Woo. *Eng.Frac. Mech*.**1992**, 42(6): 971-976
[58] LÜ Nian-Chun, Cheng Yun-Hong，LI Xin-Gang, Cheng Jin. *App. Math. Mech*. **2008**, 29, (10): 1279-1290.
[59] N. C, Lü, Y. H. Cheng. Y. T. Wang J. Cheng, *Mechanics of Advanced Materials and Structures*. **2010**, 17,(3): 215–224.
[60] Nian-Chun Lü, Yun-Hong Cheng, Yun-tao Wang, Jin Cheng.. *Applied Mathematical designling*. **2011**, (35): 2499-2507.
[61] Nian-Chun Lü, Yun-Hong Cheng, Yun-tao Wang, Jin Cheng. *Nonlinear Dynamics*. **2011**, 63(4): 793-806.
[62] Wang Yue-shen, Wang Duo, *Engineering Fracture Mechanics*., vol 55(5), 717-725, **1996**
[63] Kuang-chong Wu, *Int. J. Solids Struct.*, Vol 40(8), 1811-1823, **2003**
[64] C. Atkinson. I*nt. J. Eng. Sci.* **1975**, 13(5**):** 491-506
[65] Lü Nian-chun, Yang Ding-ning, CHENG Yun-hong，CHENG Jin. *App. Math. Mech*. **2007**,28,(4):501-510
[66] Lü Nian-chun, CHENG Jin, TIAN Xiu-bo CHENG Yun-hong. *App. Math. Mech*. **2005**,26,(9): 1212-1221
[67] Nian-Chun Lü, Yun-Hong Cheng，Jin Cheng. *International Journal for Computational Methods in Engineering Science and Mechanics,* **2008**, 9(4):246-253
[68] N. C, Lü, Y. H Cheng. X. G. Li, J .Cheng, *Composite Interfaces*. **2010**, 17,(1): 37–48.
[69] Lü Nian-chun, Cheng Jin, Cheng Yun-hong. *App. Math. Mech*. **2001**, Vol.22,(12):1429-1435
[70] N. C, Lű, Y. H Cheng. H. L. Si, J .Cheng, *Theo. App. Frac. Mech*. **2007**, 47(3):260 –273.

Appendix

In this appendix we intend to do the necessary algebra involved in D and D_1 for orthotropic anisotropy, and also for isotropy, changing the notation in this latter case in order to make a direct comparison with Broberg [22,25]. We also intend to show $D_1 \equiv 0$ when $T_1 = -T_2$ as mentioned in paper, and also that D / D_1 is purely imaginary for the possible crack velocities involved in the problem.

Calculation of D, D_1 *for orthotropy* (for the subsonic speeds)

Now, in order to illuminate these representations for universal orthotropy, we are going to refer to the Eq.(7) in literatures [16,20,22,25-27,70], where η is replaced by τ, and we write it as

$$MT^4 + NT^2 + P = 0 \tag{A.1}$$

where $M = C_{66}C_{22}$

$$P = (C_{11} - \rho\tau^2)(C_{66} - \rho\tau^2) \tag{A.2}$$

$$N = C_{66}(C_{66} - \rho\tau^2) + C_{22}(C_{11} - \rho\tau^2) - (C_{12} + C_{66})^2$$

Now write down certain properties of the roots, i.e. the sums and products, etc. From (A.1) we will obtain

$$T_1^2 + T_2^2 = -N/M \quad , \quad (T_1 - T_1)^2 = -(N - 2\sqrt{PM})/M$$

$$T_1^2 T_2^2 = P/M \quad , \quad (T_1^2 - T_2^2)^2 = (N^2 - 4MP)/M^2 \tag{A.3}$$

Write $C_r = \sqrt{C_{66}/\rho}$, $C_d = \sqrt{C_{11}/\rho}$, $a = (C_{66} - \rho\tau^2)$, $b = (C_{11} - \rho\tau^2)$

At $\tau^2 > C_d{}^2$, when $a < 0,\ b < 0$; presumed $C_{66} < C_{11}$, from Eq.(A.2) we will obtain

$$N^2 - 4PM = (C_{66}a + C_{22}b)^2 - 2(C_{66}a + C_{22}b)\ (C_{12} + C_{66})^2$$

$$+(C_{12}+C_{66})^4-4abC_{66}C_{22} \tag{A.4}$$

Evidently $0<N^2-4PM<N^2$, and $N<0$, reduce $-N\pm\sqrt{N^2-4PM}>0$, this denotes T_1^2, T_2^2 are both positive real, therefore all of the four roots of (A.1) are real. This tests that for $\tau^2>C_d^2$, we will write $\mathrm{Im}[D(\tau)/D_1(\tau)]=0$, which indicates that the disturbance of elastic wave cannot overrun $C_d t$.

Then putting Eqs.(6), (8) into (13) in literature [17,22], there results

$$D/D_1=(T_1-T_2)S(T_1,T_2,\tau)/[(T_2^2-T_1^2)(C_{12}+C_{66})(C_{11}+\rho\tau^2)]$$
$$S=T_1T_2(C_{12}+C_{66})[C_{12}^2-C_{22}(C_{11}-\rho\tau^2)]-C_{66}(C_{11}-\rho\tau^2)(T_1^2+T_2^2)$$
$$+C_{12}C_{22}C_{66}T_1^2T_2^2+(C_{11}-\rho\tau^2)[C_{12}^2+C_{12}C_{66}-C_{22}(C_{11}-\rho\tau^2)] \tag{A.5}$$

It is not difficult to test that for $|\tau|<C_r$, D/D_1 is purely imaginary for the subsonic speeds. We can give

Case (1). For $C_r<|\tau|<C_d$, remembering that we have taken the positive square root, then we will obtain

$$\mathrm{Im}[D/D_1]=\frac{Mg\sqrt{P'/M}(C_{12}+C_{66})[C_{12}^2-C_{22}(C_{11}-\rho\tau^2)+MhS'}{\sqrt{N^2-4PM(C_{12}+C_{66})(C_{11}-\rho\tau^2)}} \tag{A.6}$$

where $P'=(C_{11}-\rho\tau^2)(-C_{66}+\rho\tau^2)$

$$g=\left(\frac{-N+\sqrt{N^2+4PM}}{2M}\right)^{1/2}, \quad h=P'\left(\frac{-N+\sqrt{N^2-4PM}}{2}\right)^{1/2}$$
$$S'=C_{12}P+(C_{11}-\rho\tau^2)[C_{12}^2+C_{12}C_{66}-C_{22}(C_{11}-\rho\tau^2)]+(C_{11}-\rho\tau^2)N/C_{22}$$

Case (2). For $|\tau|<C_r<C_d$, then we can find

$$\mathrm{Im}[D/D_1]=\left(\frac{M}{N+2\sqrt{PM}}\right)^{1/2}\frac{S''}{(C_{12}+C_{66})(C_{11}-\rho\tau^2)} \tag{A.7}$$

$$S''=(P/M)^{1/2}(C_{12}+C_{66})[C_{12}^2-C_{22}(C_{11}-\rho\tau^2)]-(C_{11}-\rho\tau^2)N/C_{22}$$
$$+C_{12}P+(C_{11}-\rho\tau^2)[C_{12}^2+C_{12}C_{66}-C_{22}(C_{11}-\rho\tau^2)]$$

Case (3). For isotropy

Isotropy is regarded a special example as orthotropy, from isotropy, we will have

$$C_{11}=C_{22}=\rho C_1^2, \quad C_{66}=0.5(C_{11}-C_{12})=\rho C_2^2 \tag{A.8}$$

where C_1, C_2 are the wave velocities in the isotropic medium, simply gives D/D_1 as a function of τ, then substituting them into Eqs.(5) in literature [17, 22] and (A.6), we can obtain:

$$\frac{D(\tau)}{D_1(\tau)}=\frac{i\rho C_1[(2C_2^2-\tau^2)^2-4C_2^3C_1^{-1}\sqrt{(C_1^2-\tau^2)(C_2^2-\tau^2)}}{\tau^2\sqrt{C_1^2-\tau^2}} \tag{A.9}$$