



Analysis of fault tree importance of CNC machine tools based on BDD

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ABSTRACT

BDD technique is applied to analyze the parts importance of the CNC machine tools. Through calculating, parts importance is listed according to the values. The structure importance and probability importance of the power system are calculated in the paper. From the results we can see that the values of the two methods are perfectly close. But BDD methods are more effective than the FTA methods and can be calculated with the computer.

Keywords: BDD, CNC machine tools, Fault Tree, Importance

INTRODUCTION

The part importance refers to that when a part goes wrong it plays the role to the probability of the summit event and it shows the quantity to the importance of the subsystems or the parts. It is useful to improve the system design according to the importance of the summit events. It can improve the reliability index of the system and set up the optimized maintenance check rules [1].

Fault Tree Analysis (FTA) was firstly put forward by H.A.Watson of U.S. Bell Lab and got success in analyzing the reliability of missile sending controlling system in 1962. Using FTA on quantitative analysis was developed quickly in 70's and it became an indispensable method in the systems such as the departments of the space, nuclear power, chemical industry and others which have special demands on reliability and safety [1-2]. It was used widely on the reliability analysis of the CNC machine tools in the recent years and has made marked progress.

When we use the traditional FTA to calculate the importance, we use the minimum cut-sets and identify the improved points. But the traditional FTA can't be competent to the analysis of the large and complicated fault trees. Because of the complex nature of the fault trees, it can't be realized in the computer [1-3].

Binary Decision Diagrams (BDD) was firstly put forward by Sheldon B.Akers in 1978 which can directly reflect the logic structure of the function and was widely used on computer network, electronic circuit, logic integration and logic verification. Since 1993, it has been used in fault tree analysis with efficient and accurate calculation. It can be programmed and gets increasing attention on the system analysis with a lot of applications [1-3].

BDD is used to analyze the structure importance and probability importance of the CNC machine tools.

The Importance Analysis of the CNC Machine Tools the Structure Importance

The structure importance refers to the importance of the parts in the system. The structure importance is unrelated with the failure probability of the parts and only is related with the position in the system. The formula is given as below:

$$I_i^\Phi = \frac{1}{2^{n-1}} \sum_{2^{n-1}} [\Phi(1_i, q) - \Phi(0_i, q)]$$

Where: I_i^Φ refers to the structure importance of the part i ; n refers to the quantities of the parts and units. $\Phi(1_i, q) = (q_1, q_2, \dots, q_{i+1}, \dots, q_n)$ refers to the failure probability of the system when part i in failure. $\Phi(0_i, q) = (q_1, q_2, \dots, q_{i-1}, \dots, 0, q_{i+1}, \dots, q_n)$ refers to the failure probability of the system when part i in normal.

Lambert [4] put forward a new method to analyze the structure importance. For part i ,

$$q_j = \begin{cases} 1 & j = i \\ 1/2 & j \neq i \end{cases}$$

The formula is changed as below:

$$I_i^\Phi = G(q) = \Phi(1_i, 1/2) - \Phi(0_i, 1/2)$$

The Probability Importance

The probability importance refers to the degree of the unreliability of the system caused by the unreliability of part i .

Suppose that the system contains m static and independent parts. $q_i(t)$ is the unreliability of part i at time t . $i = 1, 2, 3, \dots, m$; $q(t) = (q_1(t), q_2(t), \dots, q_m(t))$ is the unreliability vector. $\Phi(q(t))$ is the unreliability of the system at time t . The formula of the probability importance of part i at time t is given as below:

$$I(i|t) = \frac{\partial \Phi(q(t))}{\partial q_i(t)}$$

$$i = 1, 2, 3, \dots, m$$

From the total probability formula we can know that the unreliability of the system is given as below:

$$\Phi(q) = (1 - q_i)\Phi(0_i, q) + q_i\Phi(1_i, q)$$

The formula of the probability importance of part i at time t is given as below:

$$I(i|t) = \frac{\partial(\Phi)}{\partial(q_i)} = \Phi(1_i, q) - \Phi(0_i, q)$$

Analysis of Fault Tree Importance of CNC Machine Tools Based on BDD

BDD shows BDP in graph which is an acyclic directed graph and it is composed of terminal nodes, non-terminal nodes and branches as shown in figure 1.

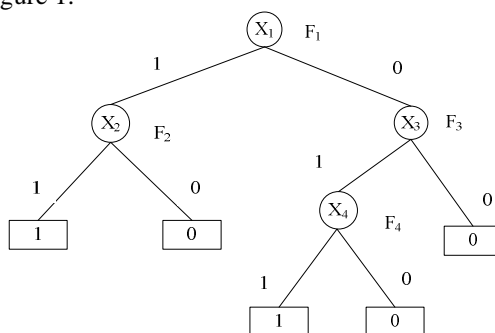


Fig.1 the basic structure of BDD

In BDD figure, the middle node means the bottom event of the fault tree and the way of the root node and the leaf node shows the non-cross combination of the failure of the bottom event.

The terminal node (the box in the figure) is also called leaf node which shows the state of the top event containing 0 and 1. 0 shows the system in the normal state and 1 shows the system in the fault state.

Non-terminal node (the circle in the figure) means the basic events of the system. X_i is the code of the basic event corresponding to the non-terminal node. X_i may be identical to the different non-terminal node. F_i is the sign corresponding to the non-terminal node. Different non-terminal nodes are correspondent to different F_i . Each non-terminal node connects two branches which are signed as 1(THEN) and 0(ELSE).

Branches (the lines in the figure) mean the state which the branch is correspondent to the father non-terminal node. 0 means the basic event will not occur. 1 means the basic event will occur.

Ite (IF-THEN-ELSE) structure based on Shannon formula is a powerful way to transfer fault trees into BDD and the mathematical expressions are given as below:

$$ite(X, M_1, M_2) = XM_1 + \bar{X}M_2$$

According to the following steps, we transfer the fault trees into BDD[5-8].

Firstly, transfer the fault trees into standard trees containing AND, OR and NON-LOGIC gates. If the middle event of the standard trees contains NAND gates, we apply De-Morgan rules to get rid of the NAND gates in the middle events. Thus only the bottom event contains NAND gate which is convenient to handle.

Secondly, replace the middle events with basic events from the middle events in the bottom layer and layer and encode them.

Thirdly, calculate BDD of the top event. Combine them according to the following two rules. Suppose $J = ite(X, M_1, M_2)$; $H = ite(Y, N_1, N_2)$.

If $X < Y$, then $J < OP >$

$$H = ite(X, M_1 < OP > H, M_2 < OP > H)$$

If $X = Y$, then $J < OP > H = ite(X, M_1 < OP > N_1, M_2 < OP > N_2)$.

Where $< OP >$ means Boolean operators as AND or OR.

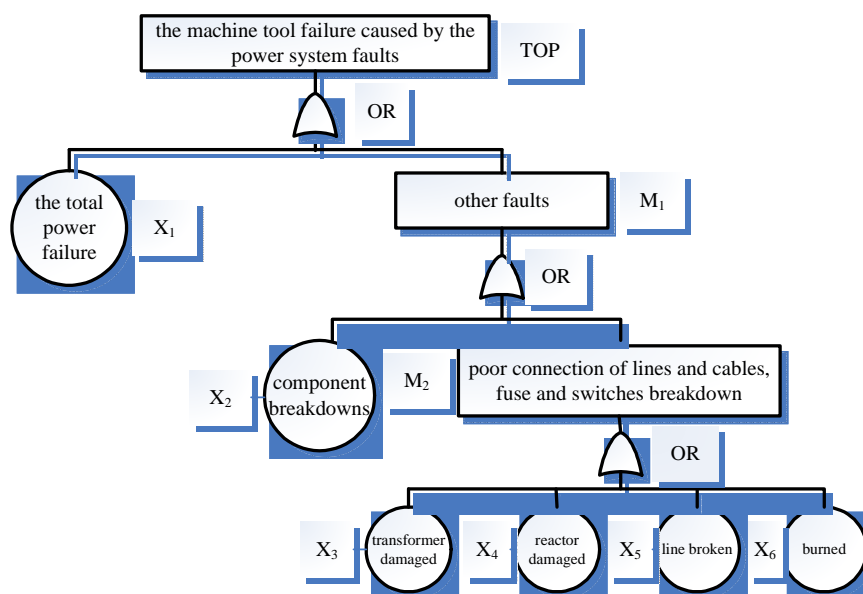


Fig.2 the fault tree of the CNC machine tools caused by the power failure

The orders of the basic events will affect BDD.

Fig.2 is the fault tree of the CNC machine tools caused by the power failure.

We analyze the parts importance of the power system of the CNC machine tool and determine the improvements.

From the reliability data collected on the spot, we determine the basic events sequence as $X_1 < X_2 < X_3 < X_4 < X_5 < X_6$ in figure 2 and *ite* structure is given as below:

$$X_i = ite(X_i, 1, 0), \text{ and } i = 1, 2, \dots, 6$$

$$M_2 = X_3 + X_4 + X_5 + X_6$$

$$M_1 = X_2 + M_2$$

$$\Rightarrow TOP = ite\{X_1, [ite(X_2, 1, ite(X_3, 1, ite(X_4, 1, ite(X_5, 1, ite(X_6, 1, 0)))))] 0\}$$

We get BDD of figure 2 shown as figure 3.

Apply BDD to Analyze the Importance of the CNC Machine Tools

To ensure no loss of generality, we discuss a random basic event Xi in BDD and suppose the situation of Xi as shown in figure 4.

We define key function Gi(q) of part i (corresponded to basic event Xi) as the fault probability transferred from the normal state which is given as below:

$$G_i(q) = \Phi(1_i, q) - \Phi(0_i, q)$$

$$TOP = X_1 M_1$$

We can calculate (1i, q) and (0i, q) of basic event Xi according to the characteristics of BDD in figure 4.

$$\Phi(1_i, q) = \sum_m \{ p_{rx_i}(q) p_{0x_i}^1(q) \} + Z(q) \quad \Phi(0_i, q) = \sum_m \{ p_{rx_i}(q) p_{0x_i}^0(q) \} + Z(q)$$

Where: m is the number of the nodes of Xi in BDD; $p_{rx_i}(q)$ is the route probability from X to Xi. $p_{0x_i}^1(q)$ is the route probability from Xi with the branch 1. $p_{0x_i}^0(q)$ is the route probability from Xi with the branch 0. Z(q) is the route probability from X to the terminal but not through node Xi.

The key function Gi(q) of basic event Xi is changed as below:

$$G_i(q) = \Phi(1_i, q) - \Phi(0_i, q) = \sum_m \{ p_{rx_i}(q) [p_{0x_i}^1(q) - p_{0x_i}^0(q)] \}$$

Shannon Decomposition Tree and BDD are all the expressions of Bull function. BDD contains less nodes in a concise style.

We realize the calculating process in the computer and get the structure importance and probability importance of each event in figure 3 with the results shown as in table 4.

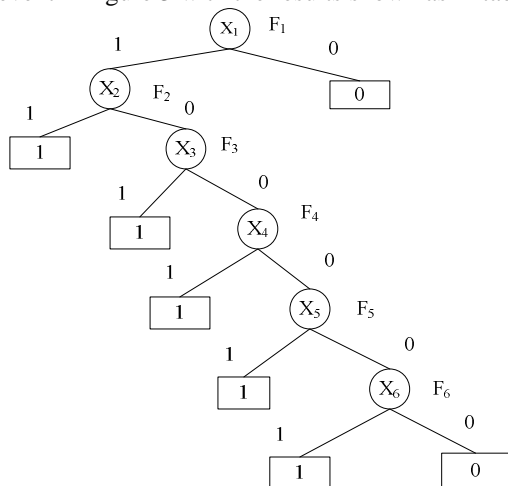


Fig.3 BDD of the CNC machine tools

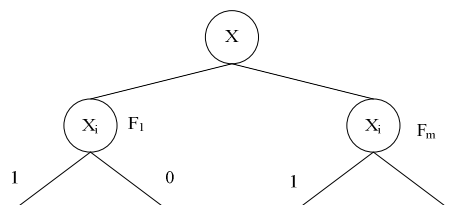


Fig.4 Basic Events caused by the power failure

Table 1 Ite structure relations of basic events in figure 3

Node number	Basic events	Unreliability at t time	Subnodes related to signal 1	Subnodes related to signal 0
F ₁	X ₁	0.011	F ₂	0
F ₂	X ₂	0.202	1	F ₃
F ₃	X ₃	0.066	1	F ₄
F ₄	X ₄	0.079	1	F ₅
F ₅	X ₅	0.132	1	F ₆
F ₆	X ₆	0.099	1	0

Notes: censoring time is 2000 hours.

Table 2 Structure importance of basic events in figure 3

Node number	Basic events	$p_{0x_i}^1(q)$	$p_{0x_i}^0(q)$	$p_{rx_i}(q)$	I_i^Φ Structure importance
F ₁	X ₁	0.95743	0.0	1.0	0.95743
F ₂	X ₂	1	0.9265	0.4999	0.03125
F ₃	X ₃	1	0.8649	0.2499	0.03125
F ₄	X ₄	1	0.7399	0.1249	0.03125
F ₅	X ₅	1	0.4999	0.0625	0.03125
F ₆	X ₆	1	0.0	0.03125	0.03125

Table 3 Probability importance of basic events in figure 3

Node number	Basic events	$p_{0x_i}^1(q)$	$p_{0x_i}^0(q)$	$p_{rx_i}(q)$	$I(A t)$ Probability importance
F ₁	X ₁	0.4029	0.0	1.0	0.4026
F ₂	X ₂	1.0	0.3003	0.011	6.8112*10 ⁻³
F ₃	X ₃	1.0	0.2691	8.3971*10 ⁻³	6.0836*10 ⁻³
F ₄	X ₄	1.0	0.1699	7.9679*10 ⁻³	6.5781*10 ⁻³
F ₅	X ₅	1.0	0.1008	7.0059*10 ⁻³	6.2855*10 ⁻³
F ₆	X ₆	1.0	0.0	6.4239*10 ⁻³	6.4266*10 ⁻³

Table 4 Probability importance of power system used BDD method and FTA method

	Structure importance		Probability importance	
	BDD method	FTA method	BDD method	FTA method
X ₁	0.95743	0.95743	0.4029	0.4029
X ₂	0.03125	0.03125	6.8112*10 ⁻³	0.00681124
X ₃	0.03125	0.03125	6.0836*10 ⁻³	0.00608361
X ₄	0.03125	0.03125	6.5781*10 ⁻³	0.00657811
X ₅	0.03125	0.03125	6.2855*10 ⁻³	0.00628553
X ₆	0.03125	0.03125	6.4266*10 ⁻³	0.00642660

CONCLUSION

From the tables we can see that the values of the two methods are very close when we calculate the structure importance and probability importance of the parts of the power system. But it is a complicated process based on the cut-sets of FTA analysis. When using BDD method in calculating structure importance and probability importance of the parts, we can effectively carry out on the computer with high efficiency and accurate results.

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