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Analysis of chaotic characteristics of the yellow river runoff

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ABSTRACT

Based on analysis of chaos theory and adaptability in runoff system, the chaotic analysis method is put forward. Monthly runoff series is reconstructed in phase space, and chaotic characteristics are identified and analyzed with Yellow River as the research object. The conclusions are as follows: natural monthly runoff is larger than the measured in saturated correlation dimension. At least 4 independent variables, up to 8 are needed to properly describe changing features of the measured monthly runoff series and build dynamic system modeling; while for natural runoff series, at least 5-6 variables， 12 at most are needed. The measured runoff series and the natural vary in chaotic characteristics in the same hydrologic station during the same period: the chaotic characteristics of downstream are stronger than upstream; the chaotic characteristics of monthly runoff from 1950s to the beginning of 21century are slightly stronger than those from 1920s to 1970s in Yellow River; The length of runoff time series has influence on the level of chaotic characteristics, the length is more longer, the level is more stronger. In a whole, the Yellow River runoff has chaotic characteristics, which provide the basis for runoff system modeling and chaotic forecasting.

Keywords: hydrology; river runoff; chaotic characteristic; Yellow River

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INTRODUCTION

The research on runoff changing law is the premise and foundation of rational development and utilization of water resources. Runoff process is a complex hydrological phenomenon, which shows a strong nonlinear characteristic. For a long time, people have been using the traditional random theory or deterministic theory or a combination of the two theories to describe the evolution of runoff which has some limitations. People are unable to describe the nonlinear complex evolution process of runoff by using the traditional theory of Euclidean geometry. In recent years, chaos theory and its applications in various fields have developed rapidly and become a powerful tool to describe the process. Chaos theory reveals the widespread complexity of nature and human society, as well as the unity of order and disorder, the unity of certainty and randomness, which expands people's horizons and deepens the understanding of the objective world. Research on chaos covers all fields of nature and social science. The progress of the research will effectively promote the development of almost all disciplines and areas of technology [1]. Analyzing chaotic time series in hydrology system is a very significant pioneering work. The new ideas and methods of chaos theory have injected new vitality into the study of hydrological sciences, especially experimental analysis. It can be said that the research has moved from the stage of past semi-empirical theory and the statistical theory into the stage of system dynamics theory [2].This research is intended for the Yellow River, using chaos theory to analyze the changing characteristics of runoff and reveal its evolution.

1.1 *Chaos Definition*

Chaos is a seemingly irregular and random-like phenomenon that appears in a deterministic system. The solution that appears in deterministic nonlinear systems and has inherent randomness is called chaotic solution. This solution can be predicted in the short term but not in the long term, thus it is different from identification solution and random solution. Chaos is not a simple disorder but no obvious periodicity and symmetry. It is ordered structure with rich interval levels, a new form of existence in non-linear system [3]. Li-Yorke definition is one of the chaos mathematical definitions with a broader impact [4].

Definition: continuous self-mapping function f is chaotic in interval [a, b], if it meets following conditions:

(1) The period of f periodic points is no upper bound.

(2)There exist uncountable subsets $S \subset [a,b]$, S does not include periodic points and meets:

1) For any x,y∈S, there is $\lim_{n\to\infty}\inf\left|f^n(x)-f^n(y)\right|=0$;

2) For any x, y∈S, there is $\lim_{n\to\infty}\sup\left|f^n(x)-f^n(y)\right|>0$;

3) For any x∈S and any periodic point y of f, there is: $\lim_{n\to\infty}\sup\left|f^n(x)-f^n(y)\right|>0$.

1.2 *Characteristics of Chaotic Motion*

The current study shows that chaotic motion has three distinct characteristics [5-6].

(1) It is extremely sensitively dependent on initial conditions. This feature means that chaos is impossible to be predicted infinitely, even if the two points are very close in initial state, chaotic motion will expand as exponential function with time, and the future stage can not be predicted with decision theory. This is the most essential characteristic of chaos.

(2) Aperiodic feature shows nonlinear and disorder of chaos.

(3) Strange attractor exists in chaos system. Attractor is a point set and a sub-space in phase space. All trajectories tend to it after the transient state dies out with time. The attractor dimension is an integer for deterministic system, but the chaotic attractor dimension is fractional, which is known as strange attractor. The existing of strange attractor in chaotic system makes the trajectory of chaotic system showing some regularity.

2 ANALYSIS OF THE ADAPTABILITY OF CHAOS THEORY'S APPLICATION IN RUNOFF SYSTEM

To analyze chaotic characteristics of runoff series, first we need determine the suitability of chaos theory applied in the runoff system, and then construct the runoff series phase. We should analyze the changing laws of runoff system by applying corresponding chaotic analysis methods. It is agreed generally that chaotic phenomenon will be produced from the non-linear, open, far-from-equilibrium, irreversible process, fluctuations and breaking system. Runoff system is a dynamic non-linear, far-from-equilibrium, and complex open system. It is a whole that is developed by a number of natural factors interacting and influencing [7]. At the same time, it is subject to impacts of different degrees of human activities, thus forming a complex change process of runoff system. Therefore, it is possible that the runoff system shows chaotic characteristics [8-11]. Usually, we can use quantitative and qualitative means, or combine the two, and use as many means as possible to recognize chaotic characteristics of time series.

3 METHODS AND STEPS OF CHAOTIC CHARACTERISTIC ANALYSIS OF RUNOFF

3.1 *Phase Space Reconstruction*

Reconstruction of phase space is the necessary condition of recognizing runoff series chaotic characteristics. Determining embedding dimension m and time delay τ [12-14] is the key factor of reconstructing phase space. Runoff time series can be seen as a power system made up of the n-variable first-order differential equations,

$$dy_i / dt = f_i(y_1, y_2, ..., y_n), i = 1, 2, ..., n \tag{1}$$

The stage space of runoff system with time changing can be showed by the n-dimensional phase space made up of coordinate y(t) and its (n-1) orders derivative.

$$Z(t) = [y(t), y^{(1)}(t), ..., y^{(n-1)}(t)]^T \tag{2}$$

We can use the discrete time series y(t) and its(n-1) time delay displacements to build a new n-dimensional phase space(i.e., embedding phase space), to replace the stage space of runoff system reflected by the continuous variable y(t) and its derivative. That is:

$$Z(t) = [y(t), y(t+\tau), ..., y(t+(n-1)\tau)]^T \tag{3}$$

Where, τ is time delay, also known as lag time. The reconstructed phase space's dimension m should be at least greater than the state space's topological dimension D's (also known as saturated correlation dimension) 2 times plus 1, that is m ≥2D +1.

For a certain observable discrete runoff time series y1, y2, …,y^n, τ is selected, then the reconstructed embedding phase space can be expressed as formula (4), of which, l=n-(m-1)τ

$$\begin{cases} Z_1 = \{y_1, y_{1+\tau}, ..., y_{1+(m-1)\tau}\} \\ Z_2 = \{y_2, y_{2+\tau}, ..., y_{2+(m-1)\tau}\} \\ ... \\ Z_l = \{y_l, y_{l+\tau}, ..., y_{l+(m-1)\tau}\} \end{cases} \tag{4}$$

3.1.1 Determination Time Delayτ of the Phase Space Reconstruction Parameters
Whether the choice of embedding time delay size is suitable will directly affect the effects of reconstructing phase space, and further affect the identification of chaotic characteristics. There are generally three types of methods to determineτvalue: phase space expansion method, serial correlation, and the (partial) multiple autocorrelation between the two.

In this study, (partial) autocorrelation and multiple autocorrelation methods are used respectively to reconstruct phase space of runoff series. Considering the length of the paper, these two methods are not introduced in detail [15].

3.1.2 Determination of Embedding Dimension m in Phase Space Reconstruction
Generally, if the inserting dimension of phase space is big enough, strange attractor of runoff system can be displayed, and motion law of runoff system that traditional methods can't show will be revealed [16]. Therefore, embedding dimension is usually chosen by m≥2D+1 in phase space reconstruction. In this study, saturated correlation dimension method (G-P) is used to determine embedding dimension of phase space [17].

(1)The definition of saturated correlation dimension: we suppose r_{ij} (m) is the absolute value of any two vectors' difference, namely the Euclidean distance, in m-dimensional phase space series Z_1, Z_2,…,Z_l.

$$r_{ij}(m) = \| \boldsymbol{Z_i} - \boldsymbol{Z_j} \| \tag{5}$$

Then, we can give a number r_0, whose value should be between the maximum and the minimum r_{ij}. Group values of $\ln r_0$ and lnC(r) can be calculated when r_0 is adjusted properly. Thus, correlation dimension dm can be figured out through formula (6).

$$d_m = \lim_{r \to \infty} \ln C(r) / \ln(r_0) \tag{6}$$

$$C(r) = \frac{1}{l(l-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{l} H(r_0 - r_{ij}) = \frac{1}{l(l-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{l} H(r_0 - \|Z_i - Z_j\|) \tag{7}$$

In which, *H(x)* is called Heaviside function, defined as follows:

$$H(x) = \begin{cases} 0, x < 0 \\ 1, x \geq 0 \end{cases} \tag{8}$$

(2) Problem-solving ideas of saturated correlation dimension method
According to the definition of correlation dimension, if there is strange attractor in the observation sequence {y (t)}, d_m correlation dimension will also increase following the increase of phase space embedding dimension. d_m will reach the saturation value D ,namely saturated correlation dimension of time series when the correlation dimension increases to a certain value. The main idea of G-P method is to choose a different area radius r_0, calculate the corresponding C(r), and put these different r_0 and C(r) into formula (6) to fit dm. The saturation value D of dm

can be obtained following the increase of m. According to m ≥ 2D +1, the appropriate embedding dimension m can be determined in the system.

3.2 *The Method of Chaotic Characteristic Analysis*

3.2.1 Saturated correlation dimension method

The diagram lnr0 ~ lnC(r) can be drawn under different conditions of embedding dimension according to the calculating result of formula （5）-（8）. If there is a straight-line part of the figure, then the slope of the straight part is the correlation dimension corresponding to the embedding dimension m. Draw the diagram of the obtained correlation dimension dm and different embedding dimension m. The correlation dimension tends to be stable when embedding dimension m reaches saturation value D, also called saturated correlation dimension. Then the system has stable chaotic attractor dimension, which indicates the runoff series has chaotic characteristics.

3.2.2 The maximum Lyapunov index

Lyapunov index is a lineage λi (i = 1,2, ..., m-1). If the only maximum value is positive in the spectrum, the system is one-dimensional chaotic; If the spectrum has two or more values being positive, the runoff system is multi-dimensional chaotic or hyper chaotic. Otherwise, the system is non-chaotic. So the max Lyapunov index can be used to analyze chaotic characteristics. The bigger its value is, the stronger the chaotic characteristics are, and the more sensitive is for initial value. Just because of the initial sensitivity of chaotic system, the long-term prediction of chaotic system is not possible. Typically, the reciprocal of the index can be used as a predictable scale of the system. For time series $y_1, y_2,\ldots,y_n$, the calculation method is as follows:

First, we can reconstruct phase space through the method described in 3.1, then according to formula (9),obtain the nearest neighbor point Z_j of Euclidean space sense Z_i and the distance between two points L_i, finally find max Lyapunov index λ according to formula (10).

$$L_i = \overset{i\neq j}{\min}\left[\left\|Z_j - Z_i\right\|\right], i, j = 1,2,...,l \quad (9)$$

$$\lambda = \frac{1}{l-1}\frac{1}{\tau}\sum_{i=1}^{l-1}\ln\frac{L_{i+1}}{L_i} \quad (10)$$

λ is system max Lyapunov index, when λ>0, indicating that the system has chaotic characteristics; when λ=0, indicating the system has a bifurcation point or periodic solution, that is ,cycle phenomenon appears in system; when λ<0, the system has a stable fixed point.

3.2.3 Principal component analysis

The method can identify chaos and noise effectively. If a given one-dimension time series is known as $y_1,y_2,y_3,\ldots,y_n$, the phase space is reconstructed when sampling interval is τand selected embedding dimension is m. So the trajectory matrix formed by the time series $Z_{l\times m}\left(l = n-(m-1)\tau\right)$[18] is as follows.

$$Z_{l\times m} = \frac{1}{l^{1/2}}\begin{bmatrix} y_1 & y_{1+\tau} & \cdots & y_{1+(m-1)\tau} \\ y_2 & y_{2+\tau} & \cdots & y_{2+(m-1)\tau} \\ \vdots & \vdots & \vdots & \vdots \\ y_l & y_{l+\tau} & \cdots & y_{l+(m-1)\tau} \end{bmatrix} = \frac{1}{l^{1/2}}\begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_l \end{bmatrix} \quad (11)$$

Covariance matrix A is:

$$A_{m\times m} = \frac{1}{l}Z_{l\times m}^T Z_{l\times m} \quad (12)$$

Calculate the eigenvalue λi(i=1,2,3,…,m) of covariance matrix A and eigenvector Ui(i=1,2,3,…,m). Sort the eigenvalues by size: λ1≥λ2≥…≥λm.

Then eigenvalue λ_i and eigenvector U_i are called principal component. Find the sum γ of all the eigenvalues.

$$\gamma = \sum_{i=1}^{m}\lambda_i \quad (13)$$

The graph with index i as abscissa and $Ln(\lambda_i/\gamma)$ as ordinate is called principal component spectrum. The principal component spectrum of chaotic sequence is a straight line with a negative slope or the points' fitting line with a negative slope. The noise spectrum is a nearly parallel line with the horizontal axis.

4 ANALYSIS OF CHAOTIC CHARACTERISTICS OF YELLOW RIVER RUNOFF

4.1 *BASIC INFORMATION*

Taking runoff series of Lanzhou， Sanmenxia，Huayuankou station of the Yellow River. The main measured and natural runoff data are 1955—2005.The total length N of the runoff sequence is 612 months. In addition, other runoff data of different lengths are experimentally analyzed. Data lengths are in line with the requirements of chaotic characteristic analysis [19-20].

RESULTS AND ANALYSIS

4.2.1. Parameter determination of phase space reconstruction of runoff series

(1) Determination of time delay τ value. The phase space reconstruction parameter τof natural and measured runoff series is calculated by using the method described in 3.1.1.The results are shown in Table 1, the calculation process shown in Figure 1-4. The paper lists only the measured runoff calculation diagram avoiding the paper being too long. According to autocorrelation features (correlation coefficient diagram through zero point first time), considering the factors that calculated value is larger than actual value, integrating the calculation of various methods, then taking coefficient τ of phase space reconstruction as 2 is more appropriate.

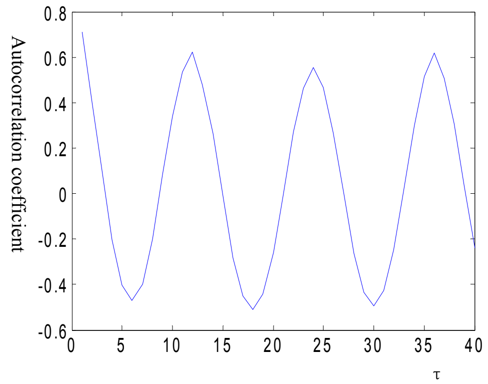

Fig. 1 the relation of autocorrelation coefficient and time delay of monthly runoff series in the Lanzhou station

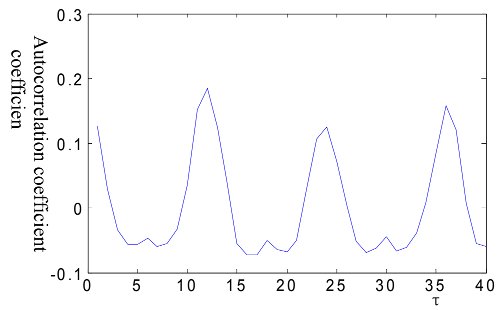

Fig. 2 the relation of autocorrelation coefficient and time delay of monthly runoff series in the Sanmenxia station

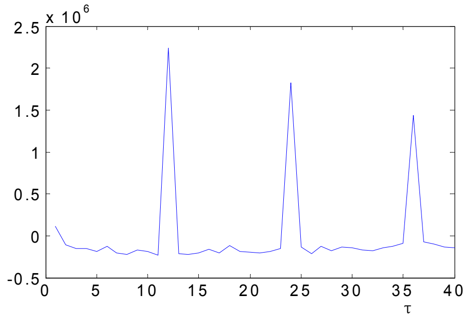

Fig. 3 the relation of complex autocorrelation and time delay of monthly runoff series in the Lanzhou station

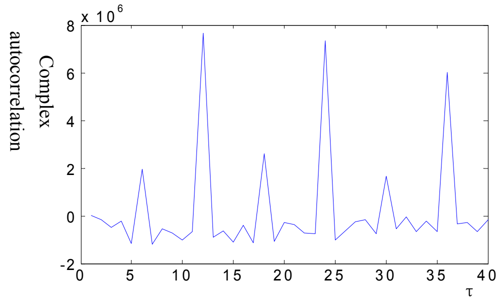

Fig . 4 the relation of complex autocorrelation and time delay of monthly runoff series (1960-2006) in the

Table.1 time delay τ

methods / station		Method 1	Method 2	Method 3
Lanzhou	Measured	3.15	1.64	1.51
	Natural	3.25	1.68	1.00
Sanmenxia	Measured	2.35	1.72	1.12
	Natural	3.05	2.13	1.05
Hyuankou	Measured	3.35	2.01	1.52
	Natural	3.45	2.12	1.54

Note: Method 1 is about Autorelation method (figure first time through zero); Method 2 is about Auto relation method (τfirst time through the initial value' (1-1/e) times); Method 3 is about multiple autocorrelation.

(2) Determination of embedding dimension m

According to 3.1.2, r_0 equals {100,150,200…, 5000}, phase space embedding dimension m equals {2, 3, 4, 5, 15}, and τequals 2, C(r) can be calculated. The runoff series $\ln r_0 \sim \ln C(r)$ figures 5-6 are drawn in the condition of different embedding dimensions. Figures of the correlation dimension and embedding dimension relation figures 7-8 are also drawn. The calculating results are in table 2.

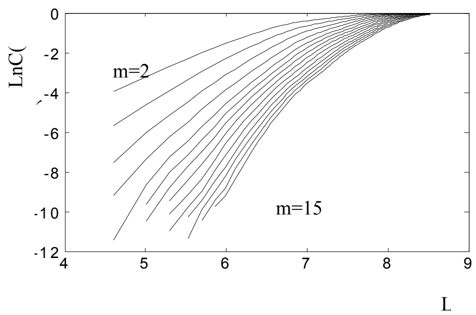

Fig. 5 Relationship between $\ln r_0$ and $\ln C(r)$

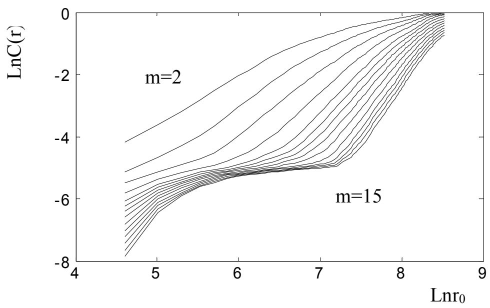

Fig. 6 Relationship between $\ln r_0$ and $\ln C(r)$ in the Sanmenxia station

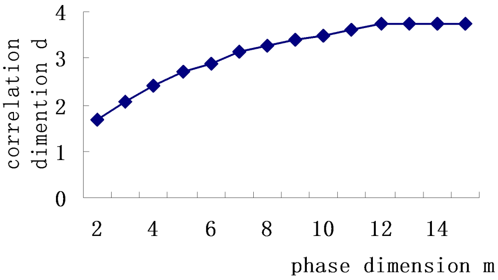

Fig.7 Relationship between m and d in the Lanzhou station

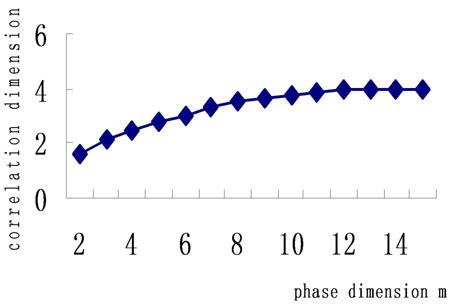

Fig.8 Relationship between m and d in the Sanmenxia station

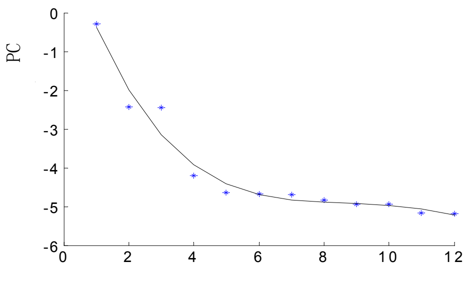

Fig13 the relation of dimension i and j PCA in the Lanzhou station

Table.2 saturation relation dimension and the most Lyapunov exponential of every station

Item Station		Time delay	Embedding dimension	Saturator relation dimension	Lyapunov index value
Lanzhou	Measured 1	2	12	3.73	0.3140
	Natural 1	2	12	5.42	0.1561
	Natural 2	2	12	5.15	0.2500
	Natural 3	2	12	5.23	0.0449
Sanmenxia	Measured 2	2	12	3.98	0.3675
	Natural 1	2	12	5.76	0.0322
	Natural 2	2	12	5.31	0.3000
Hyuankou	Measured 1	2	12	4.07	0.4423
	Natural 1	2	12	6.18	0.2206
	Natural 2	2	12	5.83	0.4166

Note: Measured 1 is about 1955-2005 measured runoff series; Measured 2 is about 1960-1998 measured runoff series; Natural 1 is about 1955-2005 natural runoff series; Natural 2 is about 1920-1970 natural runoff series; Natural 3 is about 1920-1998 natural runoff series.

4.2.2 Runoff series chaotic characteristic identification

(1) Analysis of saturated correlation dimension calculation result

According to 3.1.2， the Yellow River runoff series saturated correlation dimension is calculated, as shown in Table 2. Calculating correlation dimension has a variety of meanings: on the one hand for the time series of random changes, as the embedding dimension increases, the correlation dimension will continue to grow. For chaotic time series, when the embedding dimension m reaches value D, the correlation dimension tends to be stable and reaches a saturation value, also known as the saturated correlation dimension, while the corresponding minimum embedding dimension indicates the effective freedom degree dimension of dynamic system. When the embedding space dimension reaches D-dimensional, the system has a stable chaotic attractor dimension, indicating that the system has chaotic characteristics. On the other hand, saturated correlation dimension shows that the minimum number of independent variables using to describe the system is INT (D +1), the maximum number is INT (2D+1), and INT is the lower integral function.

Analyze the result from table 2, and take the measured monthly runoff series of Huayuankou station for example. When the embedding dimension m reaches 12, the correlation dimension tends to be stable and reaches a saturation value D= 4.07, the corresponding minimum embedding dimension shows the effective freedom degree dimension of the power system. It is clear that the measured monthly runoff series has chaotic characteristics in Huayuankou station. When the embedding dimension comes 12; the system has a stable chaotic attractor dimension 4.07. Similarly, the measured and natural monthly runoff series in Lanzhou and Sanmenxia stations have chaotic characteristics, but the dimensions of chaotic attractors are not exactly the same. The greater dimension of chaotic attractors is, the more factors affect the formation process of the runoff series, which is determined by different characteristics of runoff series itself. Data in the table show that the chaotic attractor dimension of 1955-2005 measured monthly runoff series in Lanzhou Station is relatively small, being 3.73, which indicates it needs at least 4 independent Variables to properly describe the changing characteristics of the runoff system and model dynamic system. But the chaotic attractor dimension of 1955-2005 natural monthly runoff series in Huayuankou Station is relatively large, being 6.18, which indicates it needs at least 7 independent variables to describe the changing characteristics of runoff series.

(2) The maximum Lyapunov index

According to the calculation method of the maximum Lyapunov index 3.2.2, τ equals 2, and the embedding dimension m is 12. The maximum Lyapunov index values of measured and natural monthly runoff series in Lanzhou, Sanmenxia and Huayuankou station are shown in table 2. The following conclusions can be drawn from table 2. The chaotic characteristics of measured runoff have slightly stronger than that of natural runoff at the same hydrologic station. From 1950s to the beginning of 21century, the chaotic characteristic of monthly runoff is stronger than that from 1920s to 1970s.(i.e., slightly stronger in modern time than the past). For natural monthly runoff series, the longer the series are, the stronger its chaotic characteristics are. In addition, comparing calculation results of Lanzhou station to Huayuankou station, it can be seen that the downstream runoff series has stronger chaotic characteristic than upstream runoff in both natural data and measured data.

(3) Principal component analysis

Using the principal component analysis method introduced by 3.2.3 to study the measured and natural monthly runoff series of Lanzhou, Sanmenxia and Huayuankou stations, then can draw the relationship map between the embedding dimension i and the principal components PCA ,namely the main component spectra , and fit the trend shown in figure 9-10. According to the nature of the principle component spectra of chaotic time series, the following graphs have the linear parts of negative slope, which further confirm that the runoff series has chaotic characteristics, and provide basis to building chaotic prediction model of monthly runoff. However, the difference in the strength of the chaotic property is difficult to clear from the figures.

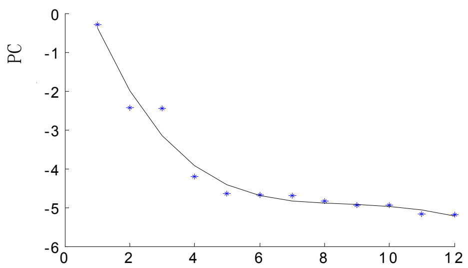

Fig9 the relation of dimension i and j PCA in the Lanzhou station

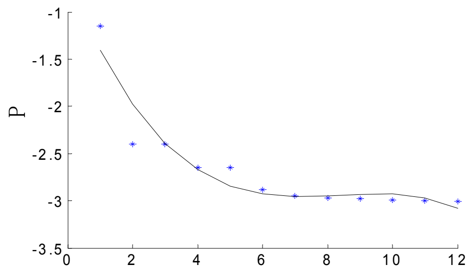

Fig10 the relation of dimension i and PCA in the Sanmenxia station

CONCLUSION

Based on the study on chaotic analysis methods of runoff time series, Yellow River monthly runoff series is analyzed in chaotic characteristics and the following conclusions are drawn.

(1)Time delay$\tau$and embedding dimension m are two important parameters in the phase space reconstruction of runoff time series and play a key role in phase space reconstruction.

(2)The time delay $\tau$of phase space reconstruction of monthly runoff time series equals 2 in main hydrologic stations of Lanzhou, Sanmenxia and Huayuankou of Yellow River. When the embedding dimension m reaches 12, system has the saturated correlation dimension.

(3) At the same hydrologic station, the saturated correlation dimension of natural monthly runoff is bigger than that of the measured one. It needs at least 4 variables (i.e. 4 factors)and at most 8 variables (i.e. 8 factors) to describe the changing characteristic of measured runoff series and build dynamic models, but least 5-6 and most 12 for natural runoff.

(4) The measured runoff has different chaotic characteristics from natural runoff at the same hydrologic station during the same period. For Yellow River mainstream, the chaotic characteristics of measured monthly runoff series are stronger than those of natural series. For different hydrologic stations, the downstream has stronger chaotic characteristics than the upstream. From 1950s to the beginning of 21 century, the chaotic characteristic is slightly stronger than that from 1920s to 1970s (i.e. now is stronger than the past).

(5) The length of runoff time series has influence on identification of chaotic characteristics. The length is longer, the chaotic characteristics are stronger. The runoff of Yellow River has chaotic characteristics, which provides basis for runoff modeling and chaotic forecasting.

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