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An efficient iterative method in numerical calculation

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ABSTRACT

Systems of linear equations appear in many areas of scientific and engineering calculations in common. The iterative method is the basic method for solving large systems of linear equations, the basic issue involved iterative method is to construct an iterative format. However, the iterative format that we have constructed is meaningful it depends on the iteration whether is convergent. For a convergent iterative scheme, its value will depend on the speed and efficiency of convergence of the iterative process. Firstly, some classical iterative methods (Jacobi method, Gauss-Seidel method, SOR method, SSOR method, etc.) are discussed in detail, and in the theoretical analysis the convergence. Then, given a practical iterative method, in this way it can depart from any approximation, and does not require any of the original equations of transformation, greatly facilitate our operations on a number of issues, and effectiveness through numerical calculation example for the iterative method was validated.

Keywords: Iterative method, Systems of Linear Equations, Convergence, Symmetric positive definite

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INTRODUCTION

Engineering numerical often encountered solving large linear equations, and the characteristics of these equations is quite large dimension, even so that people can not get solved by direct methods within the time allowed, for this iteration method introduced.

Now we have a lot of solutions of linear equations in many different iterative methods. As Jacob iterative method, Seidel iterative method, SOR method and conjugate gradient method, etc. However, an effective way of new to exploring is a task urgently.

The problem is that most of the current iteration not all types of linear equations have convergence, and to the original equation for a fundamental transformation matrix, has the potential to significantly deteriorate the condition number, thereby undermining the transformation equations and the equivalence of the original equations, there will also make the original equations lose some key advantages, such as symmetry, ribbon, as well as sparse and so on[1].

The purpose of this paper is to provide an iterative method; it can depart from any initial approximation, and, in theory, does not require any conversion of the original information. This method can also be some distortion and promotion. This paper studies the practical application of iterative methods for solving large sparse linear equations in. Often occurs when the numerical solution of partial differential equations using finite difference or finite element method such equations, the number of unknowns which can have hundreds to as many as several million.

Direct method and iterative method can be used to solve linear equations. For the equations for medium-sized, direct method has an advantage. Iterative method is solving linear equations mainly large and complex coefficients. For this kind of problem, if you use a direct method for solving, requires excessive amount of storage space and arithmetic, it is not possible or is not efficient, for example, Iterative method is usually used to solve the problem with three spatial variables, by the simultaneous nonlinear equations or discrete partial differential equations derived

from more than one spatial variable and unsteady problems[2].

The three main obstacles to effective use of iterative method is as follows:
(1) How to decide which one to use iterative methods and how to implement a given method.
(2) How to choose the iteration parameters needed some iterative method (for example, successive over- relaxation method needed relaxation factor).
(3) When should terminate the iterative process.
Because of the requirements is a wide variety of solutions to problems, also available are many iterative method chosen, so absolutely impossible to answer the above questions.

Because of the requirements is a wide variety of solutions to the problem, but also a lot of iterative methods are available, it is impossible to answer the question above absolute.

Performance when faced with a specific problem, how to choose an effective iterative method, mainly on the issue of the characteristics and the use of the computer. Therefore, it is impossible to give a general iterative method to select the best rule[3]. However, with the knowledge of the relative merits of several ordinary iterative methods can greatly simplify this task. Way we pick iterative method is to introduce several common characteristics in theory and method of calculation, and then put these ordinary iterative methods as an alternative basis in reality an effective iterative method.

For each common approaches considered, our aim is to give a better estimate can automatically determine the parameter values required iterations, and can automatically determine when to terminate the iterative process of calculation procedures. Calculation procedures are given programming language with an informal form of algorithms. In most cases, the narrative is quite complete and self-contained.

BASIC ITERATIVE METHOD

In this paper, solving linear equations

$$Ax = y \tag{1}$$

Where A is a known non- singular matrix $N \times N$, b are known real column matrix $N \times 1$.

In this method involves is a first-order linear time-invariant method, these methods can be expressed in the form

$$x^{(n+1)} = Gx^{(n)} + k,\ n = 1, 2, \cdots \tag{2}$$

Wherein G is real matrix $N \times N$ iteration, k is a vector of related known this method is a first-order, because $x^{(n+1)}$ obviously depend only $x^{(n)}$, without significant reliance $x^{(n-1)}, ..., x^{(0)}$. The method is linear, because G and k it is also not dependent on $x^{(n)}$. Steady as G and k are independent of the n. We have any form such as (2) a method known as a basic iterative method.

BASIC GAUSS-SEIDEL ITERATION METHOD

Applied to the block of linear equations

$$\begin{pmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,q} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,q} \\ \vdots & \vdots & & \vdots \\ A_{q,1} & A_{q,2} & \cdots & A_{q,q} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_q \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_q \end{pmatrix} \tag{3}$$

The Gauss-Seidel method is defined as

$$A_{i,j} X_j^{(n+1)} - \sum_{j=i+1}^{q} A_{i,j} X_j^{(n)} = F_i, \quad i = 1, 2, ..., q \tag{4}$$

Like with the Jacobi method, Gauss-Seidel iteration method for each step must be the solution of the form (4) of the equations.

Set up $A = D - C_L - C_X$,

Among

$$D=\begin{pmatrix} A_{1,1} & 0 & 0 & 0 \\ 0 & A_{2,2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & A_{q,q} \end{pmatrix},\ C_X=-\begin{pmatrix} 0 & A_{1,2} & A_{1,3} & \cdots & A_{1,4} \\ 0 & 0 & A_{2,3} & \cdots & A_{2,q} \\ 0 & 0 & 0 & \cdots & A_{3,q} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$

$$C_L=-\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ A_{2,1} & 0 & 0 & \cdots & 0 \\ A_{3,1} & A_{3,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ A_{q,1} & A_{q,2} & A_{q,3} & & 0 \end{pmatrix}$$

The formula (4) can be written as

$$(D-C_L)x^{(n+1)}=C_Xx^{(n)}+b \tag{5}$$

Therefore, Gauss-Seidel method can be expressed in matrix form

$$x^{(n+1)}=\zeta x^{(n)}+k$$

Among

$$\zeta\equiv(I-L)^{-1}U, k\equiv(I-L)^{-1}D^{-1b}$$

And

$$L\equiv D^{-1}C_L, X\equiv D^{-1}C_X \tag{6}$$

Matrix A is called Gauss-Seidel iteration matrix.

Gauss-Seidel method of splitting matrix ($D-C_L$), it is not the SPD matrix and, Gauss-Seidel method is generally not summarized.

Since the matrices A and D are SPD, Gauss-Seidel method to prove always convergent. However, although the Eigen values A of the mold is less than 1, which is typically a complex value. The ζ set of feature vectors and may contain base corresponding vector space. Then extrapolation can not be used.

However, under certain some conditions extrapolation still available. Extrapolation can improve the approximate convergence. Notably, the problem of the natural order of the extrapolation of the Gauss-Seidel method of the convergence rate can be significantly less than approximately convergence speed. This is because in this case the matrix ζ may have zero Eigen values and their corresponding second or higher order of primary vector.

Successive over relaxation (SOR) method [4]

Definition 1 Corresponding to the division

$$\begin{pmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,q} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,q} \\ \vdots & \vdots & & \vdots \\ A_{q,1} & A_{q,2} & \cdots & A_{q,q} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_q \end{pmatrix}=\begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_q \end{pmatrix}$$

The SOR method is defined as

$$A_{i,i}U_i^{(n+1)}=\omega\left\{-\sum_{j=1}^{i-1}A_{i,j}X_j^{(n+1)}-\sum_{j=i+1}^{q}A_{i,j}X_j^{(n)}+F_i\right\}+(1-\omega)A_{i,i}X_i^{(n)},\ \ i=1,2,...,q \tag{7}$$

Where ω is a real number, become the relaxation factor. ω = 1 SOR methods reduce to Gauss-Seidel method. Are called ultra- low relaxation and relaxation when ω >1and ω <1.

Definition 2 Using equation (5) of the matrix notation, SOR Method of formula (7) can be written as

$$Dx^{(n+1)}=\omega(C_Lx^{(n+1)}+C_Xx^{(n)}+b)+(1-\omega)Dx^{(n)}$$

Redrafting there

$x^{(n+1)} = \zeta_\omega x^{(n)} + k_\omega^{(F)}$

Among

$\zeta_\omega \equiv (I - \omega L)^{-1}(\omega X + (1-\omega)I)$

$k_\omega^{(F)} \equiv (I - \omega L)^{-1}\omega D^{-1}b$

L , X are given by (6). Matrix called the SOR iterative matrix.

SOR method of splitting matrix ($\omega^{-1}D^{-1}C_L$), and the Gauss-Seidel method, as it is not a SPD matrix. A SOR method is not available when the summarization. Moreover, the ultra- loose matrix a general is always a certain complex Eigen values. Therefore, ω >1extrapolation does not apply to SOR method.

Since the matrices A and B are SPD, SOR method can prove any satisfy 0< ω <2 in ω converge, and so often choose the appropriate SOR method ω fast convergence. If the coefficient matrix A for the division made compatible with the nature of the order of A and there, then make the SOR method usually gives the fastest convergence expression "optimal" ω values.

For the model problem, denoted A optimal value A, which was expressed as

$$\omega_b = \frac{1}{1+\sqrt{1-M(B)^2}} = \begin{cases} \dfrac{2}{1+\sin\pi h}\,\text{pointSOR} \\ \dfrac{2(1+2\sin^2\pi h/2)}{(1+\sqrt{2}\sin\pi h/2)^2}\,\text{lineSOR} \end{cases}$$

The ζ_ω is corresponding spectral radius equal to ($\omega_B - 1$)), and can be expressed as

$$S(\zeta_{\omega_b}) = \frac{1-\sqrt{1-M(B)^2}}{1+\sqrt{1-M(B)^2}} = \begin{cases} \dfrac{1-\sin\pi h}{1+\sin\pi h}\,\text{pointSOR}, \\ \left(\dfrac{1-\sqrt{2}\sin\pi h/2}{1+\sqrt{2}\sin\pi h/2}\right)^2\,\text{lineSOR} \end{cases}$$

From the above equation can be proved that the asymptotic convergence rate of SOR method satisfies

$$R_\infty(\zeta_{\omega_B}) \sim \begin{cases} 2\pi h\,\text{pointSOR}, \\ 2\pi h\sqrt{2}\,\text{lineSOR}, \end{cases} \quad h \to 0.$$

Astringency

Definition 3

We always assume the existence of a non singular matrix Q , will do

$$G = I - Q^{-1}A, k = Q^{-1}b \tag{8}$$

The matrix Q is called the splitting matrix.

Definition 4 Note that $\overline{x}$ is nonsingular， Assumption (8) implies that A is derived from the solution of the equation,

$$(I-G)x = k \tag{9}$$

Only if $\overline{x}$ is the sole solution,

$\overline{x} = A^{-1}b$

If the derived equation (9) has a unique solution $\overline{x}$ ， And $\overline{x}$ is the type (1) solution， we will say the method corresponding (2) is completely compatible。

Definition 5 Method (2) is convergent, If for any $x^{(0)}$, by (2) sequence determination of $x^{(1)}, x^{(2)}$…, converges to the $\overline{x}$.

Necessary and sufficient condition for the convergence is

$S(G) \prec 1$

In order to measure the linear iterative method (2) convergence, we define the error vector

$\varepsilon^{(n)} \equiv x^{(n)} - \overline{x}$

Using (1), and noted that $\overline{x}$ satisfies the equation (3). We can conclude that

$\varepsilon^{(n)} = G\varepsilon^{(n-1)} = G^n \varepsilon^{(0)}$

Therefore, for any vector β norm and β norm of the corresponding matrix, using

$\|Av\|_\partial \le \|A\|_\partial \|v\|_\partial$

We can draw

$\left\|\varepsilon^{(n)}\right\|_\beta \le \left\|G^n\right\|_\beta \left\|\varepsilon^{(0)}\right\|_\beta$

The $\left\|G^n\right\|_\beta$ gives a scale, which represents can disease the n step iterative error

We use that

$R_n(G) \equiv -n^{-1} \log \left\|G^n\right\|_\beta$

Giving a definition for method (2) of the average speed of convergence

Definition 6

If $S(G) \prec 1$, then

$\lim_{n\to\infty}\left(\left\|G^n\right\|_\beta^{1/n}\right) = S(G)$

Because of this property, we call the A asymptotic rate of convergence.

$R_\infty(G) \equiv R_n(G) = -\log S(G)$

Definition 7

The iterative method (2) is called the symmetry, If there is a non singular matrix W, make $W(I-G)W-1$ is a SPD matrix. This matrix W is called a symmetric matrix .The absence of matrix W, the iterative method (2) is called the not symmetry.

Other methods

Giving you right now a very brief overview of Approximate factorization method， Alternating direction implicit method and Fast direct method。

The basic method of approximate factorization based

This matrix Q is determined by the coefficient matrix A decomposition into $A = Q - R$， obviously，The basic iterative method can be written in the following form

$$Qx^{n+1} = Rx^n + b \tag{10}$$

The iterative matrix is now considered matrix has the form. Then， Non singular matrix H and K should make them easy to calculate, but also make the matrix is "easy inversion". [4]In addition, in order to make the convergence speed as high as possible, the matrices and selection should also make the spectral radius of the iterative matrix as small as possible.

If A is symmetric and positive definite, then an alternative approach is to take $H = L^T$ and $K = L$, where L is an upper triangular matrix by $A = L^T L$ decomposition is determined by the A of Cholesky. Then $R = 0$, process by one step iteration convergence is reached. In fact, this is just another form of Gauss direct solution. This method, when A is a large sparse matrix problems, matrix $H = L$ may not be easy to calculate.

In order to avoid the complete decomposition of A that brings these problem, one option is to take H and K were lower and upper triangular matrix, but the product of HK approximately equal to A. The usual practice is, first of all to specify particular sparse form H and K, and then determine H and K non-zero element, make the

product HK as close as possible to the A . This method includes a series of iterative technique, the difference between them mainly lies in the H and K options

If A is a symmetric positive definite matrix, then usually can use the incomplete symmetry factor decomposition $Q = HH^T$. If Q is positive, then the basic method (10) is symmetric, which can use the Chebyshev and conjugate gradient procedure to accelerate the convergence speed.

Alternating direction implicit method
If the matrix A can be expressed as

$$A = H + V \tag{11}$$

It assumes that A, H, V is SPD matrix, then the Peaceman-Rachtord method is defined as

$$\left(H + \rho_n I\right) x^{(n+1/2)} = b - \left(V - \rho_n I\right) x^{(n)} \tag{12}$$

$$\left(V + \rho_n' I\right) x^{(n+1)} = b - \left(H - cI\right) x^{(n+1/2)} \tag{13}$$

Here the assumption that for any positive integer ρ_n and ρ_n', From(12) , a known $x^{(n)}$ for $x^{(n+1/2)}$,and from (13), a known $x^{(n+1/2)}$ for $x^{(n+1)}$. If the linear equations are obtained by elliptic partial differential equations, then H and V can take three diagonal matrix, or at least narrow bandwidth matrix. For the finite difference method, H is corresponding to the horizontal difference matrix, V is the corresponding matrix to the vertical difference.

People used to recycle a specific set of parameters ρ_1， ρ_1'， ρ_2， ρ_2'， …， ρ_m， ρ_m .The value of ρ, H and V related to the Eigen values of the world. When the grid side h tends to zero, can be appropriately choosing these parameters, the number of iterations required for convergence orders of magnitude change in the $\log h^{-1}$. This is in sharp contrast with SOR method, the latter is h^{-1} .

This method is often used for solving linear equations of specific types. In the solution of rectangular domain on the constant coefficient elliptic partial differential equation, the linear equation group often appears. Rapid method is based on the discrete problems: solutions in closed form. This solution is similar to from "Fourier series separable" continuous problem. For the discrete case, the solution is expressed as having specific properties limited and form, these properties enable us to quickly find the finite. This kind of method is usually called the fast Fourier method. Fast direct method can be considered linear equations more general [5].

AN EFFECTIVE ITERATIVE METHOD

The iterative process

To the solution of linear equations (1), of which: $A = \{a_{ij}\}$ is the coefficient matrix of equations, $i = 1 \cdots n, j = 1 \cdots n;\ y = (y_1, y_2, ..., y_n)^T$ is a known vector; $x = (x_1, x_2, ..., x_n)^T$ is the requirement vector. $[a, b]$ Express the vector $a = (a_1, a_2, ..., a_n)^T$ and $b = (b_1, b_2, ..., b_n)^T$, and.

$$[a,b] = a_1 b_1 + a_2 b_2 + ... + a_n b_n$$

Therefore， $[a, a] = \|a\|^2$. The equations (1) to rewrite the vector form:

$$[a_i^T, x] = y_i, i = 1, \cdots, n \tag{14}$$

Where $a_i = (a_{i1}, a_{i2}, ..., a_{in})$ is the i matrix A

Study the following iterative process

The vector $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, ..., x_n^{(0)})^T$ as an initial approximation, to

$$x^{(1)} = x^{(0)} + W^{(1)} a_{i_0}^T, \quad W^{(1)} = \frac{r_{i_0}(x^{(0)})}{[a_{i_0}^T, a_{i_0}^T]}$$

$$\left| y_{i_0} - [a_{i_0}^T, x^{(0)}] \right| = \max_{1 \le i \le n} \left| y_i - [a_i^T, x^{(0)}] \right|, r_{i_0}(x^{(0)}) = y_{i_0} - [a_{i_0}^T, x^{(0)}]$$

$$x^{(2)} = x^{(1)} + W^{(2)} a_{i_1}^T, \quad W^{(2)} = \frac{r_{i_1}(x^{(1)})}{[a_{i_1}^T, a_{i_1}^T]}$$

$$\left| \ y_{i_1} - [a_{i_1}^Y, x^{(1)}] \ \right| = \max_{1 \le i \le n} \left| y_i - [a_i^T, x^{(1)}] \right|, \ r_{i_1}\left(x^{(1)}\right) = y_{i_1} - [a_{i_1}^T, x^{(1)}]$$

… … …

$$x^{(k+1)} = x^{(k)} + W^k a_{i_k}^T \tag{15}$$

$$W^{(k)} = \frac{r_{i_r}(x^{(k)})}{[a_{i_k}^T, a_{i_k}^T]} \tag{16}$$

$$\left| y_{i_k} - [a_{i_k}^T, x^{(k)}] \right| = \max_{1 \le i \le n} \left| y_i - [a_i^T, x^{(k)}] \right| \tag{17}$$

$$r_{i_k}(x^{(k)}) = y_{i_k} - [a_{i_k}^T, x^{(k)}] \tag{18}$$

Assume that equation (1) has a unique solution, and $x = (x^* = x_1^*, x_2^*, ..., x_n^*)^T$ says its solution. Then theorem 1

Iteration theorem

The iterative process (15) - (18) converges to the solution of equation (1), and

$$\lim_{k \to \infty} x^{(k)} = x^* \tag{19}$$

Proving $\left\| x^* - x^{(k+1)} \right\|^2 = \left[x^* - x^{(k+1)}, x^* - x^{(k+1)} \right]$

$$= \left[x^* - x^{(k)} - W^{(k)} a_{i_k}^T, x^* - x^{(k)} - W^{(k)} a_{i_k}^T \right]$$

$$= \left[x^* - x^{(k)}, x^* - x^{(k)} \right] - \left[x^* - x^{(k)}, W^{(k)} a_{i_k}^T \right]$$

$$-W^{(k)} \left[a_{i_k}^T, x^* - x^{(k)} \right] + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right]$$

$$= \left\| x^* - x^{(k)} \right\|^2 - 2W^{(k)} \left[a_{i_k}^T, x^* - x^{(k)} \right] + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right]$$

Notice the type (14) (16) (18), available:

$$\left\| x^* - x^{(k+1)} \right\|^2 = \left\| x^* - x^{(k)} \right\|^2 - 2 \ W^{(k)} \left(\left[a_{i_k}^T, x^* \right] - \left[a_{i_k}^T, x^{(k)} \right] \right) + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right]$$

$$= \left\| x^* - x^{(k)} \right\|^2 - 2W^{(k)} \left(y_{i_k} - \left[a_{i_k}^T, x^{(k)} \right] + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right] \right)$$

$$= \left\| x^* - x^{(k)} \right\|^2 - 2W^{(k)} r_{i_k}(x^{(k)}) + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right]$$

$$= \left\| x^* - x^{(k)} \right\|^2 - 2\left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right] + \left(W^{(k)} \right)^2 \left[a_{i_k}^T, a_{i_k}^T \right]$$

$$= \left\| x^* - x^{(k)} \right\|^2 - \left(W^{(k)} \right)^2 \left\| a_{i_k} \right\|^2 \tag{20}$$

Because of

$$\left(W^{(k)} \right)^2 \left\| a_{i_k} \right\|^2 > 0$$

Can draw

$$0 \le \left\| x^* - x^{(k+1)} \right\| \ \le \left\| x^* - x^{(k)} \right\| \tag{21}$$

This shows that the sequence $\left\| x^* - x^{(k)} \right\|, k = 1, 2, \cdots$ is a monotonically decreasing and the zero lower bound. So there is a limit.

If it is assumed that the $\lim_{k \to \infty} x^{(k)} = z \neq 0$, let $z = x^{(0)}$, formula (15) - (18) can be determined according to the $x^{(1)}$, the (21) type Contradiction.

$$\left\| x^* - z \ \right\| = \lim_{n \to \infty} \left\| x^* - x^{(k)} \right\| \le \left\| x^* - x^{(1)} \right\| < \left\| x^* - z \right\|$$

So $\|x^* - z\| = 0$, and $x^* = z$.

Numerical examples

Two order equations

$$\begin{cases} x_1 + 3x_2 = 4 \\ 2x_1 - x_2 = 1 \end{cases}$$

A unique solution of $x_1 = 1, x_2 = 1$, several iterations may have:

Let $x_1^{(0)} = 0$, $x_2^{(0)} = 0$ so $r_{i_0}(\mathrm{x}^{(0)}) = 4, W^{(1)} = \frac{2}{5}$; similar to $x_1^{(2)} = \frac{24}{25}$, $x_2^{(2)} = \frac{23}{25}$; $x_1^{(3)} = \frac{247}{250}$,

$x_2^{(3)} = \frac{251}{250}$, $x_1^{(4)} = \frac{1249}{1250} \approx 0.9989$,

$x_2^{(4)} = \frac{1248}{1250} \approx 0.9983$, $x_1^{(5)} = \frac{12497}{12500} \approx 0.9998$, $x_2^{(5)} = \frac{12501}{12500} \approx 1.0001$

From the above result, fourth iterations error is less than 1%, for the fifth time after iteration error is less than 0.1%.

Convergence analysis

By (20) and (16), (18), (14) type available

$$\|x^* - x^{(k+1)}\|^2 = \|x^* - x^{(k)}\|^2 - \left(W^{(k)}\right)^2 \|a_{i_k}\|^2 = \|x^* - x^{(k)}\|^2 - \left(\frac{r_{i_k}(x^{(k)})}{\left[a_{i_k}^T, a_{i_k}^T\right]}\right)^2 \|a_{i_k}\|^2$$

$$= \|x^* - x^{(k)}\|^2 - \frac{\left(y_{i_k}^k - \left[a_{i_k}^T, x^{(k)}\right]\right)^2}{\|a_{i_k}\|^2}$$

$$= \|x^* - x^{(k)}\|^2 - \frac{\left(\left[a_{i_k}^T, x^*\right] - \left[a_{i_k}^T, x^{(k)}\right]\right)^2}{\|a_{i_k}\|^2}$$

$$= \|x^* - x^{(k)}\|^2 - \frac{\left(a_{i_k}^T, x^* - x^{(k)}\right)^2}{\|a_{i_k}\|^2} \tag{22}$$

The quantity of product formula

$$[a, b] = \|a\|\|b\|\cos(a, b)$$

Can launch that

$$\|x^* - x^{(k+1)}\|^2 = \|x^* - x^{(k)}\|^2 - \|x^* - x^{(k)}\|^2 \cos^2 \phi_k$$

$$= \left(1 - \cos^2 \phi\ x\right)\|x^* - x^{(k)}\|^2 = \sin \varphi_k \|x^* - x^{(k)}\|^2 \tag{23}$$

Where ϕ_k is the angle between the vectors of $a_{i_k}^T$ and $x^* - x^{(k)}$ $k = 1, 2, \cdots$ thus

$$\frac{\|x^* - x^{(k+1)}\|^2}{\|x^* - x^{(0)}\|^2} = \sin^2 \varphi_0 \sin^2 \varphi_1 \cdots \sin^2 \varphi_k \tag{24}$$

Because (17) type, if and only if $x^* = x^{(k)}$,

$$y_{i_k} - \left[a_{i_k}^T, x^{(k)}\right] = \left[a_{i_k}^T, x^* - x^{(k)}\right] = \|a_{i_k}\|\|x^* - x^{(k)}\|\cos \varphi_k = 0$$

$|\cos \varphi_k| > 0$. So (15) - (18) the convergence speed ensured by formula (24). Though $|\cos \varphi_k| > 0$ can obtain $\sin \varphi_k$, $k = 1, 2, \cdots$ are close to or equal to zero, which is the solution of the equations which can be limited .Process (15) - (18) can have the convergent velocity of arbitrary, but also can be said to be the best[6].

The three order equations as an example, in the process (15) - (18) after one iteration can be obtained:

$$\begin{cases} 2x_1 - 3x_2 + x_3 = 1 \\ -x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + x_3 = 3 \end{cases}$$

Take $x_1^{(*)} = x_1^{(1)} = 1, x_2^{(*)} = x_2^{(1)} = 1, x_3^{(*)} = x_3^{(1)} = 1$ as the initial approximation, available

$$max\{1,1,3\} = 3, i_0 = 3, r_{i_0}(0) = 3$$

$$W^{(1)} = \frac{r_{i_0}(0)}{\left[a_{i_0}^T, a_{i_0}^T\right]} = \frac{3}{1+1+1} = 1, \quad x^{(1)} = x^{(0)} + W^{(1)} a_{i_0}^T$$

$$x_1^{(1)} = 0 + 1,1 = 1, x_2^{(1)} = 0 + 1,1 = 1, x_3^{(1)} = 0 + 1,1 = 1$$

Obviously, $x_1^{(*)} = x_1^{(1)} = 1, x_2^{(*)} = x_2^{(1)} = 1, x_3^{(*)} = x_3^{(1)} = 1$ is the solution of the equations.

CONCLUSION

This paper compared with the traditional iterative method a lot of, an effective iterative method for linear equations are studied, and proved the convergence of iterative method. This iterative method can be from any initial approximation of any transformation, and does not need the original information in theory. The can also make some deformation and extension, although not a panacea, but can solve all the problems of this type. The numerical example shows that the method is feasible, effective.

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