



Research Article

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## Adaptive parameter adjusting method for torsional vibration model of turbine shafts

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### ABSTRACT

Torsional vibration model of turbine-generator shafts is the basic for fatigue life loss analysis, for the accuracy of the fatigue analysis of the shafts depends on the precision of the inherent characteristics of the shafts model. The sensitivity of the inherent characteristics of the torsional vibration model to the structure parameters of the turbine-generator shafts is analyzed in this paper. A parameter adjusting method for the torsional vibration model is proposed based on the method of Taylor expansion. The logic structure of the adaptive process of torsional vibration model is designed. The torsional vibration model for a 1000WM turbo-generator shafts was taken as an example and the accuracy of the method is verified.

**Keywords:** Parameter adjusting method; Torsional vibration model; Inherent characteristic sensitivity.

### INTRODUCTION

Shafts, Couplings, and blades of turbine-generator units can be damaged during torsional vibration of the shafts. Modal superposition method is usually used in analyzing the fatigue life loss of shafts caused by sub synchronous oscillation (SSO). The modal speed of the gear can be obtained through a band-pass filter, and the stress of the weak sections of the shafts can be calculated by combining the torsional vibration inherent characteristic. If there was a deviation between the shafting torsional vibration model and the practical model, the torsional vibration natural frequency would deviate from the actual band-pass filter pass-band, which may affects both the filtering effect and the stress calculation of the weak sections.

The torsional vibration inherent characteristics of the shafts change with the rigidity of the rotors which is susceptible to the steam temperature. In this case, a real-time on-line adaptive adjusting torsional vibration model method based on the sensitivity computation is proposed in this paper. The torsional rigidity of the torsional vibration model can be adjusted by comparing the torsional vibration natural frequency value of the real-time monitoring and the calculation result based on the torsional vibration model, so as to ensure the accuracy of the torsional vibration analysis.

#### Sensitivity of vibration mode to structure parameters

Shafts can be stimulated as  $n$  freedom degree of mass-spring model. According to differential functions of no damping free vibration with  $n$  freedom degree, the nature frequency  $\omega_i$  and vibration mode  $\mathbf{x}_i$  ( $i = 1, 2, \dots, n$ ) can be calculated. According to vibration mechanics, frequency and vibration mode can be given:

Characteristic function

$$(\mathbf{K} - \omega_i^2 \mathbf{M})\mathbf{x}_i = \mathbf{0} \quad (1)$$

And,

$$\mathbf{x}_i^T \mathbf{M} \mathbf{x}_i = 1 \quad (2)$$

Where,  $\mathbf{x}_i$  = vibration mode of  $i$  period after orthogonality,  $\mathbf{M}$  = model mass matrix, and  $\mathbf{K}$  = model rigidity matrix.  $\mathbf{M}$  and  $\mathbf{K}$  can be written,

$$\mathbf{M} = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & & \\ -k_2 & k_2 + k_3 & -k_3 & \\ & & \ddots & \\ & & & -k_n & k_n + k_{n+1} \end{bmatrix}$$

The sensitivity equation of nature frequency to structure parameter  $P$  is

$$\frac{\partial \omega_i}{\partial p} = \frac{\mathbf{x}_i^T \left( \frac{\partial \mathbf{K}}{\partial p} - \omega_i^2 \frac{\partial \mathbf{M}}{\partial p} \right) \mathbf{x}_i}{2\omega_i} \quad (3)$$

From Eq. (3), the sensitivity of  $\omega_i$  towards  $m_j$   $j$  ( $j = 1, 2, \dots, n$ ) can be written,

$$\frac{\partial \omega_i}{\partial m_j} = -\frac{\omega_i^2 \mathbf{x}_i^T \frac{\partial \mathbf{M}}{\partial m_j} \mathbf{x}_i}{2\omega_i} = -\frac{\omega_i (\mathbf{x}_i)_j^2}{2} \quad (4)$$

Where,  $(\mathbf{x}_i)_j$  = the  $j$  element of  $i$  phase.

Make  $\mathbf{X}_i = \mathbf{x}_i / (\mathbf{x}_i)_n$ ,  $x_{i,j} = (\mathbf{X}_i)_j$ . According to transfer matrix method, when the model vibrates at phase  $i$  nature frequency  $\omega_i$ , then,

$$\begin{bmatrix} x_{i,1} \\ F_{i,1} \end{bmatrix} = \mathbf{H}_1 \mathbf{H}_2 \cdots \mathbf{H}_{n-1} \mathbf{H}_n \begin{bmatrix} x_{i,n} \\ F_{i,n} \end{bmatrix} \quad (5)$$

Where,

$$\mathbf{H}_j = \begin{bmatrix} 1 - \frac{m_j}{k_j} \omega_i^2 & -\frac{1}{k_j} \\ m_j \omega_i^2 & 1 \end{bmatrix} \quad (6)$$

From Eq. (5), it is,

$$x_{i,j} = \left( 1 - \frac{m_{j+1}}{k_{j+1}} \omega_i^2 \right) x_{i,j+1} - \frac{1}{k_{j+1}} F_{i,j+1} \quad (7)$$

And

$$\begin{aligned} \begin{bmatrix} x_{i,j} \\ F_{i,j} \end{bmatrix} &= \begin{bmatrix} f(x_{i,k-1}, F_{i,k-1}) \\ F_{i,j} \end{bmatrix} = \mathbf{H}_j \mathbf{H}_{j+1} \cdots \mathbf{H}_k \begin{bmatrix} x_{i,k} \\ F_{i,k} \end{bmatrix} \\ &= \mathbf{H}_j \mathbf{H}_{j+1} \cdots \mathbf{H}_{k-1} \begin{bmatrix} x_{i,k-1} \\ F_{i,k-1} \end{bmatrix} && (j < k \leq n) \\ &= \mathbf{H}_j \mathbf{H}_{j+1} \cdots \mathbf{H}_{k-1} \begin{bmatrix} \left( 1 - \frac{m_k}{k_k} \omega_i^2 \right) x_{i,k} - \frac{F_{i,j}}{k_k} \\ F_{i,k-1} \end{bmatrix} \end{aligned} \quad (8)$$

Then,

$$\frac{\partial x_{i,j}}{\partial k_k} = \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial x_{i,k-1}} \frac{\partial x_{i,k-1}}{\partial k_k} + \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial F_{i,k-1}} \frac{\partial F_{i,k-1}}{\partial k_k} \quad (9)$$

$$\left\{ \begin{array}{l} \frac{\partial x_{i,j}}{\partial k_k} = \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial x_{i,k-1}} \frac{(m_k \omega_i^2 x_{i,k} + F_{i,j})}{k_k^2} \quad (j < k \leq n+1) \\ \frac{\partial x_{i,j}}{\partial k_k} = \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial x_{i,k-1}} \frac{(m_k \omega_i^2 x_{i,k} + F_{i,j})}{k_k^2} - \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial F_{i,k-1}} \quad (k = n+1) \\ \frac{\partial x_{i,j}}{\partial k_k} = 0 \quad (j \leq k) \end{array} \right. \quad (10)$$

$$\left\{ \begin{array}{l} \frac{\partial x_{i,j}}{\partial m_k} = \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial x_{i,k-1}} \frac{\partial x_{i,k-1}}{\partial m_k} + \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial F_{i,k-1}} \frac{\partial F_{i,k-1}}{\partial m_k} \\ \quad = -\frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial x_{i,k-1}} \frac{\omega_i^2 x_{i,k}}{k_k} + \frac{\partial f(x_{i,k-1}, F_{i,k-1})}{\partial F_{i,k-1}} \omega_i^2 x_{i,k} \\ \frac{\partial x_{i,j}}{\partial m_k} = 0 \quad (j \leq k) \end{array} \right. \quad (j < k \leq n) \quad (11)$$

#### Parameter adjustment value calculation method

Write Eq. (11) into Taylor series, and ignore the second and more than second times modification items, it will be,

$$\Delta \omega_i = \sum_{j=1}^n \frac{\partial \omega_i}{\partial m_j} \Delta m_j + \sum_{j=1}^{n+1} \frac{\partial \omega_i}{\partial k_j} \Delta k_j \quad (12)$$

$$\Delta \mathbf{x}_i = \sum_{j=1}^n \frac{\partial \mathbf{x}_i}{\partial m_j} \Delta m_j + \sum_{j=1}^{n+1} \frac{\partial \mathbf{x}_i}{\partial k_j} \Delta k_j \quad (13)$$

Eq. (13) and Eq. (14) reflect the relationship between nature vibration parameters  $\omega_1 \cdots \omega_n$ ,  $\mathbf{x}_1 \cdots \mathbf{x}_n$  and structure parameters  $m_1 \cdots m_n$ ,  $k_1 \cdots k_{n+1}$  in turbine generation shaft vibration. If the different values of nature frequency and vibration mode between the mass-spring model and actual blade is given, the adjustment value of parameters  $\Delta m_1 \cdots \Delta m_n$  and  $\Delta k_1 \cdots \Delta k_{n+1}$  can be calculated according to the following equation.

$$\begin{bmatrix} \frac{\partial \omega_1}{\partial m_1} & \cdots & \frac{\partial \omega_1}{\partial m_1} & \frac{\partial \omega_1}{\partial k_1} & \cdots & \frac{\partial \omega_1}{\partial k_{n+1}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial \omega_n}{\partial m_1} & \cdots & \frac{\partial \omega_n}{\partial m_n} & \frac{\partial \omega_n}{\partial k_1} & \cdots & \frac{\partial \omega_n}{\partial k_{n+1}} \\ \frac{\partial \mathbf{x}_1}{\partial m_1} & \cdots & \frac{\partial \mathbf{x}_1}{\partial m_n} & \frac{\partial \mathbf{x}_1}{\partial k_1} & \cdots & \frac{\partial \mathbf{x}_1}{\partial k_{n+1}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial \mathbf{x}_n}{\partial m_1} & \cdots & \frac{\partial \mathbf{x}_n}{\partial m_n} & \frac{\partial \mathbf{x}_n}{\partial k_1} & \cdots & \frac{\partial \mathbf{x}_n}{\partial k_{n+1}} \end{bmatrix} \begin{bmatrix} \Delta m_1 \\ \vdots \\ \Delta m_n \\ \Delta k_1 \\ \vdots \\ \Delta k_{n+1} \end{bmatrix} = \begin{bmatrix} \Delta \omega_1 \\ \vdots \\ \Delta \omega_n \\ \Delta \mathbf{x}_1 \\ \vdots \\ \Delta \mathbf{x}_n \end{bmatrix} \quad (14)$$

## Analysis and Application

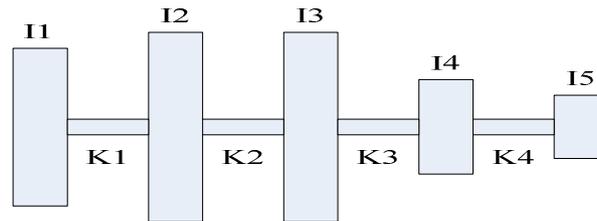


Fig.1 The mass-spring model for the turbine-generator shafts

The mass-spring model of the shafts of a 1000MW turbine-generator is shown in Fig.1. The parameters of the model will change when the condition of the turbine floats. The parameters of the accurate model and the biased model are shown in Table 1.

Table 1 Parameters of the accurate model and the biased model

Accurate model		Biased model	
Moment of Inertia	Moment of Inertia (Kg·m <sup>2</sup> )	Moment of Inertia (Kg·m <sup>2</sup> )	Relative error
I1	19398	19398	0
I2	28825	28825	0
I3	28425	28425	0
I4	3850	3850	0
I5	2150	2150	0
Torsional stiffness	Torsional stiffness (N·m/rad)	Torsional stiffness (N·m/rad)	Relative error
K1	220700000	242770000	+10%
K2	182500000	197100000	+8%
K3	153800000	161490000	+6%
K4	131000000	144100000	+10%

Adjust the torsional stiffness using the method shown in Fig. 1. According to Table 2, the parameters of the adjusted model is consistent with those of the accurate model.

Table 2 Model adjustment result

Torsional stiffness	Actual torsional stiffness (N·m/rad)	Adjusted torsional stiffness (N·m/rad)	Relative error
K1	220700000	220158396	-0.245%
K2	182500000	182779115	+0.153%
K3	153800000	154042130	+0.157%
K4	131000000	130693121	-0.234%

## CONCLUSION

It is proved that the adaptive adjustment method proposed in this paper can make sure the mass-spring model of the torsional vibration monitoring equipment is consistent with the actual turbine-generator shafts. The inherent characteristics can be accurately calculated based on the online adaptive adjustment model, which improves the reliability of the torsional vibration analysis results.

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