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Adaptive Fuzzy sliding-mode control for chaotic nonlinear systems with uncertainties based on fractional calculus

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ABSTRACT

In this paper, an adaptive fuzzy sliding-mode controller (AF-SMC) based on fractional calculus for uncertain chaotic nonlinear systems were proposed. Three fuzzy logic systems were designed to approximate the unknown system functions and the switching term of SMC, respectively. Stability of the closed-loop system was proved and adaptive laws for the tuned parameters were obtained by using Lyapunov arguments. Numerical simulations are done on the chaotic nonlinear gyroscope system and results show that the proposed fractional order controller is effective and feasible.

Key words*:* Fractional order chaotic systems, fuzzy control, sliding mode control, adaptive control.

INTRODUCTION

Due to its valuable applications in numerous and wide-spread fields, fractional calculus has received considerable popularity during past three decades [1]. According to the Poincare-Bendixon theorem [2], an integer order chaotic nonlinear system must have a minimum order of 3 for chaos to appear. However, in fractional systems (or more specifically, non-integer-order systems), it is not the case. Recently many authors began to investigate the chaotic dynamics of fractional order dynamical systems. They have found that many systems are known to display fractional order dynamics, For example, it has been shown that Chua's circuit of order as low as 2.7 can produce a chaotic attractor [3]. Non-autonomous Duffing systems of fractional order have been addressed in [4], where it is shown that a sinusoid ally driven Duffing system of order less than 2 can still behave in a chaotic manner. In Ref. [5], chaotic behaviors of the fractional order 'jerk' model was studied, in which chaotic attractor was obtained with system orders as low as 2.1. In [6], based on the cellular neural networks (CNNs) and replaces the traditional first-order cell with a non-integer-order one, a simple system showing chaotic behavior is introduced. More recently, chaos control and synchronization of fractional order system in [7] have been investigated. Address the problem of chaos control for autonomous nonlinear chaotic systems of integer and fractional orders. Where the nonlinear controller was designed using the recursive "back-stepping" method. The controller's effect is to stabilize the output chaotic trajectory by driving it to the nearest equilibrium point in the basin of attractor. In this paper the control of the fractional order chaotic systems will be studied.

Over the past decade, the variable structure control (VSC) strategy using the sliding mode concept has been widely studied and developed for control and state estimation problems since the works of Utkin. SMC is an efficient tool to control complex high-order dynamic plants operating under uncertainty conditions due to its order reduction property and low sensitivity to disturbances and plant parameter variations. In SMC, the states of the controlled system are first guided to reside on a designed surface in state space and then keeping them there with a shifting law [8]. The most prominent property of the SMC is its insensitivity to parameter variations and external disturbances. However, its major drawback in practical applications is the chattering problem. In order to eliminate chattering, Palm noted the similarity between fuzzy controller and sliding mode controller with a boundary layer, and provided a fuzzy sliding mode design approach in [9]. This design can lead to stable closed-loop system with avoiding the chattering problem in the SMC.

Numerous techniques have been proposed to eliminate this phenomenon in SMC, such as saturating approximation, integral sliding control and boundary layer technique. To tackle these difficulties, fuzzy logic controllers (FLC) are often used to deal with the discontinuous sign function in the reaching phase of SMC [10]. As well known, fuzzy logic control (FLC) is a knowledge-based control approach which can mimic human experience in controlling complex systems and has excellent capability to deal with nonlinear plants [21,22]. This method is a good choice for inferring the control gains of VSSM controller in wind turbines through fuzzy-rules-based inference. Meanwhile, the modeling error and the uncertain disturbance of wind power system can be estimated to obtain the appropriate switch gain through a fuzzy inference system with single input and single output. Many new algorithms have been proposed based on the integration of the fuzzy logic and the SMC [11]. These approaches are similar in the aspect that they directly approximate the sliding mode control law by fuzzy approximations. The main advantage of this control scheme is its ability to eliminate the chattering using a fuzzy sliding surface in the reaching condition of the SMC [12]. Recently, adaptive fuzzy SMC methods are also used for this purpose, which is shown to be quite effective [13]. In contrast to a conventional feedback control algorithm, there is a fuzzy control algorithm consists of a set of heuristic decision rules that can be represented as a non-mathematical control algorithm. This algorithm proves to be very effective especially when the precise model of the system under control is not available or expensive to prepare. This combination (i.e., F-SMC) provides the mechanism to design robust controllers for nonlinear systems with uncertainty.

Unfortunately, there are not many contributions available for the problem of the sliding mode control of fractional order systems with uncertainties. In [14], some results are obtained without using a fractional sliding manifold. In this work, we incorporate adaptive fuzzy SMC approach to control the nonlinear fractional order gyro chaotic systems with uncertainties, and new results on adaptive fuzzy SMC of fractional order systems with uncertainties are derived.

II. SLIDING MODE CONTROL OF FRACTIONAL ORDER CHAOTIC SYSTEMS

A. Mathematical model of fractional order systems

Fractional calculus is a mathematical topic more than 300 years. It is a generalization of integration and differentiation to non-integer order operator, denoted by ${}_aD_t^q$, where a and t are the limits of the operator. This operator is a notation for taking both the fractional integral and functional derivative in a single expression, and the simplest and easiest definition is Riemann-Liouville definition given as in [15]

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{1} $$

where n is the first integer which is not less α, and Γ is the Gamma function.

Consider a fractional order chaotic dynamic system

$$ \boldsymbol{x}^{(n\alpha)} = f\left(\boldsymbol{x},t\right) + g\left(\boldsymbol{x},t\right)u + d\left(t\right) \tag{2} $$

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T = [x, x^{(\alpha)}, \cdots, x^{((n-1)\alpha)}]^T$ is the state vector, $f(x,t)$ and $g(x,t)$ are smooth and bounded nonlinear system functions. In this paper, we assume that both $f(x,t)$ and $g(x,t)$ are unknown, $u(t)$ is the control input and d(t) is the external bounded disturbance, i.e. $|d(t)| \le D$.

The control task is to regulate the system output y to follow a smooth command signal y_d, which is the output trajectory of a drive system, and all signals involved must be bounded. Before the controller design, the command vector y_d and the tracking error vector e are firstly defined as following

$$ \begin{cases} e(t) = y_d(t) - x(t) \\ \boldsymbol{y}_d(t) = \left[y_d, y_d^{(\alpha)}, \cdots, y_d^{((n-1)\alpha)} \right]^T \end{cases} \tag{3} $$

In this paper, we will develop adaptive fuzzy sliding mode control of uncertain fractional order chaotic systems, i.e., the control objective is to force output trajectory of the response system to track output trajectory of the drive system.

B. Sliding mode controller design

Design of the SMC controller involves two important phases. The first phase is to design a suitable sliding surface function $\boldsymbol{\sigma}(t)$ so that once the system enters the hyper-plane $\boldsymbol{\sigma}(t) = 0$, the desired dynamic characteristics can be realized. The second is to design a proper controller $\boldsymbol{u}(t)$ so that it can drive the system's dynamics into the designed hyper plane and stay thereafter.

Two type of control law must be derived separately for those two phases described above. We first define the sliding surfaces in the space of the error states as follows

$$\boldsymbol{\sigma}(t) = -\boldsymbol{ke} = -\left(k_1 e + k_2 e^{(\alpha)} + \cdots + k_n e^{(n-1)\alpha}\right) \tag{4}$$

where $\boldsymbol{k} = [k_1, k_2, \cdots, k_n]^{\mathrm{T}}$ and the k_i is real and chosen such that the polynomial $L(s) = \sum_{i=1}^{n} k_i s^{(i-1)\alpha}, k_n = 1$, is Hurwitz, where *s* is a Laplace operator. The tracking problem will be considered as the state error vector $\boldsymbol{e}$ remaining on the sliding surface $\boldsymbol{\sigma}(t) = 0$ for all $t \geq 0$.

In order to ensure that the trajectory of the state error vector $\boldsymbol{e}$ will accross from the approaching phase to the sliding phase, the sliding mode reaching condition

$$\sigma(t)\dot{\sigma}(t) \leq 0 \tag{5}$$

must be satisfied. In the sliding mode phase, it means $\boldsymbol{\sigma}(t) = 0$ and $\boldsymbol{\sigma}^{(\alpha)}(t) = 0$. In order to force the system trajectories to remain in the sliding surface, the equivalent control $u_{eq}(t)$ can be derived as follows.

We first assume that *f* (*x*,*t*) and *g*(*x*,*t*) are known and external disturbance does not exist, i.e., $d(t) = 0$. Take the derivative of the sliding surface, we have

$$\sigma^{(\alpha)}(t) = -\sum_{i=1}^{n-1} k_i - e^{(n\alpha)} = \boldsymbol{ke}(t) + f(x,t) + g(x,t)u_{eq} - y_d^{(n)} \tag{6}$$

Hence, the equivalent control can be obtained as

$$u_{eq}(t) = \frac{1}{g(x,t)}(\boldsymbol{ke}(t) + f(x,t) - y_d^{(n)}) \tag{7}$$

On the contrary, in the sliding mode reaching phase, $\boldsymbol{\sigma}(t) \neq 0$, a sliding-mode hitting control u_{sw} should be imposed to guarantee the reaching condition (4) . Thus, the total sliding mode control can be obtained as

$$\begin{cases} u(t) = u_{eq}(t) + u_s(t) \\ u_s(t) = g^{-1}(x,t)\eta\,\mathrm{sgn}(s) \end{cases} \tag{8}$$

where the sliding mode gains $\eta > 0$ and *sgn*() is the sign function.

C. Adaptive fuzzy sliding mode controller design

When design the sliding mode control (8), the system functions *f* (*x*,*t*) and *g*(*x*,*t*) must be exactly known and the sliding mode gain η_h should be properly chosen. Unfortunately, *f* (*x*,*t*) and *g*(*x*,*t*) are always unknown and there also exist external disturbance, then the feedback linearization control effort (7) cannot be obtained. Here, two fuzzy logic systems are adopted to approximate the unknown functions *f* (*x*,*t*) and *g*(*x*,*t*), respectively, as the following

$$\begin{cases} \hat{f}(\boldsymbol{x} \mid \theta_f) = \xi^T(\boldsymbol{x})\theta_f \\ \hat{g}(\boldsymbol{x} \mid \theta_g) = \xi^T(\boldsymbol{x})\theta_g \end{cases} \tag{9}$$

where $\xi(x)$ is the fuzzy basis function, θ_f and θ_g are the adjustable parameters. In this article, we used the set of fuzzy systems with singleton fuzzifier, product inference, centroid defuzzifier, triangular antecedent membership function and singleton consequent membership function.

The indirect adaptive controller is given as follows

$$\boldsymbol{u}(t) = \hat{g}^{-1}\left(x\middle|\theta_g\right) = \boldsymbol{ke}(t) + y_d^{(n\alpha)} - \hat{f}\left(x\middle|\theta_f\right) - u_s \tag{10}$$

Since the switching control effort contains inherent high-frequency chattering, which is harmful to electrical circuits, we also adopt a fuzzy logic system to online approximate the switch control term, that is

$$\hat{\boldsymbol{u}}_s(t) = \hat{\boldsymbol{h}}(\sigma \mid \theta_h^*) = \eta\,\mathrm{sgn}(\sigma) \tag{11}$$

When replacing the unknown functions *f* (*x*,*t*) and *g*(*x*,*t*) with the estimated $\hat{f}(\boldsymbol{x} \mid \theta_f)$, $\hat{g}(\boldsymbol{x} \mid \theta_g)$ and the controller $\hat{h}$, we design the indirect adaptive fuzzy sliding mode controller as follows

$$\boldsymbol{u}(t) = \hat{g}^{-1}\left(x\middle|\theta_g\right)\left[\boldsymbol{ke} + y_d^{(n\alpha)} - \hat{f}\left(x\middle|\theta_f\right) - \hat{\boldsymbol{h}}(s)\right] \tag{12}$$

where the estimated switching controller $h(\sigma(t)) = \theta_h^T \phi(\sigma)$, θ_h^T is the adjustable parameter and $\phi(\sigma)$ is the fuzzy basis function.

The optimal parameter estimations θ_f^*, θ_g^* and θ_h^* are defined as

$$\theta_f^* = \arg\min_{\theta_f \in \Omega_f} [\sup_{x\in\Omega_x} | f(x|\theta_f) - f(x,t)|],$$
$$\theta_g^* = \arg\min_{\theta_g \in \Omega g} [\sup_{x\in\Omega_x} | g(x|\theta_g) - g(x,t)|]$$
$$\theta_h^* = \arg\min_{\theta_h \in \Omega_h} \left[\sup_{x\in\Omega_x} | h(\sigma | \theta_h) - u_s(t)| \right],$$

where Ω_f, Ω_g, and Ω_h are constraint sets of suitable bounds on θ_f , θ_g and θ_h respectively.
By using (11), (12), sliding surface equation (4) can be rewritten as

$$\sigma^{(\alpha)}(t) = \omega + [f(x|\theta_f^*) - f(x|\theta_f)] + [g(x|\theta_g^*) - g(x|\theta_g)]u_{eq}(t) - h(\sigma|\theta_h) + d(t) \tag{13}$$

where the minimum approximation error ω is defined as

$$\omega = \alpha[f(x|\theta_f^*) - f(x|\theta_f)] + [g(x|\theta_g^*) - g(x|\theta_g)]u_{eq}(t) \le \omega_{\min} \tag{14}$$

It should be noticed that in (14), the estimation error between $\hat{\boldsymbol{h}}(\sigma|\theta^*)$ and $h(\sigma|\theta^*)$ is ignored. Since the switching control effort itself is not a part of the model and is only a designed robust term to deal with the disturbance, the exact value of $\boldsymbol{u}_s(t)$ is not necessary.
The parameters' approximation errors are defined as

$$\tilde{\theta}_f = \theta_f - \theta_f^*, \tilde{\theta}_g = \theta_g - \theta_g^* \text{ and } \tilde{\theta}_h = \theta_h - \theta_h^*,$$

thus, we have

$$\sigma^{(\alpha)}(t) = -\omega - \tilde{\theta}_h^T \phi(\sigma) - \tilde{\theta}_f^T \xi(x) - \tilde{\theta}_g^T \xi(x) u_{eq}(t) - h(\sigma|\theta_h^*) + d(t)$$

II. STABILITY ANALYSIS

In this section, the Lyapunov direct argument is adopted to analysis the global stability of the proposed control scheme.

Theorem 1: *Consider the fractional order chaotic nonlinear system (2) with uncertainties, if the control input is designed as (12), and the adaptive laws are chosen as*

$$\dot{\theta}_f^{(\alpha)} = r_1 \sigma(t) \xi(x), \tag{15.a}$$
$$\dot{\theta}_g^{(\alpha)} = r_2 \sigma(t) \xi(x) u(t), \tag{15.b}$$
$$\theta_h^{(\alpha)} = -r_3 \sigma(t) g(x) \phi(x) \tag{15.c}$$

where $r_i > 0, i = 1 \sim 4$, *are the adaptive gains, then, the obtained closed-loop system will have the global stability and the tracking error will converge to zero asymptotically as well as all signals involved are uniformly bounded.*
Proof: Now consider a non-negative Lyapunov function given as follows

$$V(t) = \frac{1}{2}\sigma^2(t) + \frac{1}{2r_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2r_2}\tilde{\theta}_g^T \tilde{\theta}_g + \frac{1}{2r_3}\tilde{\theta}_h^T \tilde{\theta}_h \tag{16}$$

Noting that $\dot{\tilde{\theta}}_f = \dot{\hat{\theta}}_f$, $\dot{\tilde{\theta}}_g = \dot{\hat{\theta}}_g$ and $\dot{\tilde{\theta}}_h = \dot{\hat{\theta}}_h$, and using the control effort (11) and (12), then, we obtain the time derivative of the Lyapunov function as follows

$$\begin{aligned}
V^{(\alpha)} &= \sigma\sigma^{(\alpha)} + r_1^{-1}\tilde{\theta}_f^T \tilde{\theta}_f^{(\alpha)} + r_2^{-1}\tilde{\theta}_g^T \tilde{\theta}_g^{(\alpha)} + r_3^{-1}\tilde{\theta}_h^T \tilde{\theta}_h^{(\alpha)} \\
&= \sigma\omega - \sigma\tilde{\theta}_h^T \phi(\sigma) - \sigma\tilde{\theta}_f^T \xi(x) - \sigma\tilde{\theta}_g^T \xi(x) u(t) - \sigma h(\sigma|\theta_h^*) \\
&+ \sigma d(t) + r_1^{-1}\tilde{\theta}_f^T \tilde{\theta}_f^{(\alpha)} + r_2^{-1}\tilde{\theta}_g^T \tilde{\theta}_g^{(\alpha)} + r_3^{-1}\tilde{\theta}_h^T \tilde{\theta}_h^{(\alpha)} \\
&\le \sigma\omega + r_1^{-1}\tilde{\theta}_f^T \left(\tilde{\theta}_f^{(\alpha)} - r_1\sigma\xi(x)\right) + r_2^{-1}\tilde{\theta}_g^T \left(\tilde{\theta}_g^{(\alpha)} - r_2\sigma\xi(x)u(t)\right) \\
&+ r_3^{-1}\tilde{\theta}_h^T \left(\tilde{\theta}_h^{(\alpha)} - r_3\sigma\phi(\sigma)\right) - \sigma(D+\eta)\operatorname{sgn}(\sigma)
\end{aligned} \tag{17}$$

When using the adaptive laws given above and after simple computation, we can deduce that the following inequality holds

$$\begin{aligned} V^{(\alpha)}(t) &\le \sigma\omega - \sigma\eta_h \operatorname{sgn}(\sigma) \\ &= \sigma\omega - |\sigma|\eta_h \end{aligned} \quad (18)$$

The inequality (18) indicates that $\dot{V}(t)$ holds when $\sigma(t) \neq 0$, then V_1 and $\sigma(t)$ will approach to zero in finite time. If a proper value of the gains are selected, the trajectories of system (2) will reach the sliding surface and the system motion enters the so-called "sliding mode". Therefore, when the control law is used in (12), and the adaptive gains are designed as above, $\dot{V}(t) \le 0$ is satisfied. Using the corollary of Barbalat's Lemma, we have $\lim_{t\to\infty} |\sigma(t)| = 0$. Thus, $\lim_{t\to\infty} |e(t)| = 0$, and the control law drives the state error trajectories of the system in (2) onto the sliding surface (4) and the system is stable.
This completes the proof.

III. NUMERICAL SIMULATION

In this section, simulations are done on the chaotic nonlinear gyroscope system to demonstrate the effectiveness of our fractional order adaptive fuzzy sliding mode controller. The control object is to force the response of the system to track the trajectory of the drive system. Consider the fractional order chaotic gyroscope drive and response systems as follows:

$$\begin{cases} x_1^{(\alpha)}(t) = x_2(t) \\ x_2^{(\alpha)}(t) = -20x_1(t) + 0.125x_1^2(t) - 0.15x_2^3(t) - 0.5x_2(t) \\ -0.05x_2^2(t) + 2\sin(0.5x_1(t)) - 4\cos(2t)x_1(t) + \Delta f(x(t),t) + u(t) + d(t) \end{cases}$$

where the system uncertainty, for the response system, the uncertainties nd the external disturbance are

$$\Delta f(\boldsymbol{x},t) = 0.1\sin(2x_1) + 0.05\cos(x_2),\ d(t) = 0.6\cos(\pi t).$$

while for the drive system

$$\Delta f(\boldsymbol{x},t) = 0 \text{ and } u(t) = 0.$$

The control objective is to force the trajectories of the response system to track the reference trajectories of the drive system. The initial states of response and drive systems are set as $\mathbf{y}(0) = [1,\ -1]^T$ and $x(0) = [1.5,\ 0.75]^T$. The value of the fractional order α=0.75, and all design constants are chosen as $k_1 = k_2 = 10,\ r_1 = 200$, $r_2 = 50$, $r_3 = 2$.
The membership functions for the states and the sliding function are selected as follows:

$$\mu_{A_1^i}(x_i) = e^{-(x_i-5)^2/8},\ \mu_{A_2^i}(x_i) = e^{-(x_i-2.5)^2/8},\ \mu_{A_3^i}(x_i) = e^{-(x_i-1)^2/8},$$

$$\mu_{A_4^i}(x_i) = e^{-x_i^2/8},\ \mu_{A_5^i}(x_i) = e^{-(x_i+1)^2/8},\ \mu_{A_6^i}(x_i) = e^{-(x_i+2.5)^2/8},$$

$$\mu_{A_7^i}(x_i) = e^{-(x_i+5)^2/8},\ \mu_{A_7^i}(x_i) = e^{-(x_i+5)^2/8}.$$

The membership functions for σ are selected as follows:

$$\mu_{A^1}(\sigma) = 1/(1+e^{5(\sigma+2)}),\ \ \mu_{A^2}(\sigma) = e^{-(\sigma+1.5)^2},\ \ \mu_{A^3}(\sigma) = e^{-(\sigma+0.5)^2},$$

$$\mu_{A^4}(\sigma) = e^{-(\sigma-0.5)^2},\ \ \mu_{A^5}(\sigma) = e^{-(\sigma-1.5)^2}.$$

If the control input isn't imposed on the system, the 3-D phase portrait of the drive and response systems is given in Figure 1. It is obvious that the tracking performance is poor for the lacking of control effort imposed on response system. Figure 2 and Figure 3 shows the trajectories of the states *x*1, *y*1 and *x*2, *y*2, respectively. Trajectory of the sliding surface is given in Figure 4. Figure 5 gives the control effort. From the figures, it can be seen that the initial tracking error is obvious, when the sliding mode occurs, the tracking error diminishes. The estimations of *f* (*x*; *t*), *g*(*x*; *t*) and their corresponding real values are shown in Figure 7 and Figure 8.

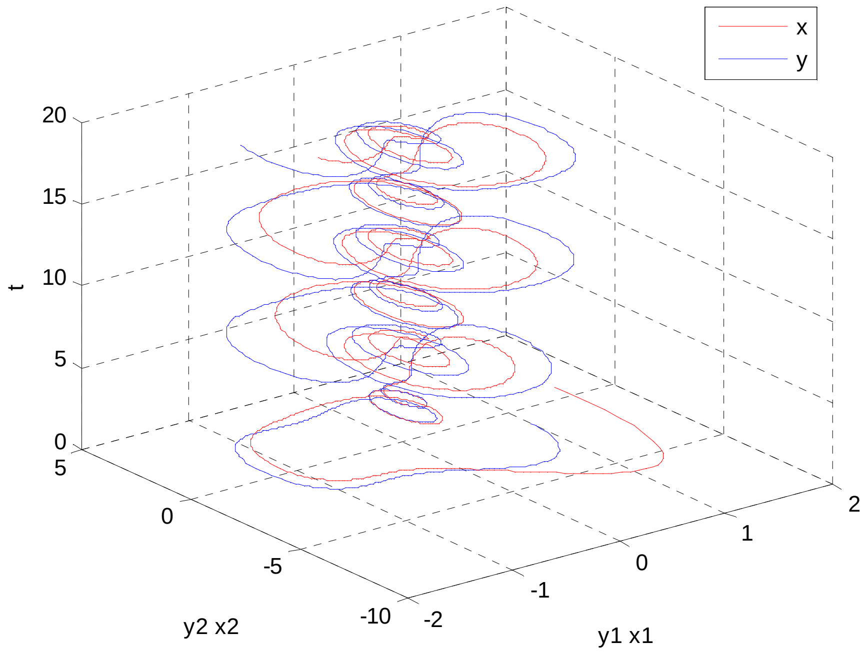

Figure 1. Phase portrait of chaotic drive and response systems

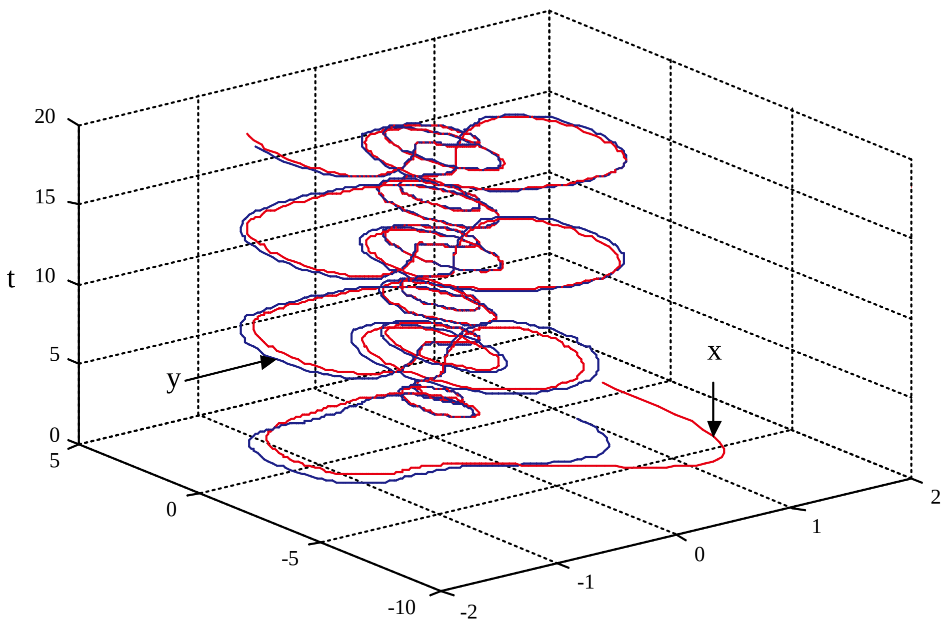

Figure 2. State tracking of the drive and response systems

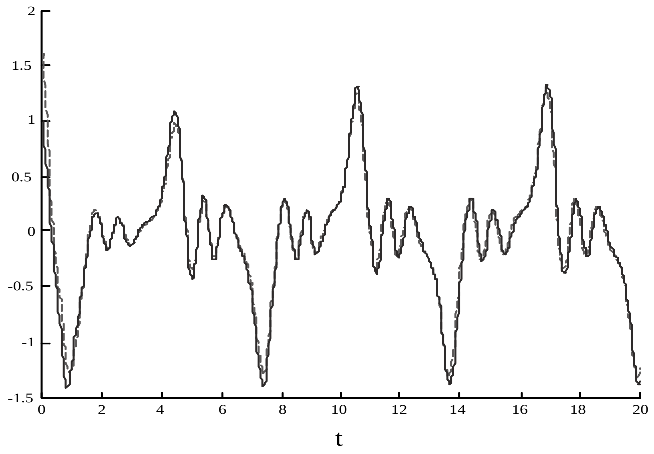

Figure 3. Trajectories of the states x_1 and y_1

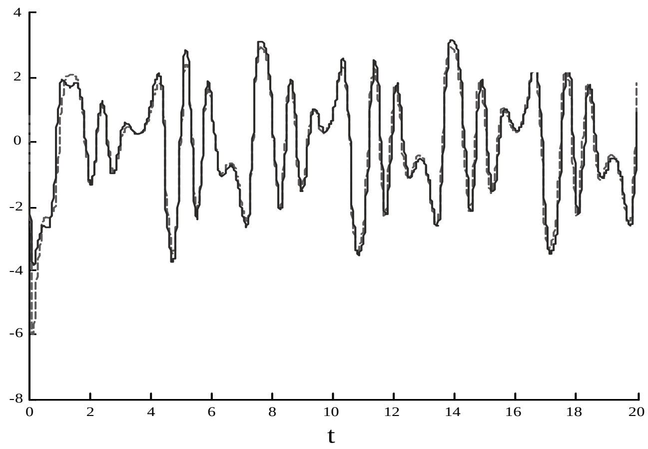

Figure 4. Trajectories of the states x_2 and y_2

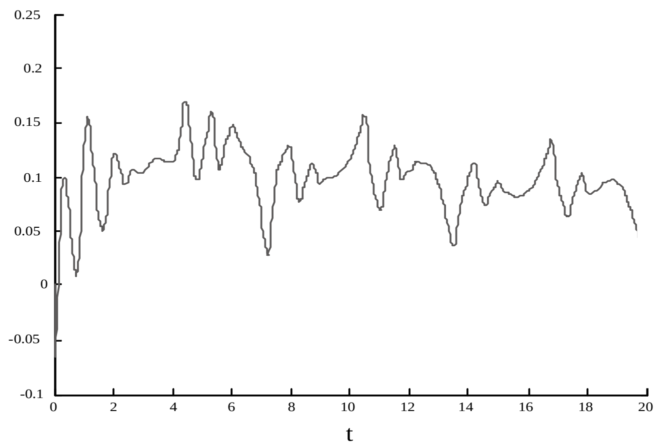

Figure 5. Control effort

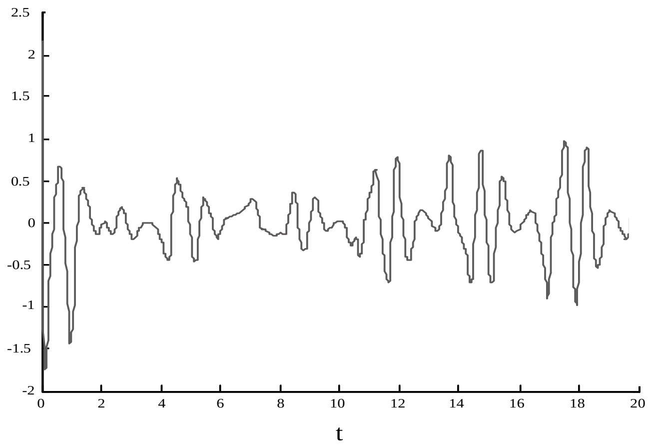

Figure 6. Trajectory of the sliding surface

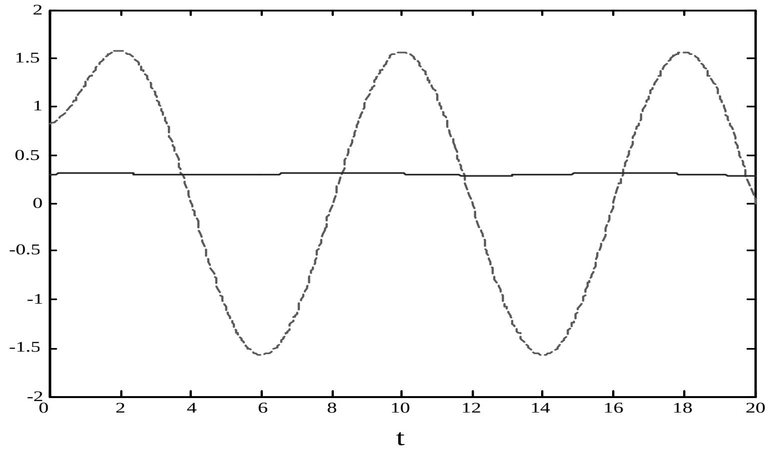

Figure 7. Estimations of *f* (*x*,*t*) and its real values

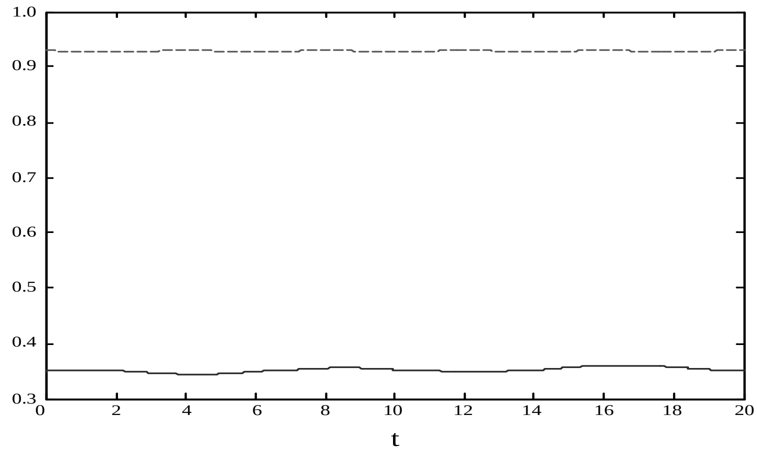

Figure 8. Estimations of *g*(*x*,*t*) and its real values

CONCLUSION

Since many systems are known to display fractional order dynamics, this paper has accordingly proposed an adaptive fuzzy sliding mode controller based on fractional calculus for a class of nonlinear chaotic systems. The scheme integrates fuzzy logic approximation technique with sliding mode control method, and has been proposed for the chaotic systems with unknown parameters and external disturbance. Moreover, by means of Lyapunov arguments, the adaptive laws and the robustly stability of sliding motion have been derived. It has been shown that both the switching surface and the FSMC controller have been obtained. Numerical simulation on chaotic nonlinear gyroscope system has been done and results validate the correctness and the effectiveness of the proposed scheme.

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