



## A two-stage damage identification method for space truss structure

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### ABSTRACT

To space truss structure, a two-stage damage identification method was proposed. First, using the modal force vector difference with small and low order modes to identify the damage bar of the space truss structure, then identifying the extent of damage through the analysis method of eigenvalue sensitivity by use of the modal frequency with easy-get and high accuracy characteristics. Numerical analysis show that this method can effectively identify different types, different quantity as well as varying degrees of injury bar. the algorithm of the method is simple, small computational workload, and is apply to actually truss structure damage identification.

**Keywords:** space truss; damage identification; modal shape; eigenvalue sensitivity

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### INTRODUCTION

Space truss structure as a common structural form has a very wide application in the construction, communication, electric power and other fields, under the influence of using the loading as well as the environment, it is extremely easy to cause the accumulation of damage and injury, which can be a serious threat to the structure's safe operation, thus, researches on the development of the structure of the space truss with the structural damage identification and safety evaluation, as well as timely maintenance and remedial measures, which have very high application value.

At present, using the method of structural damage data to identify the structure vibration data can be divided into dynamic fingerprint and inversion method. Dynamic fingerprint method is method which is mainly based on frequency, vibration mode, the modal strain energy and modal flexibility; while inversion method is mainly including analysis matrix optimization and sensitivity. As we know: the damage identification method based on frequency, vibration mode of the whole structure damage judgment is more effective<sup>[1-2]</sup>, which is poor to the damage identification results of symmetric and complex structure;<sup>[3-4]</sup> while using strain damage identification method, it is sensitive to the bar and other fine damage, but it is very difficulty that a modal testing accuracy, order and freedom degree in the current measurement can be achieve, therefore, its actual application is limited. By using inversion method, matrix optimization, sensitivity analysis method can be better identified with slight injury, which also can realize the damage localization and quantification, but with the complexity of structure increases, the required test data and the recognition difficulty are increased rapidly, moreover, the algorithm is very complex with large amount of computation<sup>[5-6]</sup>.

Transmission tower structure is complex, which is with strong symmetry, many rods as well as more degrees of freedom, therefore, at present, whether power fingerprint method or inversion method has applied little research on damage identification of transmission tower, thus the effect is not ideal. Therefore, this paper puts forward space truss structure damage identification methods with two stages, which is firstly utilized to obtain the structure of small, low order modes, through the modal force vector difference to realize the damage element of preliminary location of space truss structure, on the basis of the characteristics of the modal frequency that is easy to obtain with high measuring accuracy, it is combined with the characteristic value sensitivity analysis method, so as to solve the damage degree. This method can avoid measuring a large number of high order modal shape data, thus the algorithm is simple with small workload, which is easy to be applied in engineering. Finally, through a numerical example, it

can verify the effectiveness of the method of damage identification of truss with two stages.

### 1 Modal force vector difference and damage location

With reference to the residual force vector in the form of  $^{[7]} \Delta K \phi_i^d$ , defining the modal force vector difference, namely the index for damage localization:

$$F = K \phi_i^u - K \phi_i^d \quad (1)$$

In the formula superscript  $u$  represents the undamaged structure,  $d$  represents the damaged structure, while.  $K$  is the stiffness matrix of the whole structure.

The structure before and after damage, its parameters can be approximately expressed as the form of the first order perturbation:

$$\begin{aligned} K^d &= K^u + \Delta K, \quad \phi_i^d = \phi_i^u + \Delta \phi_i, \\ \lambda_i^d &= \lambda_i^u + \Delta \lambda_i \end{aligned} \quad (2)$$

In the formula,  $\Delta K$ ,  $\Delta \phi_i$  can respectively represent the stiffness matrix of the whole structure, the perturbation matrix of structure  $i$ ,  $\Delta \lambda_i$  can represent the perturbation value of  $i$  square value of frequency.

$\Delta \phi_i$  can be expressed as a linear combination of the intact structure of various modes, according to the perturbation theory of the first order, the modal force vector difference can be expressed as:

$$F_i = -K \Delta \phi_i = -K \sum_{j=1}^n c_{ij} \phi_j^u \quad (3)$$

In the formula,  $c_{ij}$  is the combination coefficient. It can prove <sup>[8]</sup>:

$$c_{ij} = \begin{cases} -\frac{\phi_j^{uT} \Delta K \phi_i^u}{\lambda_j^u - \lambda_i^u}, & j \neq i \\ 0, & j = i \end{cases} \quad (4)$$

Modal force vector difference can be expressed as:

$$F_i = K \sum_{j=1}^n \frac{\phi_j^{uT} \Delta K \phi_i^u}{\lambda_j^u - \lambda_i^u} \phi_j^u, \quad j \neq i \quad (5)$$

By formula (5), with the right end of it, we can see, all items except for  $\Delta K$  are intact structure parameters, therefore, the modal force vector difference only depends on the overall stiffness of the first-order perturbation matrix  $\Delta K$ , which can reflect the modal force vector difference positioning injury. In the early period of structural damage, the damage is often occurred in a few units, in the  $\Delta K$  matrix, its damage element namely the corresponding nodes are nonzero elements, thus, the modal force vector difference value can be the structural damage degrees of freedom most likely.

By formula (1), which is known, modal force vector difference requires intact structure of the overall stiffness matrix before and after damage as well as the first-order complete mode shape vector difference, in practical application, when the measured degrees of freedom is less than structural degrees of freedom, it can use the extended model <sup>[9]</sup> or reduction model <sup>[10]</sup>.

### 2 The quantification of eigenvalue sensitivity and damage <sup>[9]</sup>

In actual engineering tests, modal testing is difficult, with low precision, and the frequency measurement can be obtained more than a dozen order or even dozens of order of the exact value, therefore, obtaining the damage

location of truss structure preliminary can use the eigenvalue sensitivity for more accurate quantitative study of damage.

By formula (2). We can know, the change of  $j$  eigenvalues can be determined by measuring the frequency before and after damage, i.e.:

$$\Delta\lambda_j = \lambda_j^d - \lambda_j^u \quad (6)$$

As for the eigenvalue, by using Taylor series expansion and ignoring the two items. It can be obtained:

$$\Delta\lambda_j = \sum_{i=1}^{N_e} \Delta a_i \frac{\partial\lambda_j}{\partial a_i} \quad (7)$$

In the formula,  $N_e$  is the total number of structural units,  $\Delta a_i$  is damage degree of  $\frac{\partial\lambda_j}{\partial a_i}$  unit,  $j$  is known as the first order sensitivity of the eigenvalues of E. Assuming structure does not cause quality change before and after damage, thus, the eigenvalue sensitivity is:

$$\frac{\partial\lambda_j}{\partial a_i} = -\phi_j^T K_i \phi_j \quad (8)$$

In the formula,  $K_i$  is the whole stiffness matrix of coordinates unit.

If  $N$  eigenvalue is acquired by measurement, then in the formula, (7) can be expressed as:

$$\begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_N \end{bmatrix} = S \times \begin{bmatrix} \Delta a_1 \\ \Delta a_2 \\ \vdots \\ \Delta a_{N_e} \end{bmatrix} \quad (9)$$

In the formula,  $S$  is the characteristic value of the first-order sensitivity matrix, which can be obtained by finite element model:

$$S = \begin{bmatrix} \frac{\partial\lambda_1}{\partial a_1} & \frac{\partial\lambda_1}{\partial a_2} & \dots & \frac{\partial\lambda_1}{\partial a_{N_e}} \\ \frac{\partial\lambda_2}{\partial a_1} & \frac{\partial\lambda_2}{\partial a_2} & \dots & \frac{\partial\lambda_2}{\partial a_{N_e}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial\lambda_N}{\partial a_1} & \frac{\partial\lambda_N}{\partial a_2} & \dots & \frac{\partial\lambda_N}{\partial a_{N_e}} \end{bmatrix}_{N \times N_e} \quad (10)$$

Thus, solving the damage degree vector can be expressed:

$$\begin{bmatrix} \Delta a_1 \\ \Delta a_2 \\ \vdots \\ \Delta a_{N_e} \end{bmatrix} = S^+ \times \begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_N \end{bmatrix} \quad (11)$$

In the formula,  $S^+$  is the inverse matrix of eigenvalue sensitivity of Moore-Penrose.

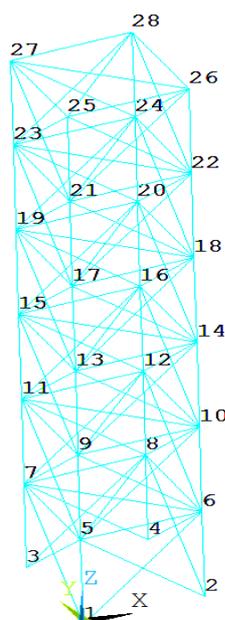
Supposing, the truss structure damage unit initially is identified, damage element number is  $L$ , thus, the eigenvalues of matrix of the first-order sensitivity can be simplified:

$$S = \begin{bmatrix} \frac{\partial \lambda_1}{\partial a_1} & \frac{\partial \lambda_1}{\partial a_2} & \dots & \frac{\partial \lambda_1}{\partial a_L} \\ \frac{\partial \lambda_2}{\partial a_1} & \frac{\partial \lambda_2}{\partial a_2} & \dots & \frac{\partial \lambda_2}{\partial a_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \lambda_N}{\partial a_1} & \frac{\partial \lambda_N}{\partial a_2} & \dots & \frac{\partial \lambda_N}{\partial a_L} \end{bmatrix}_{N \times L} \quad (12)$$

By formula (12), we can know, after the damage element preliminary is determined, the characteristic value of the first-order sensitivity matrix will be significantly simplified, which is conducive to improve the accuracy of solution element damage degree, so as to reduce the number of feature frequency measurements which are of great help.

### 3 Numerical example

This paper selects six segments (6 higher) of space truss structure as the object of study, in order to verify the effectiveness of the proposed method. The space truss cantilever structure bottom can be node consolidation, which can be consisted of 28 nodes, 108 bars, a total of 144 degrees of freedom, the structure is with finite element model which is as shown in Figure 1. Truss rod is circular cross section which diameter is  $R = 0.005m$ , it has the following characteristics: the material elastic modulus  $E = 210GPa$ , Poisson's ratio is 0.3, while the density of the material is  $\rho = 7850kg/m^3$ .



**Fig 1** truss structure mode

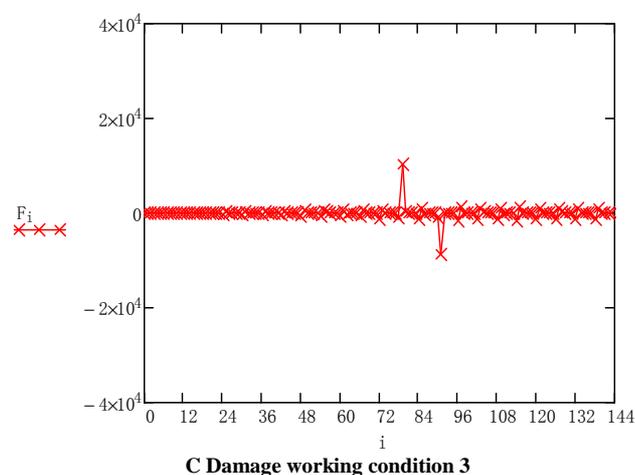
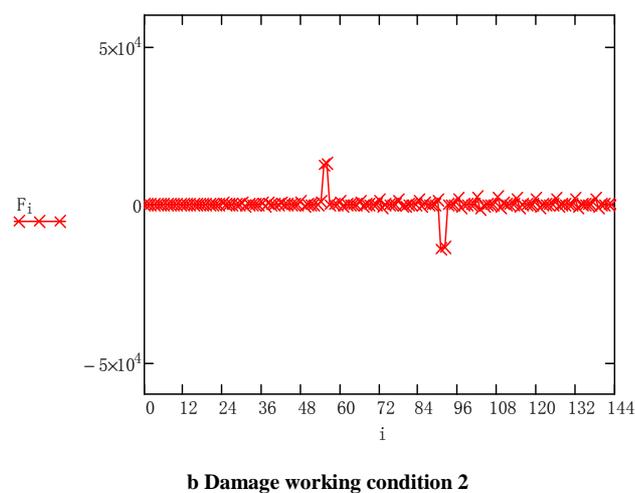
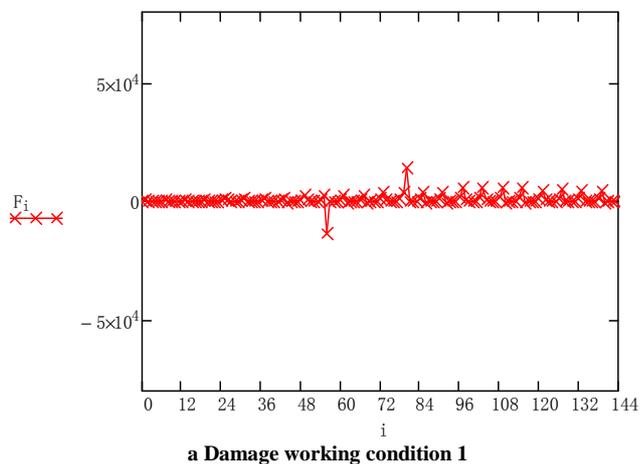
**Table 1** the damage cases

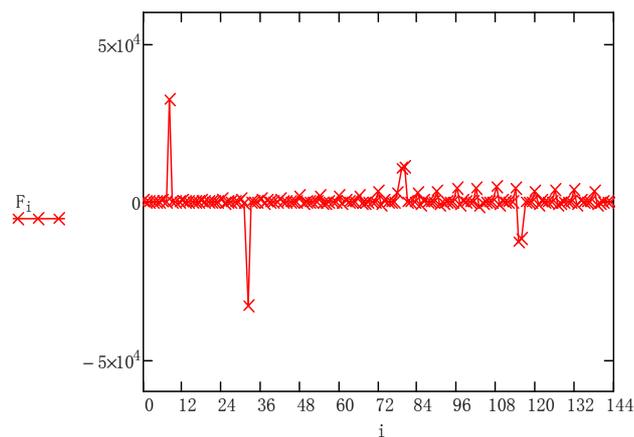
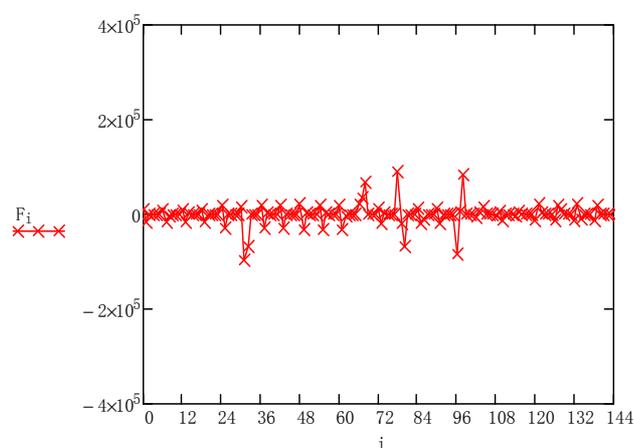
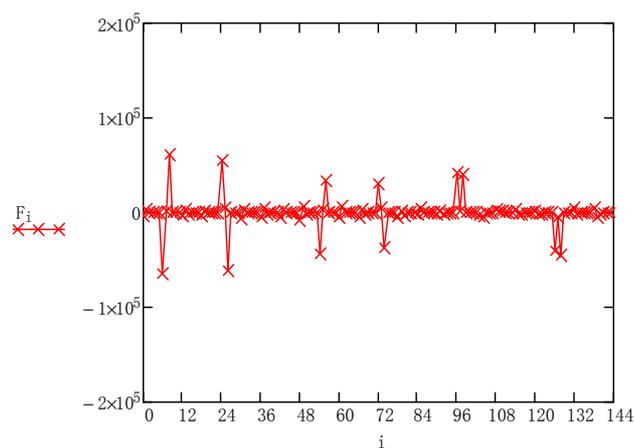
Working Condition	Rod type	Injury Rod	The corresponding node Freedom rod	The degree of injury
Single damage	1 Chord	1418	54-59,78-83	-20%
	2 The ventral rod	1420	54-59,90-95	-50%
	3 Cross bar	1820	78-83,90-95	-50%
Multiple injury	4 Chord	0610	6-11,30-35	-20%
	The ventral rod	1824	78-83,114-119	-50%
	The ventral rod	1016	30-35,66-71	-30%
	The ventral rod	1821	78-83,96-101	-50%
		0609	6-11,24-29	-20%
	6 The ventral rod	1417	54-59,72-77	-50%
		2126	96-101,126-131	-30%

Truss structure is with large number of statically indeterminate, at the same time, the bar fine damage is smaller changed to the overall vibration characteristics, damage can be set with 20% of chord injury, above 30% abdominal

bar can be considered, in order to consider the possible damage as many as possible, in the numerical example, by lowering the bar elastic modulus on the truss made loss, six damage cases can be set up, including single damage and multi-damage (shown in Table 1).

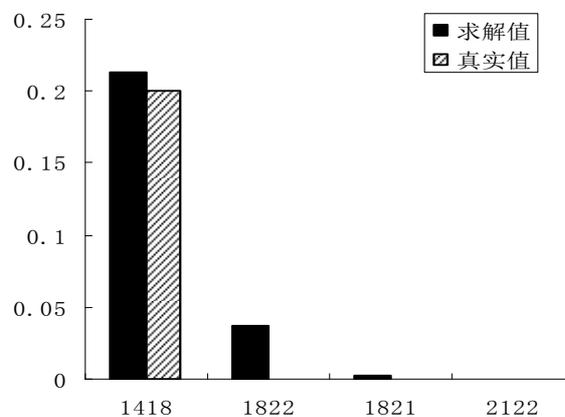
The structure is space truss structure, its vibration characteristics can be included multiple directions and torsional vibration, the example shows us that, the cross bar and the ventral rod can be mainly influenced by the direction of plane internal vibration type, which has not significant impact on the tangential direction vibration type, therefore, in the use of modal force vector difference of damage localization, in order to fully consider the planar bar and abdominal rod damage, this paper mainly uses the superposition of one or two modes, the damage condition of the truss rod position recognition cab shown in Fig. 2.



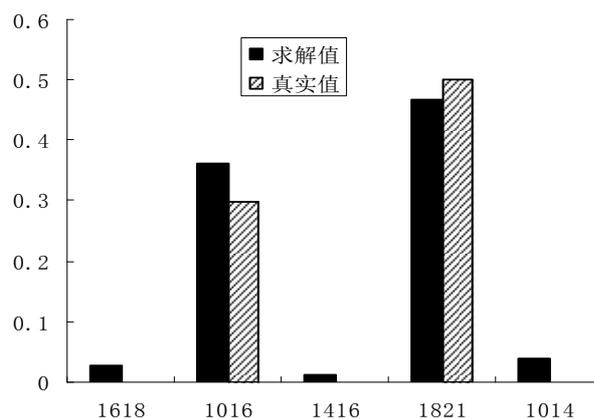
**d** Damage working condition 4**e** Damage working condition 5**f** Damage working condition 6**Fig 2** the results of damage location identification

After determining the damage element of rods, by using eigenvalue sensitivity to damage with damage quantitative research, as well as the rod damage identification process, during to the symmetry of space truss structure, it can be caused by damage characteristics of symmetrical rod of the value change of sensitivity which can be the same, therefore, in order to eliminate the influence of the symmetry, in the process of Setting up models, slight artificial rod damage can be added<sup>[11]</sup> (1216 bar damage rate is 1%, 0205 bar damage rate is 1.5%, 0208 bar damage rate is 0.5%). Based on the sensitivity analysis of structural damage under all conditions before and after the first 30 frequencies of being damaged, it can determine vibration direction of the structural fundamental frequency, the

direction of Z and the reverse direction can be the modal frequency which is sensitive to the frequency, the extent of damage identification during the following process can mainly carry on the member damage assessment by using sensitive modal frequency. If you need to get the damage identification result more accurate, the iteration can use frequency change strategy to improve [12]. During the process of determining working condition 1 and 5 with the damage location discrimination, the value of undamaged degree of freedom modal force vector is larger, which has a certain interference, therefore, this paper only discusses the damage extent identification under working condition 1 and working condition 5.



a Damage working condition 5



b Damage working condition 6

Fig 3 the results of damage severity identification

Condition 1 (Fig. 2a), node 14, 18, 21, 22 as well as the corresponding degree of freedom with its modal force vector is quite different, therefore, the possible damage bar is 1418 chord, 1822 chord, 1821 rail brace and 2122 rail brace, by using the eigenvalue sensitivity method for solving the above 4 bars with the extent of the damage, we can get the degree of damage, which can be shown in Fig. 3a, among them, 1418 rod is 21.25%, 1822 bar is 3.68%, 1821 bar is 0.26%.

## CONCLUSION

This paper puts forward a space truss structure damage identification methods, this method with two stages, which is firstly utilized to obtain the structure of small, low order modes, through the modal force vector difference, judging the truss structure possible damage degree of freedom, realizing the damage element of preliminary location of space truss structure, moreover, on the basis of using the eigenvalue sensitivity analysis method, it can solve the solution of the degree of damage. Through numerical example, the following conclusions are acquired: (1) through the recognition method with two stages, it can effectively identify the space truss structure, different location, different degrees of damage; (2) the method only needs a small amount, with low order modes and as much as possible the modal frequency, which can be easier used in the practical work; (3) this method requires

complete modal shape, when the measured degree of freedom can not meet the requirements, it can use the condensation model or modal expansion method.

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