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A response analysis method for shaft-blade combined vibration due to torsional vibration of turbine-generator shafts

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ABSTRACT

A shaft-blade combined vibration model for the long blades of the low pressure turbine was presented in this paper using the method of considering the blade as a lateral vibration system with one end fixed to a shaft section. The shaft-blade combined model of the entire turbine-generator shafts for torsional vibration analysis was given by analyzing the interaction among the shaft sections during torsional vibration, and the vibration analysis response calculation method was given as well.

Keywords: Shaft-blade combined vibration; Torsional vibration; Turbine-generator; Last stage blade.

INTRODUCTION

Mass-spring vibration model is usually used in torsional vibration analysis of the turbine-generator shafts. As the blades of the turbine can be damaged during torsional vibration as well as the shafts, a shaft-blade combined model and the response calculation method is needed for analyzing the fatigue life loss of the long blades of the low pressure turbine due to the torsional vibration of the shafts.

The blade vibration can be considered as lateral vibration relative to the blade root during shaft torsional vibration. Each blade can be considered as a mass-spring model with one end fixed on the shaft, which has only x direction vibration. Then, the acting stress on the blade imposed by the shroud is a function of the relative displacement between the blade tip and root, acting as an external stress at the blade tip. When one end of blade was fixed, the acting stress on the blade imposed by the shroud can be considered as a function of the blade tip displacement.

After equivalent stimulation of shaft vibration model to x direction vibration model, it can be combined with other models at the same position to form a shaft-blade system model, as shown in the Fig.1. The model shown in the figure has two free ends. The acting stress on the blade tip imposed by shroud acts as outward stress placed at the blade tip. And its value is proportional to the displacement of blade tip relative to root. And the sub-system torque from shaft also working outward stress imposed on the mass block in present of the shaft. The shaft-blade combined torsional vibration model for the entire turbine-generator shafts can be built based on the model shown in Fig. 1.

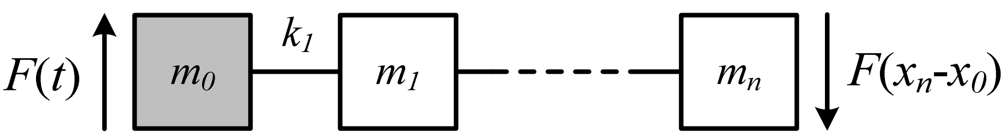

Fig. 1 shaft-blade sub-system model

Shaft-blade combined vibration model and dynamic response stimulation method

Analyze the stress of i mass block in Fig.1, then,

$$\Delta F_{i+1}^{L} = \Delta F_i^{R} = \Delta F_i^{L} + m_i \Delta \ddot{x}_i + c_i \Delta \dot{x}_i - \Delta F_{Li} \tag{1}$$

Where, ΔF_i^{L} = stress increment value left spring at x direction within single unit time,
ΔF_i^{R} = stress increment value right spring at x direction within single unit time,
$\Delta \dot{x}_i$ = speed increment at x direction, $\Delta \ddot{x}_i$ =accelerated speed increment at x direction,
c_i = mass block damping coefficient, ΔF_{Li} = mass block external stress increment at x direction.

Introduce Newmark-β method in the response analysis of torsional vibration:

$$\begin{cases} \Delta \ddot{x}_i = \dfrac{1}{\beta \Delta t^2} \Delta x_i - \dfrac{1}{\beta \Delta t} \dot{x}_i - \dfrac{1}{2\beta} \ddot{x}_i \\ \Delta \dot{x}_i = \dfrac{\gamma}{\beta \Delta t} \Delta x_i - \dfrac{\gamma}{\beta} \dot{x}_i - (\dfrac{\gamma}{2\beta} - 1) \ddot{x}_i \Delta t \end{cases} \tag{2}$$

Where, Δt =unit time, β and γ are Newmark-β parameters.

Apply average acceleration method, then β =0.5, γ =0.25.
Apply Eq (2) to Eq. (1), thus,

$$\Delta F_{i+1}^{L} = \Delta F_i^{L} + A_i \Delta x_i + B_i \tag{3}$$

Where,

$$\begin{cases} A_i = \dfrac{m_i}{\beta \Delta t^2} + \dfrac{\gamma}{\beta \Delta t} \cdot c_i \\ B_i = -m_i (\dfrac{1}{\beta \Delta t} \dot{x}_i + \dfrac{1}{2\beta} \ddot{x}_i) - c_i [\dfrac{\gamma}{\beta} \dot{x}_i + (\dfrac{\gamma}{\beta} - 1) \ddot{x}_i \Delta t] - \Delta F_{Li} \end{cases}.$$

According to transfer matrix method,

$$\begin{Bmatrix} f \\ e \end{Bmatrix}_{i+1}^{L} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{Bmatrix} f \\ e \end{Bmatrix}_{i}^{L} + \begin{Bmatrix} F_f \\ F_e \end{Bmatrix}_{i} \tag{4}$$

Where $U_{11} = 1$, $U_{12} = A_i$, $U_{21} = 1/k_{i+1}$, $U_{22} = 1 + A_i / k_{i+1}$, $F_{f,t} = B_{i,t}$, $F_{e,t} = B_{i,t} / k_{i+1}$; $f_i = \Delta F_i$ is the left side stress increment of no. i Mass block at x direction. f_{n+1} Is the right side stress increment of no? n Mass block. $e_i = \Delta x_i$, means the increment of no. i Mass block at x direction.

Apply Ricatti method, Riccati will be

$$f_i = S_i e_i + P_i \tag{5}$$

Where S_i and P_i are coefficients.
Eq (4), then,

$$S_i = \frac{U_{22} S_{i+1} - U_{12}}{U_{11} - U_{21} S_{i+1}} \tag{6}$$

$$P_i = \frac{P_{i+1} + S_{i+1}F_{ei} - F_{fi}}{U_{11} - S_{i+1}U_{21}} \tag{7}$$

$$e_{i+1} = e_i[U_{21}S_i + U_{22}] + [U_{21}P_i + F_{ei}] \tag{8}$$

The blade tip is a free end, it always have $f_{n+1} = S_{n+1}e_{n+1} + P_{n+1} = 0$, thus $S_{n+1} = 0$, $P_{n+1} = 0$. Apply Eqs. (6) And (7), S_0 and P_0 can be calculated. If e_0 is known, all the displacement increment value of each mass block can be calculated through Eq. (8).

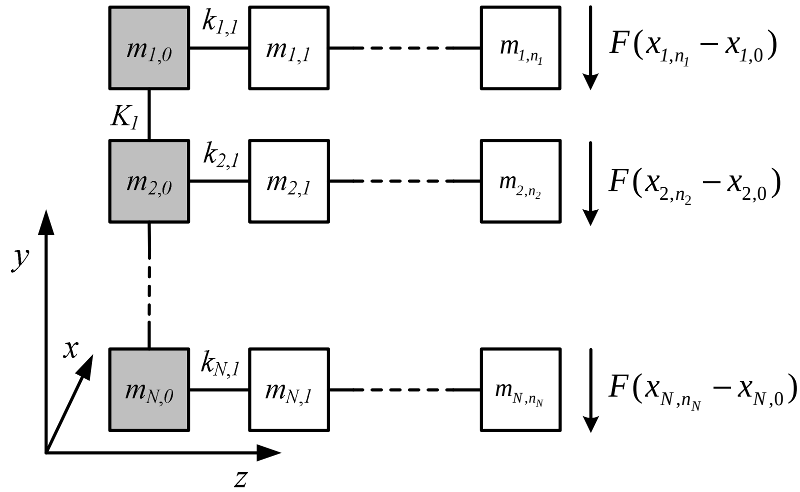

Fig.2 Shaft-blade system mass-spring model

Stimulate the whole shaft-blade system into mass-spring model, as shown in the figure. The stress on the no. i Piece of shaft imposed by blade can be expressed as $f_{i,0} = S_{i,0}e_{i,0} + P_{i,0}$. For sub mass-spring system combined by $m_{1,0}, m_{2,0}, \cdots, m_{N,0}$,

$$(e_{i+1,0} - e_{i,0})K_i - (e_{i,0} - e_{i-1,0})K_{i-1} = S_{i,0}e_{i,0} + P_{i,0} \tag{9}$$

Therefore,

$$\mathbf{Ke} = \mathbf{Se} + \mathbf{P} \tag{10}$$

Where $\mathbf{e} = [e_{1,0}, e_{2,0}, \cdots, e_{N,0}]^T$; $\mathbf{S} = \mathrm{diag}(S_{1,0}, S_{2,0}, \cdots, S_{N,0})$ $\mathbf{P} = [P_{1,0}, P_{2,0}, \cdots, P_{N,0}]^T$

And,

$$\mathbf{K} = \begin{bmatrix} -K_1 & K_1 & & \\ K_1 & -K_1 - K_2 & K_2 & \\ & & \ddots & \\ & & K_{N-1} & -K_N \end{bmatrix},$$

If the solutions of Eq. (10) are known ($e_{1,0}, e_{2,0}, \cdots, e_{N,0}$), all the displacement increment value of all mass blocks in unit time can be calculated through Eq. (8).

This method assumes that the result has linear relation with Δt. Therefore, there is accumulative error during the calculation. In order to eliminate the linear accumulative error, the accelerated speed increment value can be calculated through system increment movement formula but not Eq. (2). It will be,

$$\Delta \ddot{x}_{i,j} = \frac{f_{i,j+1} - f_{i,j} - c_i \Delta \dot{x}_{i,j}}{m_{i,j}} \tag{11}$$

When the original state of shaft-blade combined vibration model, and impose the equivalent stress of electromagnetic torque and steam torque on the mass block. Then, the model dynamic response can be calculated.

CONCLUSION

From the above, the shaft-blade fatigue life loss calculation procedure is following: Monitor the generator bus bar current, voltage and instantaneous rotational speed. And, calculate generator electromagnetic torque. After that, calculate steam torque through electromagnetic torque before torsional vibration happens. Then, according to the dynamic response stimulation of shaft-blade torsional vibration model, calculate the relative displacement among each mass block. The torsional stress value of shaft journal is proportional to the relative displacement between the left and right cross-section. Apply finite element method to calculate the dangerous point displacement- stress curve. According to this curve, the blade dangerous point stress loading experience can be obtained. And finally, according to the rotor torsional S-N curve and blade material S-N curve, calculate rotor and blade dangerous point fatigue life loss through rain-flow method.

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