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A novel pricing model to new financial products based on the game theory

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ABSTRACT

The pricing problem of financial products is an important issue to the researchers. For this reason, this paper proposed a novel pricing model to new financial products based on the game theory. The experimental results suggest that this model is feasible and correct.

Key words: Pricing Model, Financial Products, Game Theory

INTRODUCTION

In 1880, the first composite structured investment vehicle was born and since then, structured financial products have gone into public eyes. One of the most important types is equity-linked notes, also called equity-linked structured products in this paper. Modern structured products are dated from 1970s to 1980s and grew explosively during 1990s. With the constant development of structured financial products, theoretical researches on equity-linked structured products are being carried on in many countries.

Chen classified SPIN (standard poor's 500 in. dexed note) products into bonds and European call options and then respectively employed standard bonds pricing models and Black-Scholes option pricing formula to makes researches on pricing and hedging [1]. His instance analysis shows that price of this product in American market has been overvalued. Chen classified MICD products into call options and put options two kinds through maturity yielding award function and investment portfolio copying and made pricing and hedging researches [2]. His instance analysis shows that price of this product in American market has been overvalued. Carlin [3] has obtained the relation between pricing and complexity through three-phase and four-phase complex pricing game models: there are differences in pricing even products are of the same quality. The more complex products are, the higher the price is. Price of products will not go to the marginal cost even if large amounts of corporations take part in marketing. Brian [4] and Stoimenov [5] have also noted the above-mentioned relations. In their opinions, product pricing is usually higher than their theoretical value and products are generally over-priced and highly complicated. Pricing of market makers is beneficial to them but usually deviates from the theoretical value and their pricing mechanism is often confidential from investors, so private investors can hardly estimate true value of the product due to its complex design. Brown [6] also made analysis on the heavy deviation of warrants pricing in Australian market.

Mallier [7] used the Vasicek model to calculate the risk-free interest rate and made pricing on equity-linked notes respectively based on bonds and options from the angle of investors through Green's function. During their research, they discounted bonds and options with the same random interest rate in order to avoid the influence of interest rate on option value. The research shows that the option value depends on the random interest rate and the stock price index and obeys the partial differential equation of the Vasicek model. Martin [8] explored the pricing rationality of the most successful product in Swiss market - the multi-asset convertible bonds embed with barrier options - through the value-based multi-branch tree model of Chen and found that the prices of the most popular 468 products in 2007 are higher than their theoretical values by 3.4%. His analysis shows that, high prices are related with bonds, and investors usually over-value the price of risk-free bonds and underestimate the vale of risk products, which has explained why it is so important to release this product. Carole [9] analyzed two kinds of equality-linked products based on Black-Scholes framework through benefit award functions and the risk-neutral preferential assumption. He proposed that the reason for the phenomenon that investors show more interests in complicated, over-priced and high-commission products is that they have overvalued the probability of high rewards of complex products.

Deng [10] made pricing and hedging researches on the spread option products of basket risk assets through expanding Kirt approximation method and second-order boundary approximation method. The instance analysis shows that the two methods are fairly accurate and the latter are better. Generally speaking, researches on structured products at abroad mainly focus on product pricing, exploring the pricing model selection on a micro level, the relation between the design complexity and the pricing, the heavy deviation of product prices from their theoretical value and reasons for pricing deviation.

ANALYSIS VIA GAME THEORY

In the problem of game between companies and customers, players are financial companies and customers. As to a financial company, the set of feasible solutions is the set of prices from the acceptable minimum discount price to the full price. As to a customer, the feasible solutions are purchasing the new financial product. If customers have purchased the new financial products, financial companies will obtain revenues in the product price. Since customers may select other financial products, their revenues are the difference between the total costs.

All players and sets of feasible solutions for all players and their revenues are common information of players. During the financial product presale process, firstly the company will make decisions on the presale price, then customers will decide whether to buy the financial product or not according to the given price, so this problem is a complete information dynamic game.

Player 1 is financial company and player 2 is customer. Suppose $f_i (i = 1, 2, \cdots)$ is feasible solution of the financial company. U and V are functions about the price f_i and respectively the revenue. Y refers to ACCEPT and N refers to REFUSAL. The tree diagram for the dynamic game is shown in Fig. 1.

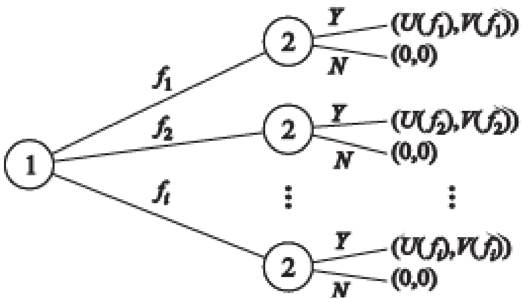

Fig. 1. The tree diagram for the dynamic game

In this tree diagram, any sub-tree formed by node 1 towards the right will form a sub-game with only player 2. Obviously, player 2 is rational and his decision depends on the value of the second component in the parenthesis, namely the optimal decision of player 2 is

$$\begin{cases} Y, & V > 0, \\ N & V < 0. \end{cases} \tag{1}$$

Whether player 1 will obtain the revenue or not depends on the decision of player 2. The game between companies and customers is a non-antagonistic cooperative game and players will try their best to obtain the most revenue. As to airline, the best result is that his revenue U is as big as possible on the basis that $V > 0$. Therefore, the optimal decision f^* of player 1 should satisfy

$$U(f^*) = \max(U(f)),\ V(f^*) > 0 \tag{2}$$

In this dynamic game, player 1 acts first and then player 2 makes decisions according to the decision of player 1, so any player should take measures to form the situation (f^*, Y), or, they will suffer loss. The situation (f^*, Y) is actually the Nash Equilibrium of this dynamic game.

NOVEL PRICING MODEL

Based on the above analysis, we know that the set of the optimal financial products during the presale period is the Nash Equilibrium of the airline:

$$f^{*}=(f_{t}^{*},f_{t-1}{}^{*},\cdots) \tag{3}$$

When the number of customers is bigger than 1 in each time node, the company's decision will turn to its influence on the number of customers which will book financial products. At this time, the original game will degrade to a single game of financial company in which, company will control the number of customers in each time node through price control so as to maximize its revenue.

Based on features of the new financial product sale, discretize the presale period by day. Time node $1\le t\le T$ is the number of days from it begins to sale, k^{t} is the number of customers need in the same day, then the number of customers booking new financial product before t days from it begins to sale is

$$k_{d}^{t}=\left[F(f_{t}^{*})k^{t}\right] \tag{4}$$

Here, $[\bullet]$ refers to round down. Then the sale revenue of financial company in the same day is

$$s^{t}=k_{d}^{t}f_{t}^{*} \tag{5}$$

Due to the influence of many factors, there are two possibilities as to customers that have booked the new financial product. k_{n}^{t} refers to the number of customers with No Show that booked the financial product in the t day and they can get a refund f_{n}. If stochastic No Show is considered during the presale process, a situation that the number of customers may be larger than the number c. Suppose the loss (including refund and compensation) caused by each DB passenger is f_{b}, then the total revenue function is

$$U(f^{*},k_{d},k_{n})=\begin{cases}\sum_{t=1}^{T}s^{t}-k_{n}^{t}f_{n}, & \sum_{t=1}^{T}(k_{d}^{t}-k_{n}^{t})-c\le 0\\ \sum_{t=1}^{T}s^{t}-(\sum_{t=1}^{T}(k_{d}^{t}-k_{n}^{t})-c)f_{b}, & \sum_{t=1}^{T}(k_{d}^{t}-k_{n}^{t})-c>0\end{cases} \tag{6}$$

Since all variables are stochastic and the number of No Show customers is related with the number of customers booking tickets, it is hard to get the revenue expectation through the above function. Since the refunding has no relations with the booking time and product holding period of customers, namely that the probability of No Show is the same no matter how long the ticket has been bought and whenever it is bought. The function (6) can be simplified as:

$$U(f^{*},k_{d},k_{n})=\begin{cases}\sum_{t=1}^{T}s^{t}-k_{n}f_{n}, & E(\sum_{t=1}^{T}k_{d}^{t})-k_{n}-c\le 0\\ \sum_{t=1}^{T}s^{t}-(E(\sum_{t=1}^{T}k_{d}^{t})-k_{n}-c)f_{b}, & E(\sum_{t=1}^{T}k_{d}^{t})-k_{n}-c>0\end{cases} \tag{7}$$

Here, $E(\bullet)$ refers to the expectation of $\bullet$ .The total revenue is composed of sale revenue U^{+} and loss U^{-}, then the total revenue expectation is

$$E(U) = E(U^{+}) - E(U^{-}) \quad \textbf{(8)}$$

The number of customers that will buy the financial product at the t day is regarded as the Poisson process (a discrete Markov process in a continuous time status) with the strength λ_t, then

$$\begin{aligned} E(U^{+}) &= E\Big(\sum_{t=1}^{T} s^{t}\Big) = \sum_{t=1}^{T} f_t^{*} E(k_d^{t}) = \\ &\sum_{t=1}^{T} f_t^{*}\left[F(f_t^{*})E(k^{t})\right] = \\ &\quad \sum_{t=1}^{T} f_t^{*}\left[F(f_t^{*})\lambda_t\right]. \end{aligned} \quad \textbf{(9)}$$

No Show of customers obeys a binomial distribution. Suppose No Show is an independent event and the probability that No Show happens on each customer is the same p_n, then the mathematical expectation of the loss caused by No Show and DB is

$$\begin{aligned} E(U^{-}) &= \sum_{j=0}^{\beta-c} (\beta - j - c) f_d C_\beta^j p_n^j (1-p_n)^{\beta-j} + \\ &\sum_{j=\beta-c}^{\beta} j f_n C_\beta^j p_n^j (1-p_n)^{\beta-j}, \\ &\beta = \sum_{t=1}^{T}\left[F(f_t^{*})\lambda_t\right] \end{aligned} \quad \textbf{(10)}$$

In a word, the Nash Equilibrium of financial product price problem can be solved through the optimization:

$$\begin{cases} \max E(U(f_1, f_2, \cdots, f_T)), \\ \text{s.t.} \quad f_t \in [\, f_{\min}, f_{\max}\,], \\ f_1 \ge f_2 \ge \cdots \ge f_{T^*} \end{cases} \quad \textbf{(11)}$$

In reality, the price control is expressed with several kinds of discounts, so the feasible region of this optimization is not continuous and this optimization problem belongs to one with discrete feasible region for which there are multiple solving methods. For the convenience of discussion, the feasible region is regarded as continuous and optimization functions in Matlab are used to solve the model.

RESULTS

Pricing principles are generally the same once revenue functions for products with multiple stock options are determined, so this paper only takes NO. 0809 product of Ying Feng Financial Service in Shenzhen Commercial Bank for example and studies its pricing method. Relevant product specifications are as follows: This financial product totally promises the safety of principal. Its yield is linked with six stocks listed in Hong Kong Stock Exchange.

The initial price of each linked stock is defined as the closing price of the stock at the initial observation day. The maturity price of each linked stock is defined as the closing price of the stock at the last observation day.

Financial income calculation is as follows:
1) When the product is at maturity, respectively calculate the absolute value of the performance of stock price as to six linked stocks. The calculation formula is

$$\text{absolute value of the performance of single stock}=\left|\frac{\text{maturity stock price}}{\text{initial stock price}}-1\right|$$

The initial observation date is Jul. 23, 2007 and the maturity observation date is Jul. 18, 2008.

2) Multiply the minimal absolute value of the six ones by the participation rate and we can get the final yield rate of the financial product. The detailed calculation formula is: yield rate of the financial product at maturity (Hong Kong dollar)=50%×the minimal absolute value.

3) Terminate the product and redeem the principal and interest.

According to the above-described yield calculation method, the total yield rate of this product is 0% at least and has no ceiling. This product is a principal-protected structured financial product linked with multiple underlying stock portfolios, issued by Shenzhen Ping An Bank. According to its specifications, $\lambda = 0, F = 1000, \delta = 0.5$, so the revenue function can be represented with

$$f(S)=\begin{cases} F(1+\lambda) & \text{if } S_{iT}=S_{it} \quad \text{as to all } i=1,2,\cdots,6 \\ F\left[1+\delta\min\left|\dfrac{S_{iT}}{S_{it}}-1\right|\right] & \text{if } S_{iT}\neq S_{it} \quad \text{there is certain } i\in\{1,2,\cdots,n\} \end{cases}$$

In order to release the relevance among products, this paper adopts the maturity closing prices of each stock during the period from Jul. 1, 2005 to Jul. 1, 2007 as the sample data to evaluate the drift rate and fluctuation rate of stock price.

Based on the above data, make one hundred simulations on the maturity value of each stock price, calculate the revenue functions respectively and we get the yield expectation 1 022. 886 356 yuan, so this product can be priced as

$$C_t = e^{-0.0333}\times 1022.886\,356 = 982.385\,131\,9 \ yuan$$

Therefore, the theoretical price of the product should be 989.3851 (round to four decimal places) yuan. We can see that, this product is priced high actually and its actual yield rate will be lower than the expected one which is actually on the high side compared with the risk investors should take. Actually, the yield rate for this product at maturity is only 0.974%.

CONCLUSION

The key to pricing multi-asset principal-protected equity-linked products lies on the maturity revenue function for the product. Once the revenue function is determined, Monte Carlo Simulation can be used to simulate the underlying asset price and finally we obtain the product price. Therefore, Monte Carlo Simulation is fairly practical, convenient and flexible as to pricing products linked with multiple but not too many underlying assets. However, the pricing study in this paper is based on rather rigorous assumptions, namely a perfect market, but most of current markets are not perfect ones which will challenge the applicability of the product pricing. Therefore, further researches can be made on pricing principal-protected equity-linked products based on multi-asset options on an imperfect market.

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