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A Mathematical Modeling Self-Focusing Of Langmuir Waves in Relativistic Plasma

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ABSTRACT

In this paper we study the self-focusing of a Langmuir wave with a Gaussian distribution of intensity along its wave front. The relativistic oscillation of the mass of the electrons in the field of Langmuir waves and also the relativistic electron ponderomotive force are shown to have a major effect on nonlinear dynamics of Langmuir waves. Both the relativistic oscillation of the mass and the electron ponderomotive force promote self-focusing of the Langmuir wave. The Landau damping of the Langmuir wave opposes the self-focusing effect. When damping is unimportant, the beam gets more and more focused as it advances. A paraxial ray theory of self-focusing, through relativistic nonlinearity, reveals that the self focusing length $R_n \sim a\, v_{th}/ v_{osc}$ *where* a *is the radial width of the Langmuir wave and* v_{th} *and* v_{osc} *are the electron thermal and oscillatory velocities.*

INTRODUCTION

The propagation of an ultra-short intense laser pulse through an under dense plasma is an important area of research[1-17]. Collective charged – particle accelerators have received considerable theoretical and experimental interest over last one decade[2-12]. In this scheme two laser beams of slightly different frequencies ω_1 and ω_2 and wave numbers $\vec{k}_1$ and $\vec{k}_2$ are launched in plasma. They exert a ponderomotive force of the electrons, resonantly exciting a Langmuir wave of frequency $\omega_1 - \omega_2 \approx \omega_p$ and wave number $k_1 - k_2 \approx \omega_p / c$. As the plasma wave grows, the relativistic effect on the frequency mismatch becomes important and the plasma wave attains saturation, at a potential much bigger than the ponderomotive potential $\phi_p(\phi >> \phi_p)$. The electrons trapped in the Langmuir wave acquire excessively large velocities and can be accelerated to several MeV energies. In this scheme, however, any nonuniformity in the intensity distribution of the Langmuir wave can have important repercussions on the particles – acceleration process; hence a careful study of the propagation of nonuniform Langmuir waves is to be carried out.

In this paper we study the self – focusing of Langmuir wave with a Gaussian distribution of intensity along its wave front. The incoming laser beams may also have some contributions to the self-focusing process, however, that may be neglected as long as the oscillatory electron velocity due to the Langmuir wave exceeds that due to the laser beams, i.e., $\phi > (\omega_p/\omega_{laser})E_{laser}/k$, where E_{laser} and ω_{laser} are the field and frequency of a laser, and k is the wave vector of the plasma wave. Recent computer simulations have indicated that Langmuir waves play an important role in causing self-focusing. Nevertheless, the present treatment is inconsistent to some extent as the refractive index modification induced by the Langmuir wave influences the propagation and intensity distribution of the laser beams, which in turn should influence the process of Langmuir wave excitation. The propagation of Langmuir wave in collisionless homogeneous plasma can result in self-focusing by creating a density depression in the plasma as well as by increasing the electron mass by relativistic effects . The density depression is due to transverse ponderomotive forces, which tend to expel plasma from high field regions. This density depression creates a local increase in the effective index of refraction and acts as an optical guide for the radiation beam. In addition to this self-focusing mechanism, a further reduction in the plasma frequency occurs in regions of high field intensity due to relativistic mass increase of the electrons in the presence of the Langmuir wave the self-focusing due to the density depression occurs on a longer time scale than does the relativistic mass increase self-focusing effect. Present treatment also neglects an important effect, viz the parametric coupling of Langmuir mode to the ion acoustic wave. This restricts the analysis to $\upsilon_{osc} < \upsilon_{oscth}$, where υ_{oscth} is the threshold value of oscillatory velocity for parametric excitation. In the opposite case the eikonal theory of coupled plasma modes should be used instead of the eikonal theory for a single mode.

In sec. 2. we present a simple analysis of self-focusing using the eikonal approach. We obtain an expression for the beam width parameter and discuss its behavior with the distance of propagation in Sec. 3.

2. Beam Width Parameter

Consider the propagation of a Langmuir wave in homogeneous plasma of equilibrium density n_o^o. For harmonic time dependence of the wave potential ϕ(as $e^{i\omega t}$), The Langmuir wave causes an oscillatory velocity of electrons:

$$\upsilon_{osc} \simeq -\frac{ek}{m_0\gamma\omega}\phi(r), \quad (1)$$

which for a nonuniform Langmuir wave is space dependent. This causes two effects: (i) the electron mass is modified, m = mo $\left(1-\upsilon_{osc}^2/c^2\right)^{-1/2}$, where m_o is the electron mass, and (ii) The electrons experience a ponderomotive force $\vec{F}_p = -m\gamma/2\nabla\left(\upsilon_{osc}^2\right)$, which leads to the redistribution of electron density on a time scale ~ a/c_s, where a_o is the radial extent of the Langmuir wave and c_s is the ion sound speed. The Langmuir wave dispersion relation, $\omega^2 = \omega_p^2 + k^2\upsilon_{th}^2$, on replacing k by $i\nabla$ operator, gives the following equation for ϕ :

$$\left(\nabla^2 + \frac{\omega^2 - \omega_p^2}{\upsilon_{th}^2}\right)\phi = 0 \tag{2}$$

where

$\omega_{po} = \left(\frac{4\pi n_o e^2}{\gamma_0 m_o}\right)^{\frac{1}{2}}$ is the ambient plasma frequency , γ (r, z) $= \left(1 + a^2(r,z)\right)^{\frac{1}{2}}$ is the relativistic mass factor a^2(r, z) = I (r, z) $= \frac{e^2 k^2 \phi\phi^*}{m_0{}^2 \omega^2 c^2}$ is the normalized Langmuir wave field intensity , n_o is the time independent electron density in the presence of the Langmuir wave,m_0 and e are the electron rest mass and charge, respectively, and υ_{th} is the electron thermal speed . A radiation field a^2 (r, z) at r = 0 will produce an index of refraction profile peaked on axis. Further, we take the rapid phase variation of ϕ as e^{-ikz}. In the steady state the modified electron density can be written as

$$\begin{aligned} n_o &= n_o^o \exp\left(-\upsilon_{osc}^2/\upsilon_{th}^2\right), \\ &= n_o^o \left(1 - \upsilon_{osc}^2/\upsilon_{th}^2\right). \end{aligned} \tag{3}$$

Using Eq. (3) in Eq. (1), we obtain

$$\nabla^2\phi = n_o^o \exp \frac{\omega^2}{\upsilon_{th}}\left(\in_o + \in_2 \phi\varphi^*\right) = 0, \tag{4}$$

where

$$\upsilon_{th} = \left(2Te/\gamma m\right)^{1/2}, \qquad \in_o = 1 - \omega_{po}^2/\omega^2 ,$$

$$\in_2 = e^2k^2/m_0{}^2\gamma^2\omega^2\upsilon_{th}^2 , \quad \omega_{po} = \left(4\pi n_o^o e^2/m_0\right)^{1/2}.$$

Expressing ϕ as

$$\begin{aligned} \phi &= A\,(r, z)\, e^{i(\omega t - kz)}, \\ k &= \omega/\upsilon_{th} \in_o^{1/2} . \end{aligned} \tag{5}$$

The wave equation in the WKB approximation can be written as

$$-2ki\frac{\partial A}{\partial z} + \nabla^2 A + \omega^2/\upsilon_{th}^2 \in_2 AA^* = 0. \tag{6}$$

Employing an eikonal S, the complex amplitude A can be written as

$$A = A_o\,(r,z) \exp\left[-ik\, S(r,z)\right],$$

where A_o and S are real functions of r and z given by

$$2\frac{\partial s}{\partial z} + \left(\frac{\partial s}{\partial r}\right)^2 = \frac{1}{k^2}\left(\frac{\partial^2 A_o}{\partial r^2} + \frac{1}{r}\frac{\partial A_o}{\partial r}\right)\frac{1}{A} + \frac{\in_2}{\in_o}A_o^2, \tag{7}$$

$$\frac{\partial A_o^2}{\partial z} + \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right)A_o^2 + \frac{\partial A_o^2}{\partial r}\frac{\partial s}{\partial r} = 0. \tag{8}$$

We assume that z = 0,

$$A_o^2 = A_{oo}^2 \, e^{-r^2/a^2} \, .$$

For z>0, we express A_o^2 and S in the paraxial ray approximation as [9,13]

$$S = r^2/2\beta(z,t) + \psi(z,t),$$

$$A_0^2 = \frac{A_{oo}^2}{f^2} e^{-r^2/a^2 f^2} \approx \frac{A_{oo}^2}{f^2}\left(1 - \frac{r^2}{a^2 f^2}\right), \quad (9)$$

$$\beta = 1/f \, \frac{df}{dz},$$

The parameter f in Eq. (9) is a measure of the axial intensity as well as the width of the beam and is known as the beam width parameter, at z=0, f=1, and $df/dz = 0$ for an initially plane wave front.

The equation governing f eventually turns out to be

$$\frac{d^2 f}{dz^2} = \left(\frac{1}{k^2 a^4} - \frac{\epsilon_2 A_{oo}^2}{\epsilon_o a^2}\right) \Big/ f^3 , \quad (10)$$

Giving

$$f^2 = z^2 \left(\frac{1}{R_d^2} - 1/R_n^2\right), \quad (11)$$

where

$R_n = a \sqrt{\epsilon_o / \epsilon_z A_{oo}^2}$ is the focusing length and $R_d = a^2 k$ is the diffraction length.

3. Effect of Landau Damping

If one incorporates the Landau damping term for the Langmuir wave, then ϵ_o in Eq. (4) takes over to $\epsilon_o + i\epsilon_i$,

where $$\epsilon_i = 2\sqrt{\pi}\left(\frac{\omega_p}{k\upsilon_{th}}\right)^2 \exp\left\{-\omega^2 \Big/ \left[\omega^2 - \omega_p^2\left(1 - \upsilon_{osc}^2/\upsilon_{th}^2\right)\right]\right\}. \quad (12)$$

Following a similar procedure [8] the intensity distribution of the Langmuir wave can be written as

$$A_o^2 = A_{oo}^2 \Big/ f^2 e^{-2k_i z} \, e^{-r^2/a^2 f^2} \quad (13)$$

Where $k_i = k\epsilon_i / 2\epsilon_o$. Then the equation governing the beam – width parameter turns out to be

$$\frac{d^2 f}{dz^2} = \frac{1}{R_d^2 f^3} - \frac{\epsilon_2 A_{oo}^2 \exp(-2k_i z)}{\epsilon_o a^2 f^3} \quad (14)$$

We have solved Eq. (14) for the following set of parameters: $n_o = 10^{14}$ cm^{-3}, T = 10eV, a = 200 μm, $\omega_p = 10^{11}$ Hz, $k\upsilon_{th0}/\omega_{p0} = 0.25$, and $\upsilon_{osc}/\upsilon_{th0} \approx 0.1$. The results are shown in Fig. 1

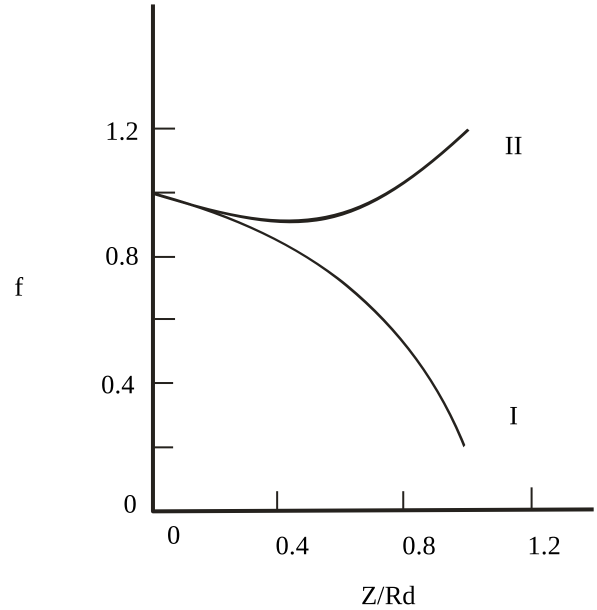

Fig. 1. Variation of the beamwidth parameter with the distance of propagation in a plasma. Curves I and II correspond to undamped and damped $\left(k\upsilon_{th0}/\omega_p\, 0 \approx 0.25\right)$ Langmuir waves

DISCUSSION

Langmuir waves of duration larger than a/c, and amplitude such that $\upsilon_{osc}/\upsilon_{th} > \sqrt{\in_o}/ka \sim 0.3/ka$, undergo self–focusing in a plasma. The nonlinearity arises through the ponderomotive force. The Landau damping of the Langmuir wave opposes the self–focusing effect.

When damping is unimportant the beam gets more and more focused as it advances. However, near the focal spot when the intensity attains large values the wave frequency is completely detuned from the local plasma frequency and WKB approximation fails. The wave suffers huge self-distortion which is highly undesirable in a charged particle accelerator.

In the context of collective charge particle accelerator the present calculation is valid only when the oscillatory velocity of electrons due to the laser waves is smaller than that due to the Langmuir wave, i.e. $\upsilon_{osc} > \upsilon_{o\ laser}$ of $k\phi > \left(\omega_p/\omega_{laser}\right) E_{laser}$ since, the potential ϕ of the Langmuir wave (driven as a beat wave by the ponderomotive potential ϕ_p two laser waves) can be estimated to be ϕ - $\phi_p/\in_o$, this condition is satisfied when $\upsilon_{laser} > \in_o \omega_p/k \sim 0.1\, \omega_p/k$. The density channel created by the Langmuir wave will focus the laser beams also and influence the process of Langmuir wave excitation. The present calculation does not account for these effects.

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