Transfer property and separate principle of finite time observers in linear control systems

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ABSTRACT

A parametric design problem of the finite time observers for the linear time-invariant systems and some important properties of finite time observers are investigated in this paper. Based on the parametric solutions for a class of Sylvester matrix equations, the parametric expressions of all the gain matrices for the finite time observers are proposed. Based on the above design results of finite time observers, the corresponding transfer property and separate principle for the finite time observers are proved, respectively. Finally, a numerical example shows the effectiveness of the proposed design method, the transfer property and the separate principle of the parametric finite time observers in the linear time-invariant systems.

Key words: Linear control system, finite time observer, parameterization, transfer property, separate principle
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INTRODUCTION

Since state estimation is initially proposed in the references [1-2], the theory of observers that can reconstruct the state from measurements of the input and output of the linear control systems, has gain much attentions of researchers, such as [3-5]. However, in most control engineering realizations that pursuit some good system performances, it is necessary and important that the estimation error can be convergent in predefined finite time. Thus the convergence in finite time becomes an attractive feature in the literature of state estimation theory and also attracts the attentions of researchers, such as [6-11]. In [6], a continuous-time observer that comprises two standard observers and a delay is design and can observe the state of the linear system in predefined finite time exactly and almost any convergence time can be assigned, independent of the observer eigenvalues. In [7], a finite time functional observer for linear systems that can estimate directly linear functions of the state of a linear system in a given finite time is proposed. In [8], a finite time unknown input observer for linear systems with unknown input disturbances in a predefined finite time is designed. In [9], besides deriving general conditions that guarantee finite time convergence, the design of observers with finite convergence time for continuous time linear time varying systems is studied. In [10], the finite convergence time of the impulsive observer is achieved by updating the observer state based on current observer data in a finite time interval. In [11] and [12], the parametric design of finite time functional observers in linear time-invariant systems and robust finite time functional observers in uncertain systems are investigated, respectively.

From the above knowledge of the finite time observers in linear control systems, we can find that the researchers focus on the design of the finite time observers and there are few results about the properties of the finite time observers. Thus in this paper we will give a parametric design method of the finite time observers in the linear time-invariant systems. Based on the results of the parametric solution to a class of Sylvester matrix equations in our preliminary researches of [13-14], we will design the parametric expressions of all the gain matrices for the finite time observers and propose the corresponding algorithm of designing the finite time observers. Based on the design results of the finite time observers, we will prove the corresponding transfer property and separate principle for the finite time observers.

PROBLEM FORMULATION

Consider the linear time-invariant system in the following form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}, x(t_0) = x_0, t \geq t_0 \tag{1}$$

where $x \in \Re^n$, $u \in \Re^p$ and $y \in \Re^q$ are the state vector, the input vector and the output vector, respectively; $A \in \Re^{n \times n}$, $B \in \Re^{n \times p}$ and $C \in \Re^{q \times n}$ are known matrices satisfying two assumptions as

Assumption A1: $\mathrm{rank}(B) = p$ and $\mathrm{rank}(C) = q$;

Assumption A2: The matrix pair (A, C) is observable, that is

$\mathrm{rank}\begin{bmatrix} sI - A^T & C^T \end{bmatrix} = n$, $\forall s \in \square$

Consider a pair of observers of system (1)

$$\dot{z}_i(t) = (A - L_i C) z_i(t) + L_i y(t) + Bu(t), \quad i = 1, 2, t \geq t_0 \tag{2}$$

where $L_i \in \Re^{n \times q}$ are the gain matrices and $z_i \in \Re^n$, $i = 1, 2$ are the state of the observers. Set

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$$

The pair of observers (2) can be written in the following compact form

$$\dot{z}(t) = Nz(t) + Ly(t) + Hu(t) \tag{3}$$

Where $N = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$, $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$, $H = \begin{bmatrix} B \\ B \end{bmatrix}$, $N_i = A - L_i C \tag{4}$

Assume that the state estimation of system (1) via observer (3) is designed as follows

$$\hat{x}(t) = M[z(t) - e^{ND} z(t - D)] \tag{5}$$

where $D > 0$ is a suitable chosen finite time.

Definition 1. Given the linear time-invariant system (1) satisfying Assumptions A1 and A2. If there exist constant gain matrices $L_i \in \Re^{n \times q}$ ($i = 1, 2$) in (2) or $L \in \Re^{2n \times q}$ in (3) and $M \in \Re^{n \times 2n}$ in (5) such that the estimation error

$$e(t) = \hat{x}(t) - x(t) \tag{6}$$

can be convergent to zero in a finite time D and stays bounded during the convergence interval $[t_0 \quad t_0 + D]$, the observer system (2) with (5), or the observer system (3) with (5) is called as a finite time observer of the linear time-invariant system (1).

Theorem 1[6]. Given the linear time-invariant system (1) satisfying Assumptions A1 and A2. Problem DPFO has a solution, if and only if the matrices $N_i (i = 1, 2)$ are Hurwitz and $\begin{bmatrix} T & e^{ND}T \end{bmatrix}^{-1}$ with $T = \begin{bmatrix} I_n \\ I_n \end{bmatrix}$ exists. In addition, there holds

$$M = \begin{bmatrix} I_n & 0_n \end{bmatrix} \begin{bmatrix} T & e^{ND}T \end{bmatrix}^{-1} \tag{7}$$

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Based on the existing condition of finite time observers for linear systems in Theorem 1, the design problem of parametric design, transfer property and separate principle problem for the finite time observers in linear time-invariant systems can be described as follows, which can be denoted by Problem DPFO.

Problem DPFO (Design of Parametric Finite-time Observers). Given the linear time-invariant system (1) satisfying Assumption A1 and Assumption A2. If the finite time observers (2) with (5) or (3) with (5) exits, determine the parametric expressions of the gain matrices $L_i \in \Re^{n \times q}$ ($i = 1, 2$) in (2) or $L \in \Re^{2n \times q}$ in (3), and $M \in \Re^{n \times 2n}$ in (5) such that the estimation error (6) converges to zero in a finite time D and stays bounded during the convergence interval $[t_0 \quad t_0 + D]$ for a suitable chosen finite time delay D. And then based on the above design results of finite time observers, prove the transfer property and separate principle for the finite time observers in linear time-invariant systems.

SOLUTION TO PROBLEM DPFO

1) SOLUTIONS TO MATRICES L_i, $i = 1, 2$ AND M

Without loss of generality, we assume that the Hurwitz matrices N_i, $i = 1, 2$ have distinct and self-conjugate eigenvalues. We denote the eigenvalues and their corresponding eigenvectors of matrices N_i^T by $s_{ki} \in \Box$, and $v_{ki} \in \Box^n$ ($k = 1, 2, \cdots, n$, $i = 1, 2$), respectively. Then there exist

$$N_i^T v_{ki} = (A - L_i C)^T v_{ki} = s_{ki} v_{ki}, \quad (k = 1, 2, \cdots, n, \quad i = 1, 2) \tag{8}$$

Denote
$$\Lambda_i = \mathrm{diag}\left(s_{1i}, s_{2i}, \cdots, s_{ni}\right), \quad i = 1, 2$$
and
$$V_i = [v_{1i} \quad v_{2i} \quad \cdots \quad v_{ni}], \quad i = 1, 2$$

By writing (8) into a compact form, we can obtain

$$A^T V_i - C^T W_i = V_i \Lambda_i, \quad i = 1, 2 \tag{9}$$

where $W_i = L_i^T V_i, \quad i = 1, 2 \tag{10}$

Further, equation (9) can be written into the following matrix form

$$\left[s_{ki} I_{n \times n} - A^T \quad C^T \right] \begin{bmatrix} v_{ki} \\ w_{ki} \end{bmatrix} = 0, \quad k = 1, 2, \cdots, n, \quad i = 1, 2 \tag{11}$$

Based on the dual theory, the matrix pair (A, C) is observable is equivalent with that the matrix pair (A^T, C^T) is controllable. Therefore, there exist two polynomial matrices $N(s) \in \Re^{n \times q}[s]$ and $D(s) \in \Re^{q \times q}[s]$ such that

$$\left[sI - A^T \quad C^T \right] \begin{bmatrix} N(s) \\ D(s) \end{bmatrix} = 0, \quad \forall s \in \Box \tag{12}$$

Comparing (11) with (12), we know that there must exist a group of parametric vectors $g_{ki} \in \Box^q$ ($k = 1, 2, \cdots, n$, $i = 1, 2$) such that[13-14]

$$\begin{bmatrix} v_{ki} \\ w_{ki} \end{bmatrix} = \begin{bmatrix} N(s_{ki}) \\ D(s_{ki}) \end{bmatrix} g_{ki}, \quad k = 1, 2, \cdots, n, \quad i = 1, 2 \tag{13}$$

Based on equations (10) and (13), we can give the parametric expressions of V_i and W_i, ($i=1,2$) in (9) as

$$V_i = \begin{bmatrix} N(s_{1i})g_{1i} & N(s_{2i})g_{2i} & \cdots & N(s_{ni})g_{ni} \end{bmatrix}, \quad i=1,2 \quad (14)$$

and

$$W_i = \begin{bmatrix} D(s_{1i})g_{1i} & D(s_{2i})g_{2i} & \cdots & D(s_{ni})g_{ni} \end{bmatrix}, \quad i=1,2 \quad (15)$$

Based on (10), (14) and (15), we can obtain

$$L_i = (W_i V_i^{-1})^T, \quad i=1,2 \quad (16)$$

In order to guarantee that the gain matrices L_i ($i=1,2$) are real, the free parameter vector $g_{ki} \in \Box^{\,q}$ ($k=1,2,\cdots,n$, $i=1,2$) must satisfy the following constraint:

Constraint C1. $\quad s_{ki} = \overline{s_{li}} \Leftrightarrow g_{ki} = \overline{g_{li}}, \quad l,k=1,2,\cdots,n, i=1,2$

If the matrix $\begin{bmatrix} T & e^{ND}T \end{bmatrix}$ is inverted, from (7) and (14) we can obtain the parametric expression of $M \in \Re^{n \times 2n}$ as

$$M = \begin{bmatrix} I_{n \times n} & 0_{n \times n} \end{bmatrix} \begin{bmatrix} T & e^{ND}T \end{bmatrix}^{-1} \quad (17)$$

where the matrix e^{ND} can be parameterized by

$$e^{ND} = \mathrm{diag}(V_1^T, V_2^T)^{-1} \mathrm{diag}\left(e^{\Lambda_1 D}, e^{\Lambda_2 D}\right) \mathrm{diag}(V_1^T, V_2^T) \quad (18)$$

and V_i, $i=1,2$ are determined by (14).

2) TRANSFER PROPERTY OF THE FINITE TIME OBSERVERS

Theorem 2. Given the linear time-invariant system (1) satisfying Assumption A1 and Assumption A2. Then there holds that the closed-loop transfer function of system (1) under state feedback $u = K\hat{x} + v$ of the finite time observer (2) with (5) or (3) with (5) is equal to that of system (1) under state feedback $u = Kx + v$.

Proof. The closed-loop system of system (1) under state feedback $u = Kx + v$ can be given by

$$\begin{cases} \dot{x}(t) = (A + BK)x(t) + Bv(t) \\ y(t) = Cx(t) \end{cases} \quad (19)$$

It is easy to obtain its closed-loop transfer function as follows

$$W(s) = C[sI - (A + BK)]^{-1}B \quad (20)$$

The closed-loop system of system (1) under state feedback $u = K\hat{x} + v$ of the finite time observer (2) with (5) or (3) with (5) can be given by

$$\begin{cases} \dot{x}(t) = Ax(t) + BKMz(t) - BKMe^{ND}z(t-D) + Bv(t) \\ \dot{z}(t) = (N + HKM)z(t) + LCx(t) - HKMe^{ND}z(t-D) + Hv(t) \end{cases} \quad (21)$$

We can rewrite (21) into a matrix form as follows

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A & BKM \\ LC & N+HKM \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$

$$- \begin{bmatrix} BKMe^{ND} \\ HKMe^{ND} \end{bmatrix} z(t-D) + \begin{bmatrix} B \\ H \end{bmatrix} v(t) \tag{22}$$

By using the Laplace transform to (22), we can obtain

$$s \begin{bmatrix} X(s) \\ Z(s) \end{bmatrix} = \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix} \begin{bmatrix} X(s) \\ Z(s) \end{bmatrix}$$

$$+ \begin{bmatrix} B \\ H \end{bmatrix} V(s) \tag{23}$$

Then system (23) can be written in a simple form as

$$\left(\begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix}, \begin{bmatrix} B \\ H \end{bmatrix}, [C \quad 0] \right)_{(24)}$$

Denote

$$P = \begin{bmatrix} I & 0 \\ T & I \end{bmatrix}, \quad Q = \begin{bmatrix} I & 0 \\ -T & I \end{bmatrix}$$

Based on (7), we can obtain

$$Q \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix} P$$

$$= \begin{bmatrix} I & 0 \\ -T & I \end{bmatrix} \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix} \begin{bmatrix} I & 0 \\ T & I \end{bmatrix}$$

$$= \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC-TA & -TBKM +TBKMe^{ND}e^{-Ds} + N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix} \begin{bmatrix} I & 0 \\ T & I \end{bmatrix}$$

$$= \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC-TA & N \end{bmatrix} \begin{bmatrix} I & 0 \\ T & I \end{bmatrix}$$

$$= \begin{bmatrix} A+BKMT - BKMe^{ND}e^{-Ds}T & BKM - BKMe^{ND}e^{-Ds} \\ LC-TA+NT & N \end{bmatrix}$$

$$= \begin{bmatrix} A+BK & BKM - BKMe^{ND}e^{-Ds} \\ 0 & N \end{bmatrix} ,$$

and

$$Q \begin{bmatrix} B \\ H \end{bmatrix} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_c P = [C \quad 0]$$

Under the transformation the closed-loop system (21) or (24) is equivalent with the following system

$$\left(\begin{bmatrix} A+BK & BKM - BKMe^{ND}e^{-Ds} \\ 0 & N \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}, [C \quad 0] \right)_{(25)}$$

It is easy to find that the transfer function of the closed-loop system (21) or (24) is same with that of (25), and the transfer function of (25) is equal to (20), which is the transfer function of (19), because the following result holds:

$$
\begin{bmatrix} C & 0 \end{bmatrix} \left(sI - \begin{bmatrix} A+BK & BKM - BKMe^{ND}e^{-Ds} \\ 0 & N \end{bmatrix} \right)^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} [sI-(A+BK)]^{-1} & * \\ 0 & (sI-N)^{-1} \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix}
$$

$$
= C[sI-(A+BK)]^{-1} B
$$

3) SEPARATE PRINCIPLE OF THE FINITE TIME OBSERVERS

Theorem 3. The eigenvalues of the closed loop system for the linear time-invariant system (1) via the state feedback $u = K\hat{x}+v$ of the finite time observer (2) with (5) or (3) with (5)) are equal to the sum of the eigenvalues $\lambda(A+BK)$ of the closed loop system for the linear time-invariant system (1) via state feedback $u = Kx+v$ and the eigenvalues $\lambda(N)$ of the finite time observer.

Proof. From the proof of Theorem 2, there holds

$$
Q \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix} P = \begin{bmatrix} A+BK & BK \\ 0 & N \end{bmatrix},
$$

$$
A_c = \begin{bmatrix} A & BKM - BKMe^{ND}e^{-Ds} \\ LC & N+HKM - HKMe^{ND}e^{-Ds} \end{bmatrix}
$$

We can know that the matrix   is equivalent with the matrix $A+BK$, thus there holds

$$
\lambda(A_c) = \lambda(A+BK) \bigcup \lambda(N)
$$
.

A NUMERICAL EXAMPLE
Consider the linear system (1) with the following parameters as

$$
A = \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
$$

It is easy to see that the matrix pair (A,C) is observable. Thus we can utilize the proposed parametric design method of finite time observers in this example. We can calculate the polynomial matrices $N(s)$ and $D(s)$ satisfying (24) as

$$
N(s) = \begin{bmatrix} s+3 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad D(s) = \begin{bmatrix} -s^2 - 5s - 6 & s+2 \\ s+3 & -s-2 \end{bmatrix}
$$

For simplicity, we select the eigenvalues of the Hurwitz matrices N_i, $i=1,2$ by $s_{11} = -2$, $s_{21} = -1+8j$, $s_{31} = -1-8j$ and $s_{12} = -10$, $s_{22} = -3+3j$, $s_{32} = -3-3j$, respectively, and

$$
g_{11} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad g_{21} = \begin{bmatrix} 2+3j \\ 1+6j \end{bmatrix}, \quad g_{31} = \begin{bmatrix} 2-3j \\ 1-6j \end{bmatrix}, \quad g_{12} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad g_{22} = \begin{bmatrix} -1+2j \\ 6+5j \end{bmatrix}, \quad g_{32} = \begin{bmatrix} -1-2j \\ 6-5j \end{bmatrix}
$$

We can obtain

$$L_1 = \begin{bmatrix} -4.0074 & -0.8744 \\ 26.4113 & 2.0074 \\ 19.2167 & 3.3153 \end{bmatrix}$$

and

$$L_2 = \begin{bmatrix} 8.7849 & 0.208 \\ 13.948 & 1.2151 \\ 14.2695 & -10.2057 \end{bmatrix}$$

By selecting $D = 0.8$, we can obtain

$$e^{ND} = \begin{bmatrix} -0.1452 & -0.1332 & -0.0601 & 0 & 0 & 0 \\ 1.6752 & 0.2285 & -0.1548 & 0 & 0 & 0 \\ 1.5072 & 0.1619 & 0.314 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1379 & 0.0759 & 0.0035 \\ 0 & 0 & 0 & -0.151 & 0.0674 & -0.0407 \\ 0 & 0 & 0 & -0.9897 & 0.524 & -0.0022 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.2855 & 0.0089 & 0.0495 & 0.7145 & -0.0089 & -0.0495 \\ 0.0662 & -0.0744 & 0.1151 & -0.0662 & 1.0744 & -0.1151 \\ 1.8578 & 0.0189 & 0.338 & -1.8578 & -0.0189 & 0.612 \end{bmatrix}$$

Substituting the above matrices into the finite time observer (2) with (5) or (4) with (5), we finish the design of the finite time observer.

Assume that $u(t) = \sin t$ is the input signal of the system (1), $x_0 = [1 \quad 2 \quad -1]^T$ is the initial state of the system (1) and $z_0 = [1 \quad 2 \quad 4 \quad 5 \quad 7 \quad 8]^T$ is the initial state of the finite time observer. By using the function ODE45 in Matlab, we can give the simulation figure to compare the state of system (1) and the finite time observer with $D = 0.8$ in Fig. 1. From Fig. 1, we can find that the errors between the linear system and the designed finite time observer almost converges to zero in the finite time $D = 0.8$.
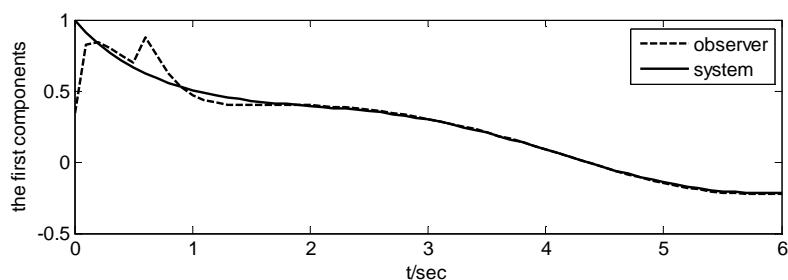

Figure 1. The first state components of the linear system and the finite time observer with $D = 0.8$

In addition, we select the eigenvalues of the closed-loop system for system (1) under state feedback $u = Kx$ as $-6 + 7i$, $-6 - 7i$ and -4. The corresponding gain matrix of state feedback is obtained as

$$K = [-130 \quad 56 \quad -66]$$

By Theorem 2, the closed-loop transfer functions of both system (1) under state feedback $u = K\hat{x} + v$ of the finite time observer (2) with (5) or (3) with (5) and system (1) under state feedback $u = Kx + v$ are

$$\begin{bmatrix} \dfrac{s+2}{s^3+72s^2+283s+2} \\ \dfrac{s+4}{s^3+72s^2+283s+2} \end{bmatrix}$$

And by Theorem 3, the eigenvalues of the closed-loop system for the linear time-invariant system (1) via the state feedback $u = K\hat{x} + v$ of the finite time observer (2) with (5) or (3) with (5)) are same with the sum of the eigenvalues $\lambda(A+BK)$ of the closed loop system for the linear time-invariant system (1) via state feedback $u = Kx + v$ and the eigenvalues $\lambda(N)$ of the finite time observer, which can be given by

$\{-6 \pm 7i, -4, -1 \pm 8j, -10, -3 \pm 3j\}$.

CONCLUSION

In this paper the design problem and its properties of a finite time observer for linear time-invariant systems is studied. The proposed finite time observer merges the advantages of the individual observers, such as finite time convergence. And the transfer property and separate principle for the finite time observers are still held for the finite time observers. Finally, the parametric design method of the finite time observer has been demonstrated on a numerical example and the simulation results show that this parametric design method, transfer property and separate principle for the finite time observers are effective.

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