ABSTRACT

The generalized Weng model is widely used in oil production prediction. At present, there are linear iterative trial and error method, regression method and genetic algorithm, etc. Linear iterative trial and error method and regression method have uncertainty influenced by artificial factors, which likely cause bigger errors. Genetic algorithm is more complex. In this paper, we solve the generalized Weng model parameter by making use of the nonlinear curve fitting and Levenberg-Marquard iterative method, and establish iterative initial value by the method of multiple regression. Through an example, this method effectively enhances the precision of parameter calculation, and is simple in calculation. It is suitable for parameters calculation of all kinds of time quantum oil and gas reservoir.

Keywords generalized Weng model; linear iteration trial and error method; multiple regression; nonlinear curve fitting; Levenberg-Marquard iteration method

INTRODUCTION

Study on the decline of oil, to development index calculation of oil field, prediction of recoverable reserves, planning and deployment of oil and gas field, and the reservoir parameter evaluation, has play a decisive role. Since Arps[1] proposed the production decline theory in 1945, domestic and foreign scholars have put forward a lot of production decline models, such as the Logistic[2] model, Weibull model[3,4], Rayleigh model[5] and HCZ model[6], etc. In China, famous geophysical experts Mr. Weng Wenbo in 1984 first proposed the Poisson cycle model (referred to as Weng model)[7] in the 《Prediction Theory Basis》 monographs. It laid a solid foundation for the prediction of oil production in China, and caused attention of domestic petroleum engineer. In 1996, Chen Yuanqian deduced the model[8], extended it to the generalized Weng model, and put forward the linear iterative trial and error method. Using this method to solve the model, because the parameter B value and the selected regression period are uncertainty, it will lead to bigger error. Therefore, this paper proposes using the nonlinear curve fitting method with higher accuracy to solve the model parameters.

2 Generalized Weng Model [8]

Equation of the model:

\[ Q = A t^B e^{-\frac{t}{C}} , \] (1)

with \( Q \)—year yield, \( 10^4 t / a \) (oil), \( 10^8 t / a \) (gas);
\( t \)—relative development time, \( a \) (year);
\( A, B, C \)—Predictive model constant.
3 Nonlinear Curve Fitting

Assuming there are experimental data \((x_i, y_i) (i = 1, 2, ..., n)\), we search function \(f(x)\) which makes the squares sum of the deviation of function value in the point \(x_i (i = 1, 2, ..., n)\) and the observed data minimize. Also is to search function \(f(x)\), it makes \(\sum_{i=1}^{n} (f(x_i) - y_i)^2\) minimize.

Making use of nonlinear curve fitting to solve the generalized Weng model parameters, the function \(f(x)\) equal to Eq. 1:

\[ f(t) = QT e^{\frac{-t}{C}}. \]  

Then we look for the optimal estimation \(\hat{A}, \hat{B}, \hat{C}\) of the parameters \(A, B, C\), it may make the following objective function minimize:

\[ F(X) = \sum_{i=1}^{n} g_i^2(X) = \sum_{i=1}^{n} (Q(t) - q_i)^2, \]  

With \(X = [A, B, C]^T\), \(q_i\) is the actual yield of annual oil, \(t\) is the relative development time. This is a nonlinear multivariable optimization problem, we commonly use Newton iterative method for solving.

The iterative formula of the Newton iterative method for [9]:

\[ X^{(k+1)} = X^{(k)} - \nabla^2F^{-1}(X^{(k)})\nabla F(X^{(k)}). \]  

In the formula: \(X^{(k)}\) is the current approximate point of optimal solution; \(X^{(k+1)}\) is the following approximate point of optimal solution; \(\nabla F(X^{(k)})\) is the gradient of \(F(X)\) in the point \(X^{(k)}\); \(\nabla^2F(X^{(k)})\) is the Hesse matrix.

The Hesse matrix calculation is relatively complex, we may make use of the first derivative replacing the second derivative, make \(G(X) = [g_1(X), g_2(X), ..., g_n(X)]^T\). can get the iterative formula of Gauss-Newton [9]:

\[ X^{(k+1)} = X^{(k)} - [J(X^{(k)})^T J(X^{(k)})]^{-1} J(X^{(k)})^T G(X^{(k)}), \]  

with \(J(X)\) is Jacobian matrix of \(G(X)\) in the point \(X^{(k)}\).

In the Gauss-Newton iterative method, matrix \(J(X)^T J(X)\) is sometimes ill conditioned (singular or near singular), then finding inverse matrix is difficult, even not to come out. In addition, sometimes the search direction \(p^{(k)} = -J(X^{(k)})^T G(X^{(k)})\) in the formula and gradient \(\nabla F(X^{(k)})\) of the point \(X^{(k)}\) are near orthogonal, then it can result in slow progress or false convergence. So, in this paper, we adopts Levenberg-Marquard [9] iterative method. The iterative formula for concrete:

\[ X^{(k+1)} = X^{(k)} - [J(X^{(k)})^T J(X^{(k)}) + \lambda_k I]^{-1} J(X^{(k)}) G(X^{(k)}), \]  

with \(I\) is the unit matrix; \(\lambda_k\) is a positive real number, controls the direction of search.

Levenberg-Marquard algorithm is sensitive to the initial value of parameters, and initial value determines the accuracy of the estimated parameters. The Binary regression method for solving the generalized Weng model is put forward in the [10]. Disadvantages of this method is the regression period selected in three-dimensional space. So, if evaluators have different observation angles, the data length observed good linear relationship is different. The human factors will
impact on the precision of the results. In this paper, all the data is regressed using the binary regression method. Although the approximate value of parameters obtained has more poor accuracy, it is better than arbitrary value as the initial value of the Levenberg-Marquardt algorithm.

4 Application Examples

Example 1
Select example in literature [11]. Romanshkinian oil fields is one of the large oil fields of the former Soviet Union, in 1952 put into development. In the table 1, the actual oil production is oil production data of this oil field from 1952 to 1979.

Using genetic algorithm to solve the model parameters in literature [11]. In this paper, we firstly make use of the binary regression method in reference [10] for all data to get the approximate parameters value as the initial value of iteration method: $A_0 = 127.75$, $B_0 = 1.78$, $C_0 = 16.07$. The above initial values are brought into Levenberg-Marquard iterative method, and through programming to calculate approximate three parameter value. Fitting results are shown in table 1 and table 2. By comparison, using this method, in addition to the relative error of initial 5 data points are large, the relative error of the rest each point is controlled within 5%, and the residual squares sum than genetic algorithm is smaller, simpler calculation. The solution developed in the paper is more precise, and simpler and easier for calculation.

CONCLUSION

In this paper, we solve the parameters of the generalized Weng model by using nonlinear curve fitting and Levenberg-Marquard iterative method, and determine the initial value of iteration by using binary regression method. After example validating, the method effectively improves the precision of parameter calculation, reduces man-made factors, and the calculation is simple. This method is suitable for the model parameters of oil gas reservoir of various time periods.

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