The optimal design of a medicine cabinet

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ABSTRACT

Design of medicine cabinet is facing the problems of how to reduce redundancy and improve the storage of medicine. According to the different specifications of the medical box, design the best scheme of medicine cabinet layout in the condition of minimum redundancy. Based on the analysis of a number of problems we build the concrete models and find results of individual problems. This model can be expanded to the largest problem of storage capacity.

Key words: Optimization model; medicine cabinet; redundancy

INTRODUCTION

At present, the automatic drug delivery system(Fig. 1) can be used widely in hospital [1]. It is mainly used to solve the problem of mismanagement in pharmacy now. Among them, and the role of medicine cabinet is very important. It must put drug centrally, smoothly push, facilitate to take and put medicine. Moreover, the volume of a medicine cabinet cannot too much to run the system very well, and quantity of medicine cabinet is not too much for cost reasons. Therefore, medicine storage slots in a medicine cabinet must be carefully designed. The 2012 CUMCM questions D is about the optimal design of a medicine cabinet [2]. There is a general discussion today about the issue [3-5]. The key is correctly to build mathematical model and to design algorithm. In this paper the problems are given a proper solution.

PROBLEM DESCRIPTION

A medicine cabinet, similar to a bookcase in structure, usually consists of a series of medicine storage slots divided by several horizontal and vertical plates (Fig.2). In order to ensure the accuracy of the drug-sorting and prevent distribution errors, each slot stores only one kind of drug with same box size. Fig.3 shows the arrangement of the medicine cabinet. The arrangement of pillboxes in a medicine cabinet is shown in fig.4.
A gap, at least 2mm, between the pillbox and the vertical or horizontal division plates is required to make the pillbox move smoothly in drug storage slots. The gap should be determined to avoid the overlap, stuck, roll or rotate when pillboxes move in a slot. Please construct mathematical models to solve following problems under the simple assumption that the thickness of the horizontal and vertical division plates can be neglected.

1. There are many drugs with different pillbox sizes in a hospital. Using the data given in Appendix 1[2], design a medicine cabinet with the minimum number of slot width types. The solution should include the corresponding pillbox ID stored in each slot.

2. If the gap between the pillbox and the vertical plate is bigger than 2mm, it leads to the width redundancy. A cabinet with more width types will increase the manufacture costs and decrease the adaptive ability of slots, though adding the number of the width types can effectively reduce the width redundancy. Based on the data in Appendix 1[2], please design the medicine cabinet again to minimize the width redundancy and the number of the slot width types. The number of the slot width types and the corresponding pillbox ID containing in each slot should be given in the solution.

3. The width of a cabinet can’t exceed 2.5m, and the height can’t exceed 2m, respectively. The maximal available height of a cabinet is 1.5m since the bottom 0.5m is taken by the conveyer belt. If the gap between the pillbox and the vertical or horizontal division plates exceeds the required 2mm, then we call the extra gaps as "the width or height redundancy". The "total redundancy" is defined to be the product of the width redundancy and height redundancy. Based on the solution in 2, determine the minimum number of the slot height types, such that the total redundancy of the cabinet is minimized. The solution should include a detailed cabinet design scheme, including the distances between both the vertical or horizontal division plates, and the corresponding pillbox ID for each slot.

4. The maximal daily demand for each drug is given in Appendix 2[2]. It is assumed that the length of the cabinet (storage slot) is 1.5m, and all the drugs are stored in the cabinet once a day. The pillboxes of a drug can occupy more than one storage slots to satisfy the daily needs. Using the data in Appendix 1 and 2[2], compute the slot number for each drug and the number of the cabinets for the hospital.
MODEL AND SOLUTION

1. Question 1

1.1 Our model

Our model, in the most general form, is

$$\min \ n$$

$$\begin{cases} 
  w + 2 \leq d < 2w \\
  d \leq \sqrt{w^2 + l^2} \\
  d \leq \sqrt{w^2 + h^2} \\
  1 \leq n \leq 47, n \in N
\end{cases}$$

where

- $n$ is the number of slot width types,
- $d$ is the of slot width,
- $l$, $w$ and $h$ are the dimensions of the pillbox.

For solving the model, firstly, we should determine the tolerance width interval of each pillbox under the constraints (see Table 1).
Table 1. The tolerance width interval of pillbox

<table>
<thead>
<tr>
<th>width (mm)</th>
<th>tolerance interval</th>
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<th>tolerance interval</th>
<th>width (mm)</th>
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<th>width (mm)</th>
<th>tolerance interval</th>
<th>width (mm)</th>
<th>tolerance interval</th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>[12,20)</td>
<td>20</td>
<td>[22,36.8)</td>
<td>30</td>
<td>[32,42.4)</td>
<td>40</td>
<td>[42,56.6)</td>
<td>50</td>
<td>[52,70.7)</td>
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<tr>
<td>11</td>
<td>[13,22)</td>
<td>21</td>
<td>[23,37.4)</td>
<td>31</td>
<td>[33,43.8)</td>
<td>41</td>
<td>[43,57.9)</td>
<td>51</td>
<td>[53,72.1)</td>
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<td>12</td>
<td>[14,24)</td>
<td>22</td>
<td>[24,38)</td>
<td>32</td>
<td>[34,45.2)</td>
<td>42</td>
<td>[44,59.3)</td>
<td>52</td>
<td>[54,73.5)</td>
</tr>
<tr>
<td>13</td>
<td>[15,26)</td>
<td>23</td>
<td>[25,37.8)</td>
<td>33</td>
<td>[35,46.6)</td>
<td>43</td>
<td>[45,60.8)</td>
<td>53</td>
<td>[55,74.9)</td>
</tr>
<tr>
<td>14</td>
<td>[16,28)</td>
<td>24</td>
<td>[26,41.6)</td>
<td>34</td>
<td>[36,48)</td>
<td>44</td>
<td>[46,62.2)</td>
<td>54</td>
<td>[56,92.4)</td>
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<tr>
<td>15</td>
<td>[17,30)</td>
<td>25</td>
<td>[27,37.5)</td>
<td>35</td>
<td>[37,49.4)</td>
<td>45</td>
<td>[47,63.3)</td>
<td>55</td>
<td>[57,77.7)</td>
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<tr>
<td>16</td>
<td>[18,32)</td>
<td>26</td>
<td>[28,42.8)</td>
<td>36</td>
<td>[38,50.9)</td>
<td>46</td>
<td>[48,65)</td>
<td>56</td>
<td>[58,79.1)</td>
</tr>
<tr>
<td>17</td>
<td>[19,34)</td>
<td>27</td>
<td>[29,43.4)</td>
<td>37</td>
<td>[39,52.3)</td>
<td>47</td>
<td>[49,66.4)</td>
<td></td>
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</tr>
<tr>
<td>18</td>
<td>[20,34.9)</td>
<td>28</td>
<td>[30,39.5)</td>
<td>38</td>
<td>[40,53.7)</td>
<td>48</td>
<td>[50,65.7)</td>
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</tr>
<tr>
<td>19</td>
<td>[21,35.5)</td>
<td>29</td>
<td>[31,43.1)</td>
<td>39</td>
<td>[41,55.1)</td>
<td>49</td>
<td>[51,69.2)</td>
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</tr>
</tbody>
</table>

To answer the above questions can be equivalent to find \( n \) interval, their intersection is empty, and each of the above tolerance intervals completely contain at least one of them.

1.2 Our algorithm

Step 1: For all tolerance intervals, sorted from small to large according to the lower limit of them, denoted by \( A_1, A_2 \ldots A_{47} \).

Step 2: If \( A_1 \cap A_2 = \emptyset \), then outputs \( A_1, A_2 = A_3 \); if \( A_1 \cap A_2 = A \neq \emptyset \), then \( A = A_1, A_2 = A_3 \).

Step 3: If \( A = A_{47} \), it is over, else repeat step 2.

Finally, the clusters of interval are shown in table 2.

Table 2 The clusters of interval

<table>
<thead>
<tr>
<th>No.</th>
<th>clustered interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[19,20)</td>
</tr>
<tr>
<td>2</td>
<td>[34,49,9857113690718)</td>
</tr>
<tr>
<td>3</td>
<td>[46,46,6690475583121)</td>
</tr>
<tr>
<td>4</td>
<td>[58,63,6396103067893)</td>
</tr>
</tbody>
</table>

According to the result of clustering, the minimum number of slot width types is 4 (see table 3).

Table 3 The clusters of interval

<table>
<thead>
<tr>
<th>No.</th>
<th>slot width types (mm)</th>
<th>the corresponding width range of the pillbox (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>10–17</td>
</tr>
<tr>
<td>2</td>
<td>34</td>
<td>18–32</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
<td>33–44</td>
</tr>
<tr>
<td>4</td>
<td>58</td>
<td>45–56</td>
</tr>
</tbody>
</table>

2. Question 2

2.1 Our model

Our multi-objective programming model [6] is

\[
\min \left( n, \sum_{i=1}^{n} \frac{m_i (m_i - 1)}{2} \right)
\]

\[
\begin{align*}
&w + 2 \leq d < 2w \\
&d \leq \sqrt{w^2 + l^2} \\
\end{align*}
\]

s.t.

\[
\begin{align*}
&d \leq \sqrt{w^2 + h^2} \\
&1 \leq n \leq 47, n \in N \\
&m_i \in N, i = 1, 2, \ldots, n
\end{align*}
\]
Where

- \( n \) is the number of slot width types,
- \( d \) is the number of slot width,
- \( l, w \) and \( h \) are the dimensions of the pillbox,
- \( m \) is the number of the corresponding pillbox stored in \( i \)-th slot.

### 2.2 Our algorithm

Here we use the optimal partition method [7-10], let \( w_1, w_2, \ldots, w_m \) are \( m \) types of pillbox widths, split them into two sections \((w_1, w_2, \ldots, w_p)\) and \((w_{p+1}, \ldots, w_m)\), where \( p = \left\lfloor \frac{n}{2} \right\rfloor \).

Their mean and variance are

\[
\overline{w}(1, p) = \frac{1}{p} \sum_{i=1}^{p} w_i \\
S^2(1, p) = \frac{1}{p} \sum_{i=1}^{p} (w_i - \overline{w}(1, p))^2
\]

and

\[
\overline{w}(p + 1, m) = \frac{1}{m - p} \sum_{i=p+1}^{m} w_i \\
S^2(p + 1, m) = \frac{1}{m - p} \sum_{i=p+1}^{m} (w_i - \overline{w}(p + 1, m))^2
\]

The optimal partition method has been seen as the main method suitable to the clustering of the ordering sample. When the total variation \( V \) reaches to minimize, classification achieves the best result.

\[
V = \min_{i, j=1, i \neq j} \sum_{i, j=1}^{m} V_{ij}
\]

where \( V_{ij} = (j - i + 1)S^2(i, j) \quad 1 \leq i \leq j \leq m \).

As shown in Table 4, we can obtain the number of the slot width types is 16 by applying the optimal partition method.

#### Table 4 The slot width types and the corresponding width range of pillbox

<table>
<thead>
<tr>
<th>slot width</th>
<th>width range of pillbox</th>
<th>slot width</th>
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<td>10-11</td>
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<td>18-21</td>
<td>37</td>
<td>33-35</td>
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<td>42-44</td>
<td>58</td>
<td>54-56</td>
</tr>
</tbody>
</table>

### 3. Question 3

The "total redundancy" is defined to be the product of the width redundancy and height redundancy, and the width redundancy is a constant based on the solution in 2, so we only need to optimize height redundancy.

By the similar method used in question 2, we get the number of the slot height types is 16 (see Table 5).

#### Table 5 The slot height types and the corresponding height range of pillbox

<table>
<thead>
<tr>
<th>slot height</th>
<th>height range of pillbox</th>
<th>slot height</th>
<th>height range of pillbox</th>
<th>slot height</th>
<th>height range of pillbox</th>
<th>slot height</th>
<th>height range of pillbox</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>28-38</td>
<td>66</td>
<td>60-64</td>
<td>82</td>
<td>77-80</td>
<td>127</td>
<td>101-125</td>
</tr>
<tr>
<td>46</td>
<td>39-44</td>
<td>70</td>
<td>65-68</td>
<td>86</td>
<td>81-84</td>
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<tr>
<td>51</td>
<td>45-49</td>
<td>74</td>
<td>69-72</td>
<td>94</td>
<td>85-92</td>
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</tr>
<tr>
<td>61</td>
<td>50-59</td>
<td>78</td>
<td>73-76</td>
<td>102</td>
<td>93-100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Question4

The slot number for each drug can be expressed as

\[
    n_j = \left\lceil \frac{q_j l_j}{1500} \right\rceil , 1 \leq j \leq 1919 \quad (4)
\]

Where \( q_j \) is the maximal daily demand for drug \( j \), \( l_j \) is the length of drug \( j \). \( \left\lceil \cdot \right\rceil \) is called to round up the value.

The necessary positive area to satisfy the daily needs can be expressed as

\[
    A = \sum_{j=1}^{1919} n_j d_j H_j \quad (5)
\]

Where \( n_j \) is slot number for the \( j \)-th drug, \( d_j \) and \( H_j \) are width and height of the slot where the \( j \)-th drug lay.

The number of the cabinets for the hospital can be expressed as

\[
    M = \left\lceil \frac{A}{WH} \right\rceil \quad (6)
\]

Where \( W \) and \( H \) are width and height of the cabinet.

According to the above model we get the number of the cabinets \( M = 2 \).

CONCLUSION

The proposed method can effectively solve the problem of the optimal design of drug cabinet. In our work, model and algorithm are simple and easy to understand by everyone, which is convenient for your test and application. This method can be also used for design goods shelves, bookshelf and container.

REFERENCES