

# Shot throwing technique biomechanical parameters sensitivity and optimization research 

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#### Abstract

Shot throwing problem are always affected by many factors, so the paper analyzes shot force status, utilizes release height, release speed, release angle and throwing distance relations, it analyzes shot movement status from geometric perspective, and establishes the three and shot throwing distance functional relationship shot projectile mathematical model. To make model more reasonable, it further optimize the model, and state model hypothesis rationality. By further calculation on throwing model, it fixes release height and then solves shot throwing best release angle is $\theta \pm 2 k \pi \in(0, \pi / 4], k \in N$, from which $\theta=1 / 2 \arccos \left(g h /\left(g h+v^{2}\right)\right)$ and furthest throwing distance $S=v \sqrt{v^{2}+2 g h} / g$.By researching on air resistance and throwing angle relations as well as shot sensitivity throwing problems'sensitivity analysis, finally it gets the conclusion that ignore air resistance is very necessary.


Key words: Throwing distance, biomechanics, physiology and biochemistry, parameters optimization, mathematical model

## INTRODUCTION

Shot putting event has already existed for several hundred years in Chinese history, since the event can enhance physique and improve people physical quality, it is favored by mass, therefore in early period; Chinese shot was still at the top in international. But in recent years, Chinese shot performance has appeared standstill even backward phenomenon [1-4]. For example, in Sydney Olympic Games in 2000, Chinese women shot putters hadn't achieved any medals, and even men shot kept standstill in Asian level for a long term, due to Chinese status as well as domestic and foreign pressure, it urgently needs us should research from techniques and training as well as others as soon as possible; besides, due to sports competitions are going more intense, athletes scores gap gets smaller, slight score gap will suffer a sudden decline in rank, as difference between the bronze and champion was 0.1 m in the 8th national games, the phenomenon showed contemporary competition required us should more deeply excavate technical potential in training so as to give athletes maximum potentials into play and get satisfied result [5-9]. But how to do can increase technical requests? In shot putting, lots of problems should be taken into account so as to improve techniques, such as, release angle, release height and release speed and others relationships, so that let shot arrive at best release angle and furthest distance [10-14].

For the problem, Chinese and foreign many scholars have made researches, and achieved abundant accomplishments, such as: some people has ever got best angle suited roughly range by researching on some movies; However, James. G. Hay (1978) according to actual measurement, he put forward that best release angle should be between $38^{\circ}$ and $42^{\circ}$, in addition, he got release speed and best release angle relations, with speed increasing, best release angle just slightly increased, though it got closer to $45^{\circ}$, it could not arrive at $45^{\circ}$; so $45^{\circ}$ is an ideal extreme state; Liu Ben-Liang (1984) proposed that shot flight trajectory was described by ground oblique angle. Ground oblique angle was shot release point and landing two points connection straight line and ground included angle [1-5]. According to author research, it was clear that best release angle was $45^{\circ}$, thereupon " ground oblique angle p and 0
was not the same", so best release angle should less than $45^{\circ}$ that would be more reasonable; [1] Miao Wen-Ke etc.(1984) pointed out, " for throwing release angle problem, to arrive at maximum distance and best release angle, it needs coaches and shot putters work together to change and revise shot throwing angle and throwing speed, and then it could get best performance" [2].

The paper on the basis of previous research results, with regard to how to choose maximum release speed and best release angle so that let distance be the maximum, by establishing mathematical model, it gets shot best throwing model, and verifies that it is reasonable to ignore shot throwing moment air motion suffered air resistance influence.

## SHOT THROWING MODEL

Shot during throwing process, with release angles difference, shot movement directions will also different, it mainly divides into three cases, as Figure 1 show:


Figure 1: Shot throwing case

## Shot throwing mathematical model

In the model, it ignores throwers mechanical process get involved in throwing circle, at first it carries out research on shot releasing moment throwing angle and initial speed. After shot releasing, due to it moves in a plane, shot in releasing point area vertical direction is movement height $H(\mathrm{t})$, takes time $t_{\text {as }} x$ axis to construct rectangular plane coordinate system. After shot leaving out of hand, its movement path can be expressed by rectangular plane coordinate system, when shot moves to $t_{1}$ time, then shot arrives at top point, and its speed in vertical direction is 0 . As Figure 2 shows [3]:


Figure 2: Shot-put motion path graphic
Analyze above figure, apply momentum theorem, it can get:
$f \cdot t^{3}=m v \Rightarrow v=\frac{f \cdot t^{3}}{m}$
While by:
$v \sin \theta=g t_{1}$
That
$t_{1}=\frac{v \sin \theta}{g}$
By simplifying calculation, it can get top point:
$H\left(t_{1}\right)=h+\frac{1}{2} g t_{1}{ }^{2}=h+\frac{v^{2} \sin ^{2} \theta}{2 g}$
By adding coefficient, it can get parabola equation is:
$H(t)=a\left(t-\frac{v \sin \theta}{g}\right)^{2}+h+\frac{v^{2} \sin ^{2} \theta}{2 g}$
When time arrives at $t=0$ :
$H(0)=a \frac{v^{2} \sin ^{2} \theta}{g^{2}}+h+\frac{v^{2} \sin ^{2} \theta}{2 g}=h$
So it can get:
$a=-\frac{g}{2}$
Input formula (7) into formula (6), it can get:
$H(t)=-\frac{g}{2}\left(t-\frac{v \sin \theta}{g}\right)^{2}+h+\frac{v^{2} \sin ^{2} \theta}{2 g}$
Assume $H\left(t_{2}\right)=0$, it gets:
$t_{2}=\sqrt{\frac{2 h}{g}+\frac{v^{2} \sin ^{2} \theta}{g^{2}}}+\frac{v \sin \theta}{g}$
Due to:

$$
S=v \cos \theta \cdot t_{2}
$$

In case release height is known, thrower and shot landing point distance:
$S=\sqrt{\frac{2 h v^{2} \cos ^{2} \theta}{g}+\left(\frac{v^{2 \sin 2 \theta}}{2 g}\right)^{2}}+\frac{v^{2} \sin 2 \theta}{2 g}$
According to above formula, we can get distance, speed, angle, height function relationships.

## Define best throwing mode

After given release height, to different release speeds, we should define best release angle. Obviously, it is extreme value problem, according to calculus knowledge; it should firstly solve stagnation point. By formula (11), it can get $S(v, h, \theta)$ is speed and height's one kind of monotonic function, and is also $\theta$ maximum point, therefore we can get by differential, that:
$\partial S / \partial \theta=0$
So:
$\frac{d S}{d \theta}=v^{2} \cos 2 \theta g+\frac{v^{4} \sin 2 \theta \cos 2 \theta-4 h v^{2} \cos \theta \sin \theta g}{\sqrt{\frac{v^{4} \sin ^{2} 2 \theta}{g^{2}}+8 \frac{h v^{2} \cos ^{2} \theta}{g}}}=0$
After simplifying, it is:
$v^{2} \cos 2 \theta \sqrt{v^{4} \sin ^{2} 2 \theta+8 h g v^{2} \cos ^{2} \theta}+v^{4} \sin 2 \theta \cos 2 \theta-4 h g v^{2} \cos \theta \sin \theta=0$
By converting, it gets:
$\cos 2 \theta=\frac{g h}{g h+v^{2}}=\frac{g}{g+\frac{v^{2}}{h}}$
By formula (15), it is known that to fixed release height $h$, if speed increases, then corresponding best release angle $\theta$ will also increase, in the following we make further analysis of formula (15), according to situation $\theta>0, h>0$ so $\cos 2 \theta>0$, and then $0<\theta \leq \frac{\pi}{4}$, therefore we get best release angle:
$\theta=\frac{1}{2} \arccos \left(\frac{g h}{g h+v^{2}}\right)$
Similarly it can solve throwing furthest distance is:

$$
\begin{equation*}
S=\frac{v}{g} \sqrt{v^{2}+2 g h} \tag{11}
\end{equation*}
$$

Due to $\theta$ period is $2 \pi$, when $\theta \pm 2 k \pi \in\left(0, \frac{\pi}{4}\right), k \in N, \theta \pm 2 k \pi$ is best throwing shot release angle, especially when $h=0$, at this time $\theta=45^{\circ}$.

## AIR RESISTANCE AND THROWING ANGLE BEST ANGLE RELATIONS DISCUSSION

Regarding shot throwing mathematical model establishment and solution, it discusses in case that assume ignoring air resistance, so when it has resistance, formula (11) will not at work anymore, so we reconstruct shot movement trajectory equations, their corresponding construction process is as following:
$m x=-k x \sqrt{x^{2}+y^{2}}$
$m y=-m g-k y \sqrt{x^{2}+y^{2}}$
When $t=0$ :
$y=v_{0} \sin \theta$
When $t=0$ :
$x=0$
When $t=0$ :
$y=h$
In above formula, $g$ is gravity accelerated speed.
Air resistant constant is $k$, then according to fluid mechanical relative knowledge, it can get :
$k=\frac{1}{2} c q s$
To formula (23), air resistance coefficient is $c$, air flow density is $q$, shot forward movement moment vertical movement direction projection area is $S$, from which $c$ is defined according to Reynolds number $R$, that is:
$R=\frac{g v d}{\mu}$
Among them, viscosity coefficient is $\mu$, speed is $v$, shot diameter is $d$.
By consulting relative documents, we can get $\mu=0.00001819$, $s=0.006^{2}{ }^{2} \mathrm{pi}^{2}, g=1.293\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$, and let shot throwing release speed value as $v=10-14 \mathrm{~m} / \mathrm{s}$, and then we can get $R=8.5 \times 10^{4}-1.6 \times 10^{8}$, but according to formula(23), we can get that Reynolds number actually extracts value between $2 \times 10^{4}-2 \times 10^{8}, c$ always is a constant that is 0.47 , and so according to previous stated conditions, it can solve $k=3.4 \times 10^{-3}$, by applying formula(18)-(19)we can solve different values corresponding maximum value that is $x$, in the following, combine golden section ratio with Runge - kulta method, it can get its solution by calculating. The solution is under different shot release initial speeds, different air resistances corresponding different heights, they and achieved best distance and best angle relations, except for that, we can also make comparison with case ignoring air resistance. For different heights in or without air resistance cases, we make numerical analysis of corresponding release speed, release angle and shot flight distance relations, as Table 1 and Table 3 show.

Table 1: Height 2.2m air resistance comparison table

| Speed | 14.5 | 14 | 13.5 | 13 | 12.5 | 12 | 11.5 | 11 | 10.5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ |
| Ignoring air | $42^{0} 19^{\prime}$ | $42^{0} 09^{\prime}$ | $41^{0} 57^{\prime}$ | $41^{0} 45^{\prime}$ | $41^{0} 30^{\prime}$ | $41^{0} 15^{\prime}$ | $40^{0} 58^{\prime}$ | $40^{0} 39^{\prime}$ | $40^{0} 17^{\prime}$ | $39^{0} 53^{\prime}$ |
| resistance | $23.55 m$ | $22.09 m$ | $20.68 m$ | $19.32 m$ | $18.01 m$ | $16.75 m$ | $15.54 m$ | $14.38 m$ | $13.27 m$ | $12.21 m$ |
| Considering | $42^{0} 14^{\prime}$ | $42^{0} 05^{\prime}$ | $41^{0} 52^{\prime}$ | $41^{0} 41^{\prime}$ | $41^{0} 27^{\prime}$ | $41^{0} 12^{\prime}$ | $40^{0} 53^{\prime}$ | $40^{0} 32^{\prime}$ | $40^{0} 15^{\prime}$ | $39^{0} 48^{\prime}$ |
| air resistance | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ | $23.34 m$ |

Table 2 Height 2.0 m air resistance comparison table

| Speed | 14.5 | 14 | 13.5 | 13 | 12.5 | 12 | 11.5 | 11 | 10.5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ |
| Ignoring air | $42^{0} 33^{\prime}$ | $42^{0} 23^{\prime}$ | $42^{0} 12^{\prime}$ | $42^{0} 33^{\prime}$ | $42^{0} 01^{\prime}$ | $41^{0} 48^{\prime}$ | $41^{0} 33^{\prime}$ | $41^{0} 17^{\prime}$ | $40^{0} 59^{\prime}$ | $40^{0} 17^{\prime}$ |
|  | 23.36 | $2 \mathbf{2 1 . 1 0}$ | 20.36 | $\mathbf{1 7 . 8 3}$ | $\mathbf{1 7 . 8 3}$ | $\mathbf{1 6 . 5 7}$ | 15.37 | $\mathbf{1 4 . 2 0}$ | $\mathbf{1 3 . 1 0}$ | $\mathbf{1 2 . 0 4}$ |
|  | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ |
|  | $42^{0} 32^{\prime}$ | $42^{0} 21^{\prime}$ | $42^{0} 10^{\prime}$ | $42^{0} 33^{\prime}$ | $42^{0} 00^{\prime}$ | $41^{0} 46^{\prime}$ | $41^{0} 30^{\prime}$ | $41^{0} 15^{\prime}$ | $40^{0} 33^{\prime}$ | $40^{0} 14^{\prime}$ |
| Considering air |  |  |  |  |  |  |  |  |  |  |
| resistance | 23.19 | 21.75 | 20.36 | 19.02 | $\mathbf{1 7 . 7 3}$ | 216.49 | 15.29 | 14.14 | 13.05 | 12.10 |
|  | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ |

Table 3 Height 1.8m air resistance comparison table

| Speed | 14.5 | 14 | 13.5 | 13 | 12.5 | 12 | 11.5 | 11 | 10.5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ | $m / s$ |
| Ignoring air | $42^{0} 46^{\prime}$ | $42^{0} 37^{\prime}$ | $42^{0} 28^{\prime}$ | $42^{0} 17^{\prime}$ | $42^{0} 05^{\prime}$ | $41^{0} 52^{\prime}$ | $41^{0} 37^{\prime}$ | $41^{0} 20^{\prime}$ | $41^{0} 02^{\prime}$ | $40^{0} 41^{\prime}$ |
|  | 23.18 | 21.73 | 20.32 | 18.96 | 17.65 | 16.39 | 15.19 | 14.03 | 12.93 | 11.87 |
|  | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ |
| Considering | $42^{0} 43^{\prime}$ | $42^{0} 33^{\prime}$ | $42^{0} 22^{\prime}$ | $42^{0} 14^{\prime}$ | $41^{0} 50^{\prime}$ | $41^{0} 32^{\prime}$ | $41^{0} 14^{\prime}$ | $40^{0} 56^{\prime}$ | $40^{0} 40^{\prime}$ | $40^{0} 36^{\prime}$ |
|  | 23.00 | 21.57 | 20.18 | 18.84 | 17.55 | 16.31 | 15.12 | 13.97 | 12.87 | 11.83 |
|  | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ | $m$ |

By above Table 1, Table 2, Table 3, we can see, ignoring air resistance compares with considering air resistance, the two corresponding best angles are very approximate, but when considering air resistance, corresponding horizontal distance will diminish, and it will increase with release speed increasing, but due to athlete release speed is not big in throwing shot process, therefore for release angle problem, it can ignore air resistance influence.

## ABOUT SHOT THROWING PARAMETERS PROBLEMS SENSITIVITY ANALYSIS

By formula(11), we get relations about release speed and release angle as well as release height the three, but athlete more concerned problem is making his shot throwing further so that let his performance surpass others. But we know that for athlete shot release height, it will have no big change, so we make research from shot release speed and release angle so as to pursue the two to shot throwing furthest maximum variable that is comparing the two to shot throwing distance sensitivity research.

Regarding shot throwing moment release angle and initial speed research, it has many methods, formers had ever used fast flash projections several methods, but the method exist some drawbacks, as it wastes time and strength, so it should apply a kind of simpler method to measure shot before landing flight time $t$, as well as shot threw distance $S$ and corresponding release height $H$, let formula(6)height $H(t)$ be equal to zero, that: ${ }^{H(t)}=0$, so it gets:

$$
\begin{equation*}
\tan \theta=\frac{\frac{1}{2} g t^{2}-h_{0}}{L} \tag{25}
\end{equation*}
$$

Simplified by formula(2) as:

$$
\begin{equation*}
v=\frac{L}{\cos \theta * t} \tag{26}
\end{equation*}
$$

So only measure shot above three corresponding values, and then according to formula (22), (23) connection
equations, it can decide initial speed and release angel, and it also unique determination. So we make practical measurement on different levels shot putters, and get following results as Table 4, Table 5, and Table 6:

Table 4: No. 1 testee

| Throwing <br> times | Best horizontal <br> distance | Best release <br> angle | Release <br> angle | Release initial <br> speed | Horizontal <br> distance | Shot flight <br> time | Release <br> height |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 13.055 m | $40^{\circ} 45^{\prime}$ | $24^{0} 51^{\prime}$ | $10.35 \mathrm{~m} / \mathrm{s}$ | $11.35 m$ | $1^{\prime \prime} 2$ | $1.95 m$ |
| 2 | 13.055 m | $40^{\circ} 45^{\prime}$ | $23^{\circ} 46^{\prime}$ | $10.65 \mathrm{~m} / \mathrm{s}$ | 11.70 m | $1^{\prime \prime} 2$ | 1.95 m |
| 3 | 13.055 m | $40^{\circ} 45^{\prime}$ | $24^{0} 08^{\prime}$ | $10.50 \mathrm{~m} / \mathrm{s}$ | 11.50 m | $1^{\prime \prime} 2$ | $1.95 m$ |

Table 5: No. 2 testee

| $\begin{aligned} & \text { Throwing } \\ & \text { times } \end{aligned}$ | $\begin{gathered} \text { Best horizontal } \\ \text { distance } \end{gathered}$ | $\begin{gathered} \text { Best release } \\ \text { angle } \end{gathered}$ | Release angle | $\begin{gathered} \text { Release initial } \\ \text { speed } \end{gathered}$ | Horizontal distance | Shot flight time | Release height |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 9.07 m | $39^{0} 12^{\prime}$ | $31^{0} 48^{\prime}$ | $8.14 \mathrm{~m} / \mathrm{s}$ | 8.30 m | $1^{\prime \prime} 2$ | 1.90 m |
| 2 | 9.44 m | $39^{0} 42^{\prime}$ | $24^{0} 49^{\prime}$ | $8.71 \mathrm{~m} / \mathrm{s}$ | 8.70 m | $1^{\prime \prime} 1$ | 1.85 m |
| 3 | 9.07 m | $39^{0} 12^{\prime}$ | $26^{0} 30^{\prime}$ | $8.30 \mathrm{~m} / \mathrm{s}$ | 8.20 m | $1^{1 / 1}$ | 1.87 m |
| 4 | 9.94 m | $39^{0} 42^{\prime}$ | $24^{0} 36^{\prime}$ | $8.79 \mathrm{~m} / \mathrm{s}$ | 8.80 m | $1^{\prime \prime} 1$ | 1.86 m |

Table 6: No. 3 Testee

| Throwing times | Best horizontal <br> distance | Best release <br> angle | Release angle | Release initial <br> speed | Horizontal <br> distance | Shot flight time | Release height |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 9.81 m | $40^{0}$ | $45^{0} 41^{\prime}$ | $8.68 \mathrm{~m} / \mathrm{s}$ | 9.10 m | $1^{\prime \prime} 5$ | 1.65 m |
| 2 | 9.73 m | $40^{0}$ | $36^{0} 53^{\prime}$ | $8.56 \mathrm{~m} / \mathrm{s}$ | 8.91 m | $1^{\prime \prime} 3$ | 1.60 m |

In above three testees: the first testee release angel is too small, throwing at angle $40^{\circ} 45^{\prime}$, and his available achieved or surpassed best result is $13.4 m$;the second testee performance from release area to ball drop point is 8.80 m , but his performance is 11.7 m ;the third testee performance from release area to ball drop point is 11.7 m , but his performance is 12.1 m ;

## CONCLUSION

The model obtains release angle $\alpha$, release speed $v$ factor to throwing distance $S$ influences sizes, therefore it has certain guiding values in future athletes competition and training as well as coaches guiding aspects. From above model values, by comparing whether ignoring air resistance research, finally it gets that ignoring shot throwing moment air movement suffered air resistance influence is reasonable. By shot throwing problems' sensitivity analysis, we get athlete best throwing angle and best throwing distance, so it can conclude the model is reasonable, therefore athlete should improve paper mentioned best angle exercising in future training so that can throw best result.

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