Several optimization strategies for CDT algorithm

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ABSTRACT

The Constrained Delaunay Triangulation is one of the most typical algorithms in computer graphics. But it still has some limitations. Thus we want to do some changes to make it more efficient. Our main work can be divided into two parts. Firstly, we design an algorithm to improve its readability and execution efficiency. In this part, we adopt the thought of polygon’s successive division and create a polygon class, for this mainly using C++ and OpenGL languages to realize. Secondly, we design another algorithm to expand its application scope. Main thought is the cross determination for selecting the eligible diagonals. Experiment showed we have achieved expectation effect to some extent.

Key words: CDT algorithm; optimization; execution efficiency; cross determination

INTRODUCTION

As one of the most important topics in computer graphics, triangulation has wide application in computer graphics processing, pattern recognition, curved surface description of three-dimensional geometry modeling system and many other fields. Planar polygons’ triangulation is to divide the polygon into a series of triangles and generate no more new vertexes[1-2]. It is meaningful to study the polygon triangulation. On the one hand, as to the most simple planar graph, it is more convenient to represent, analyze and process the computer data by using triangles. On the other, triangulation is the foundation when to deal with many other issues[3]. Any polygons can be transformed into triangle meshes, therefore it has a common sense.

Currently, there are many algorithms for polygon triangulation, such as the Divide-and-Conquer Algorithm, Greedy Algorithm, Incremental Inserting Algorithm, Triangulation Network Growth Algorithm, and so on. Among all of these algorithms, the CDT(Constrained Delaunay Triangulation) proposed by Bern & Eppstein is one of the most typical one. Its main thoughts is to choose the shortest edge from all of the polygon’s diagonals, then delete those that intersect with it. Keep on iteration until finish triangulation. So it is easy to understand and realize. But it still has some limitation, that is only convex polygons can use it. And the efficiency is lower because of much deleting operation. [4]

The intent of this paper is to make some optimizing for the CDT algorithm. Therefore, the following description is not exhaustive, and does not give a detailed analysis of the theories surrounding triangulation. Our contributions can be summarized as follows: Firstly, we designed a algorithm using polygon class and realize the triangulation without deleting operation. Secondly, we designed another algorithm so that it can be used in concave polygons. Lastly, we made some test to verify our algorithms.

OPTIMIZATION I: READABILITY AND EXECUTION EFFICIENCY

In this part, we will attempt to make some changes to improve the implement efficiency. Among this the biggest change is that we design the polygon class and do without delete operation. Main steps can be showed as follows: [5-7]
(1) Create the polygon class, including the information of polygon’s diagonals and vertices.
(2) Compute the length of each diagonal, sort in ascending order.
(3) Draw the shortest diagonal, therefore the polygon will be divided into two smaller ones. Respectively determine the number of the two polygons’ vertices. When the number is three or less than three, the polygons have been the triangles, and then finish the triangulation. Otherwise compute the length of polygons’ diagonals, sorting in ascending order. Find out the shortest one and draw it.
(4) Repeat the steps above until total number of vertices is three.

The following figures show the triangulation process.

![Fig.1: Polygon triangulation](image)

The following is the class polygon we designed:

```cpp
class PolyTriangulator {
public:
    struct Diag {
        float length; // diagonal length
        int pnt0ID; // diagonal end 0
        int pnt1ID; // diagonal end 1
    };

    typedef MathN::Vec3f Point; // coordinates type of polygon vertices
    typedef std::vector<Point> Polygon; // list type of polygon vertices
    typedef std::vector<Diag> Diags; // list type of diagonals

    PolyTriangulator(Polygon* _poly=0) : poly_(_poly) { } // constructed function

public:// public functions
    void renderPoly(void);
    void renderDiags(void);
    void renderTri(void);
    void setPolygon(Polygon* _poly) { poly_ = _poly; }

private:// private functions
    void _assertPoly(void);
    void _createDiagInfo(void);
    void _sortDiags(void);
    void _cullDiags(void);

private:
    Diags diags_; // list of all diagonals
    Polygon* poly_; // polygons to triangulate
};
```

**OPTIMIZATION STRATEGY : APPLICATION SCOPE**

As CDT algorithm only applies to the convex polygons, we want to do some work in order it could be used in the concave polygons. That is another important job we have done. Improved thoughts can be described as follows:[8]

Assume that the number of vertices of the polygon is \( N \), then we can get the number of the diagonals is \( N(N-3)/2 \). According to the definition of simple polygon triangulation, the line segment used for triangulation can be selected from all the diagonals. Main idea of our improved algorithm is to choose the eligible diagonals step
by step. Firstly, we can delete those which are outside of the polygon, then delete those crossing with the side of the polygon. After these, we draw the eliminating marks on those which are in the polygon and cross with each other. When finish these tasks, we have accomplished the triangulation. Main steps are as follows.

(1) Calculate the length of the diagonals and save them in an array $A$, two endpoints of the diagonal are saved as well.

(2) Judge the diagonals outside of the polygon (Such as $P_1P_2$ in Figure 2.) and make deleting mark. We can decide whether the mid-point (or any other points except endpoints) of the diagonal is inside or outside of the polygon. If it is outside, then delete it.[9]

(3) Select the diagonals which intersect with others and delete them. If two lines intersect, they must cross each other. We can apply the cross determination. If $P_1P_2$ crosses $Q_1Q_2$, then vectors $(P_1 - Q_1)$ and $(P_2 - Q_1)$ must locate on the two sides of the vector $(Q_2 - Q_1)$, according to the cross determination, we can get the following expression:

$$
(P_1 - Q_1) \times (Q_2 - Q_1) \times (Q_2 - Q_1) = 0
$$

It also can be expressed as follows:

$$
(P_1 - Q_1) \times (Q_2 - Q_1) \times (Q_2 - Q_1) = 0
$$

The limiting case is

$$
(P_1 - Q_1) \times (Q_2 - Q_1) = 0
$$

It means that $(P_1 - Q_1)$ and $(Q_2 - Q_1)$ locate in the same line, so $P_1$ must lie in the line $Q_1Q_2$; Similarly, when the following equation is true

$$
(Q_2 - Q_1) \times (P_2 - Q_1) = 0
$$

It means that $P_2$ must lie in the line $Q_1Q_2$. So the condition judging whether $Q_1Q_2$ crosses $P_1P_2$ can be expressed as:

$$
(Q_1 - P_1) \times (P_2 - P_1) \times (P_2 - P_1) \times (Q_2 - P_1) \geq 0
$$

(4) As to the diagonals which lie in the polygon, we can do as follows: sort these diagonals in an array $A$, make marks on the diagonals which intersect with the shortest one, and save the shortest one in another array $B$. Repeat the process until all the diagonals have been marked, which means we have finished selecting the needed diagonals.

(5) Lastly, diagonals stored in the array $A$ are the needed. Draw them and then we have finished the polygon’s triangulation. The following Figure 3 and 4 show the triangulation process of convex polygon and concave ones using our algorithm.
CONCLUSION

For our first algorithm, we adopt the thought of polygon successive division and the polygon class, use C++ and OpenGL languages, which make it easy to understand and realize. On the other hand, the execution efficiency will be improved without much deleting operation. But as CDT algorithm, it cannot be used in concave polygons.

The second we applied cross determination to the CDT algorithm. Because of this, the algorithm can break the limitation that it can be only used in the convex polygons. Instead it also can be used in the concave ones.

The quality of triangles’ shape is another most directive criteria to determine whether the triangulation is well or bad. Generally, we can use $\partial$ evaluation factors. It can be calculated as follows: [10]

$$\partial = \frac{4\sqrt{3}S}{a^2 + b^2 + c^2}$$  \hspace{1cm} (6)

Generally, the $\partial$ is greater than 0.1. When $\partial \leq 0.1$, we can call it narrow triangle and when $\partial = 1$, it is the regular triangle and has the best shape quality.

According to this, after computing, there are none narrow triangles by our second algorithm, shown in figure 3 and figure 4. In other simple planar polygons, this is still relatively well. So it has greatly improved the triangles’ shape quality, which also can be observed visually.
In the future, we will continue to optimize the CDT algorithm to make it be used in more complex polygons, such as the polygons with the hole. (as shown in Figure 5) Meanwhile, we will consider how to further improve the execution efficiency as well.

Fig.5: Polygon with a hole

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