Several discussions on the training methods of badminton in our country on the basis of mathematical model

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ABSTRACT

On the basis of analytical hierarchy process (AHP), this paper makes an analysis of badminton from four aspects: physical quality, spirit of cooperation, reaction quality and innovation ability. This paper concludes that humanities training accounts for 44.7% whereas physical training accounts for 55.3% in badminton, which demonstrates that badminton is not just a sport of challenging physical limits, but the one which calls for technics, skills and so forth. After a geometric model is established, this paper discusses that when an athlete spikes, he or she should try to keep the arms straight in front and vertical to the striking point. The higher he or she jumps, the more likely that ball can pass over the net. And the height of the striking point determines the trajectory and landing point of the ball. Thus, according to different vertical heights of the striking point, this paper classifies the height into six categories: 1.6, 1.7, 1.8, 1.9, 2.0, and 2.1. The angles that athletes smash the ball also differ; therefore this paper makes an analysis of spiking from the perspective of striking angle. Afterwards, on the basis of the spiking angle, the position where the ball is hit obliquely into the opponent’s court is also studied.

Key words: Analytical hierarchy process; geometric model; moment of inertia; badminton

INTRODUCTION

There are relatively few domestic studies of badminton. In recent years, research on Chinese badminton has mainly focused on technique, strategy and current condition whereas little attention has been paid to particular action.

According to the book Brief History of Chinese Badminton’s Development, badminton was brought to some of China’s advanced cities around 1910 from European and American countries. Afterwards, Chinese teams have attained great honors in World Badminton Championships and World Cups in badminton.

Nevertheless, the development of badminton in our country has experienced different periods: first being in the lead, then suffering from great sharp fluctuations and finally resting on a relatively high level and making outstanding achievements on the global stage, which is due to our country’s vigorous efforts to promote this sport.

2. THE ESTABLISHMENT OF MODELS

2.1 AHP model

On the basis of AHP, this paper makes a quantitative analysis of badminton, and establishes three layers: target layer, criteria layer and scheme layer.

The target layer refers to badminton training. The criteria layer refers to influential factors of the scheme, including physical quality \( \frac{c_1}{c_1} \), spirit of cooperation \( \frac{c_2}{c_2} \), reaction quality \( \frac{c_3}{c_3} \) and innovation ability \( \frac{c_4}{c_4} \).
The scheme layer incorporates humanities training $A_1$, physical training $A_2$ and entertainment training $A_3$. Then the hierarchical structure is established as Fig.1 shows.

2.1.2 Constructing the judgment (comparison) matrix

The judgment matrix is used to reflect the significance of factors in one layer to those in the above layer in the form of matrix. To make comparison between every two factors and get a quantified judgment matrix, scales from 1 to 9 are introduced as Table 1 demonstrates.

<table>
<thead>
<tr>
<th>Scale $a_{ij}$</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$i$ is equally important as $j$</td>
</tr>
<tr>
<td>3</td>
<td>$i$ is relatively more important than $j$</td>
</tr>
<tr>
<td>5</td>
<td>$i$ is more important than $j$</td>
</tr>
<tr>
<td>7</td>
<td>$i$ is much more important than $j$</td>
</tr>
<tr>
<td>9</td>
<td>$i$ is definitely much more important than $j$</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>the scales of the intermediate states of above descriptions</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>If $i$ is compared with $j$, then the judgment value is $a_{ji} = 1/a_{ij}$, $a_{ii} = 1$</td>
</tr>
</tbody>
</table>

Here is the figure of scales from 1 to 9.

To begin with, the judgment matrix is figured out. Then, according to the above principles and scales from 1 to 9, and with reference to the experience of experts and the author as well as a large quantity of literature, the comparison matrix between every two factors is worked out as Tables 2 to 6 illustrate.
Tab. 2: Comparison matrix

<table>
<thead>
<tr>
<th>G</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>1</td>
<td>1/3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$c_2$</td>
<td>3/8</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$c_3$</td>
<td>1/3</td>
<td>1/5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$c_4$</td>
<td>1/3</td>
<td>1/5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab. 3: Comparison matrix

<table>
<thead>
<tr>
<th>$c_1$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>$A_3$</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab. 4: Comparison matrix

<table>
<thead>
<tr>
<th>$c_2$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1/5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1/5</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab. 5: Comparison matrix

<table>
<thead>
<tr>
<th>$c_3$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1/5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1/8</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab. 6: Comparison matrix

<table>
<thead>
<tr>
<th>$c_4$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1/5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1/8</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

2.1.3 Single hierarchical arrangement and its consistency test

A consistency test is carried out with the consistency index $CI = \frac{\lambda_{\text{max}} - n}{n-1}$ in which $\lambda_{\text{max}}$ is the maximum eigenvalue and $n$ is the order of the comparison matrix. The smaller $CI$ is, the closer the judgment matrix reaches complete consistency and vice versa.
2.1.4 Total hierarchical arrangement and its consistency test

\[
A = \begin{bmatrix}
1 & 1/3 & 3 & 3 \\
3 & 1 & 5 & 5 \\
1/3 & 1/5 & 1 & 1 \\
1/3 & 1/5 & 1 & 1
\end{bmatrix}
\]

**normalization of column vectors**

\[
\begin{bmatrix}
0.214 & 0.192 & 0.3 & 0.3 \\
0.075 & 0.577 & 0.5 & 0.5 \\
0.121 & 0.115 & 0.1 & 0.1 \\
0.201 & 0.115 & 0.1 & 0.1
\end{bmatrix}
\]

**sum by row**

\[
\begin{bmatrix}
1.066 \\
2.22 \\
0.386 \\
0.386
\end{bmatrix}
\]

**normalization**

\[
W^{(0)} = \begin{bmatrix}
0.2515 \\
0.555 \\
0.0965 \\
0.0965
\end{bmatrix}
\]

\[
AW^{(0)} = \begin{bmatrix}
1 & 1/3 & 3 & 3 \\
3 & 1 & 5 & 5 \\
1/3 & 1/5 & 1 & 1 \\
1/3 & 1/5 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
0.214 \\
0.075 \\
0.121 \\
0.201
\end{bmatrix} = \begin{bmatrix}
1.012 \\
2.275 \\
0.387 \\
0.387
\end{bmatrix}
\]

\[
\lambda^{(0)}_{\text{max}} = \frac{1}{4} \left( \frac{1.012}{0.251} + \frac{2.275}{0.555} + \frac{0.387}{0.0965} + \frac{0.387}{0.0965} \right) = 4.037
\]

\[
W^{(0)} = \begin{bmatrix}
0.251 \\
0.555 \\
0.097 \\
0.097
\end{bmatrix}
\]

The judgment matrix can also be figured out likewise:

\[
B_1 = \begin{bmatrix}
1 & 1 & 1/3 \\
3 & 3 & 1
\end{bmatrix},
B_2 = \begin{bmatrix}
1 & 5 & 5 \\
1/5 & 1/5 & 1
\end{bmatrix},
B_3 = \begin{bmatrix}
1 & 5 & 8 \\
1/8 & 1/5 & 1
\end{bmatrix},
B_4 = \begin{bmatrix}
1 & 5 & 8 \\
1/5 & 1 & 5
\end{bmatrix}
\]

The corresponding maximum eigenvalue and eigenvector are:

\[
\lambda^{(1)}_{\text{max}} = 3.64, \omega^{(1)}_1 = \begin{bmatrix}
0.244 \\
0.512
\end{bmatrix}
\]

\[
\lambda^{(2)}_{\text{max}} = 3.29, \omega^{(2)}_2 = \begin{bmatrix}
0.251 \\
0.092
\end{bmatrix}
\]

\[
\lambda^{(3)}_{\text{max}} = 3.31, \omega^{(3)}_3 = \begin{bmatrix}
0.204 \\
0.148
\end{bmatrix}
\]

\[
\lambda^{(4)}_{\text{max}} = 3.31, \omega^{(4)}_4 = \begin{bmatrix}
0.204 \\
0.148
\end{bmatrix}
\]
Consistency indexes are used for a test:

\[ CI = \frac{\lambda_{\text{max}} - n}{n - 1}, \quad CR = \frac{CI}{RI} \]

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
<td>1.51</td>
</tr>
</tbody>
</table>

(1) For judgment matrix \( \mathbf{A} \), \( \lambda_{\text{max}}^{(0)} = 4.073, RI = 0.9 \)

\[ CI = \frac{4.073 - 4}{4 - 1} = 0.24, \quad CR = \frac{CI}{RI} = \frac{0.024}{0.9} = 0.027 < 0.1 \]

It means that the inconsistency of \( \mathbf{A} \) is acceptable, and its eigenvector can be replaced by weight vector.

(2) Similarly, for judgment matrices \( \mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3, \mathbf{B}_4 \), all have passed the consistency test according to the above principle.

Calculation results from the target layer to the scheme layer are illustrated in the hierarchical structure diagram as Fig. 3 shows:

The calculation structure is shown as follows:

\[ \omega^{(0)} = (\omega_1^{(0)}, \omega_2^{(0)}, \omega_3^{(0)}, \omega_4^{(0)}) = \begin{pmatrix} 0.624 & 0.185 & 0.252 & 0.575 \\ 0.234 & 0.240 & 0.089 & 0.286 \\ 0.136 & 0.575 & 0.66 & 0.139 \end{pmatrix} \]
This paper concludes that for badminton, humanities training accounts for 44.7% whereas physical training accounts for 55.3%, which demonstrates that badminton is not just a sport of challenging physical limits, but the one which calls for technics, skills and so forth.

2.2 Solution of the spiking trajectory and shot point under the geometric model

A spiking model of badminton is established according to the geometric principle, and its trajectory and shot point are also determined according to the differences of high batting and low batting. As Fig.4 demonstrates:

When an athlete spikes, he or she should try to keep the arms straight in front and vertical to the striking point. The higher he or she jumps, the more likely that ball can pass over the net. And the height of the striking point determines the trajectory and landing point of the ball and leads to either long-track ball or short-track ball. Thus, according to different vertical heights of the striking point, this paper classifies the heights into six categories: 1.6, 1.7, 1.8, 1.9, 2.0, and 2.1. Then on the basis of vertical distance between the striking point and the ball net, Table 8 is drawn.

<table>
<thead>
<tr>
<th>vertical height/H</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2.0</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>the distance between the striking point and the ball net is 0.75m</td>
<td>17.23</td>
<td>9.07</td>
<td>6.15</td>
<td>4.65</td>
<td>3.74</td>
<td>3.12</td>
</tr>
<tr>
<td>the distance between the striking point and the ball net is 0.5m</td>
<td>10.55</td>
<td>6.05</td>
<td>4.10</td>
<td>3.10</td>
<td>2.49</td>
<td>2.08</td>
</tr>
<tr>
<td>the distance between the striking point and the ball net is 0.25m</td>
<td>5.78</td>
<td>3.02</td>
<td>2.05</td>
<td>1.55</td>
<td>1.25</td>
<td>1.04</td>
</tr>
</tbody>
</table>

As can be seen from Table 8, different distances between the striking point and the net surface contribute to different landing points of badminton. Moreover, different batting heights of athletes can lead to different positions of the ball after it passes over the net.

2.2.1 Improvements of the low-landing ball’s spiking trajectory and shot point under the geometric model

Similarly, based on the different vertical heights of the striking point, this paper classifies height into six different categories: 1.6, 1.7, 1.8, 1.9, 2.0, and 2.1. Then, according to different vertical distances between the striking point and the net surface, Table 9 is drawn.
As can be learned from Table 9 and Figure 5, although the distance between the striking point and the net surface has been reduced into 0.1m, the low-batting ball’s shot positions in the opponent’s court vary greatly.

2.2.2 Changing the angle of hitting the ball

The angle of spiking is not vertical, nor is vertical for the ball to be shot to the opponent’s court. Generally speaking, when athletes spike, the ball is hit by a certain angle and shot to the opponent’s court obliquely.

Suppose that the vertical distance between the striking point and the periphery of the court is 0.5m, when an athlete hits a ball at an angle of $45^\circ$ on the right, then the distance between the striking point and shot point $S$ is:

$$S = \frac{0.5}{\cos 45^\circ} = \frac{0.5}{\sqrt{2}/2} = 0.7072m$$

Meanwhile, when the angle of spiking is $30^\circ$ offset, the distance between the shot point and the striking point’s projection $S$ is:

$$S = \frac{0.5}{\cos 30^\circ} = \frac{0.5}{\sqrt{3}/2} = 0.574m$$

As Fig. 6 shows,

Based on the data of Figure 3, Table 10 is established for analysis.

<table>
<thead>
<tr>
<th>angle</th>
<th>vertical angle</th>
<th>coordinate of landing point</th>
<th>coordinate of landing point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.65m</td>
<td>9.45m</td>
<td>12.45m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 11.15 \ 11.15 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 11.45 \ 6.24 \end{pmatrix}$</td>
</tr>
<tr>
<td>1.75m</td>
<td>6.14m</td>
<td>8.57m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 6.05 \ 6.05 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 6.02 \ 3.476 \end{pmatrix}$</td>
</tr>
<tr>
<td>1.85m</td>
<td>4.2m</td>
<td>5.6m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 4.12 \ 4.12 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 4.08 \ 2.356 \end{pmatrix}$</td>
</tr>
<tr>
<td>1.95m</td>
<td>3.15m</td>
<td>4.37m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 3.25 \ 3.25 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 3.08 \ 1.86 \end{pmatrix}$</td>
</tr>
<tr>
<td>2.05m</td>
<td>2.49m</td>
<td>3.52m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 2.18 \ 2.18 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 2.48 \ 1.42 \end{pmatrix}$</td>
</tr>
<tr>
<td>2.15m</td>
<td>2.11m</td>
<td>2.98m</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\begin{pmatrix} 1.95 \ 1.95 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 2.05 \ 1.47 \end{pmatrix}$</td>
</tr>
</tbody>
</table>
2.3 Calculation of Moment of Inertia of arms when striking a ball

Through the Lagrange equation, this paper gets the restrained particle kinetic equation in which Lagrange function $L$ is the differential value between systematic kinetic energy $K$ and potential energy $P$:

$$ L = K - P $$

The systematic dynamic equation is

$$ F_i = \frac{d}{dt} \left( \frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial \dot{q}_i} \right) \quad i = 1, 2, \ldots, n $$

In the above equation, $q_i$ is the corresponding speed, $q_i$ is the coordinate of potential energy and kinetic energy, $F_i$ is the force exerted by the $i$-th coordinate, and the angles between the thigh and lower leg with the axis are $\theta_1, \theta_2$, and the lengths are respectively $l_1, l_2$, and the distances between the gravity position of front and rear arm and the elbow center and knee are $p_1, p_2$, and the coordinate of the arm’s center of gravity $(X_i, Y_i)$ is

$$ \begin{align*}
X_i &= p_1 \sin \theta_1 + l_1 \sin \theta_1 + p_2 \sin(\theta_1 + \theta_2) \\
Y_i &= -l_1 \cos \theta_1 - p_2 \cos(\theta_1 + \theta_2)
\end{align*} $$

Similarly, the coordinate of the arm’s center of gravity of arms $(X_e, Y_e)$ can be figured out. The systematic kinetic energy $E_k$ and potential energy $E_p$ have the following expressions:

$$ E_k = E_{k1} + E_{k2} + E_{k3} + E_{k4} + E_{k5} + E_{k6} + E_{k7} + E_{k8} $$

$$ E_p = E_{p1} + E_{p2} + E_{p3} + E_{p4} + E_{p5} + E_{p6} + E_{p7} + E_{p8} $$

The above equation is then expressed in the form of Lagrange function. From the systematic dynamic equation of Lagrange, the torques on the hip joint and knee joint $M_h$ and $M_k$ are also worked out:

$$ M_h = \begin{bmatrix} D_{h1} & D_{h2} & D_{h3} & D_{h4} & D_{h5} & D_{h6} & D_{h7} & D_{h8} \\ D_{h9} & D_{h10} & D_{h11} & D_{h12} & D_{h13} & D_{h14} & D_{h15} & D_{h16} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_{h17} & D_{h18} & D_{h19} & D_{h20} & D_{h21} & D_{h22} & D_{h23} & D_{h24} \end{bmatrix} $$

$$ M_k = \begin{bmatrix} D_{k1} & D_{k2} & D_{k3} & D_{k4} & D_{k5} & D_{k6} & D_{k7} & D_{k8} \\ D_{k9} & D_{k10} & D_{k11} & D_{k12} & D_{k13} & D_{k14} & D_{k15} & D_{k16} \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} D_{k17} & D_{k18} & D_{k19} & D_{k20} & D_{k21} & D_{k22} & D_{k23} & D_{k24} \end{bmatrix} $$

$D_{ijk}$ in the above equation is worked out as
\[
D_{11} = 0 \quad D_{22} = 0 \quad D_{12} = 0
\]
\[
D_{21} = m_2 l_2 \cos \theta_2
\]
\[
D_{12} = m_2 l_2 \cos \theta_2
\]
\[
D_{21} = m_2 l_2 \cos \theta_2
\]
\[
D_i = (m_i + m_j) g \sin \theta_i + m_j g \sin (\theta_i + \theta_j)
\]
\[
D_{12} = -m_2 l_2 \sin \theta_2
\]
\[
D_{21} = m_2 l_2 \sin \theta_2
\]
\[
D_{12} = -m_2 l_2 \sin \theta_2
\]
\[
D_{21} = D_{12} + D_{21}
\]
\[
D_2 = m_2 l_2 \sin (\theta_i + \theta_j)
\]

With reference to theoretic equations, the mechanical movement on badminton athletes’ hand joints is analyzed when they spike. And spiking skills are studied with reference to mechanical analyses of shoulder and elbow joints.

REFERENCES