Revenue-sharing contract mechanism in $n$-echelon Green supply chain

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ABSTRACT

The revenue-sharing contract mechanism for $n$-echelon supply chain in green environment is researched in this paper. It analyzes the decision under the $n$-echelon green supply chain in three scenarios, namely, the decentralized decision, centralized decision and revenue-sharing contract, and their optimal policies are also proposed. Further, a numerical case is given to validate the models and conclusions. It is shown that with the raising green degree the markup rate of the green product will increase, the optimal order quantity of retailers in the green supply chain will decrease, and meanwhile, the profits for each member will firstly decrease and then increase. In addition, the profits for each member in revenue-sharing contract can be improved significantly compared to in decentralized decision.

Keywords: $n$-echelon green supply chain; green product; green degree; revenue-sharing contract;

INTRODUCTION

Environmental consciousness has become increasingly important at present, as environmental problems are affecting the living conditions of the world’s population more severely. The efforts to reduce the impact of business activities on the environment have been labeled as green supply chain management (GSCM). Enterprises engaging in GSCM have experienced both environmental and financial benefits. Wilkerson (2005b) reported that the GSCM is desirable not only for its environmental consciousness, but also due to its profit-bringing potential for the supply chain members. Pepsi saved $44 million by switching from corrugated to reusable plastic shipping containers for 1 liter and 20-ounce bottles. Commonwealth Edison emerged about $50 million from managing materials and equipment by a life-cycle management channel.

Recently, more and more scholars engaged in the related research of green supply chain. Beamon (1999) investigated the environmental factors, summarized the elemental differences between the extended supply chain and the traditional supply chain, and developed a general procedure towards achieving the green supply chain. Sarkis (2003) posits a strategic decision framework for green supply chain management. Jayaraman et al (2003) provided the models and solution procedures regarding the design network of reverse distribution. Srivastava(2008) discussed an integrated comprehensive conceptual framework which combined descriptive modeling with optimized techniques for network design in reverse logistics. Zhang et al (2013) applied game theory to study four models in view of the three-level green supply chain system in which market demand correlated with product green degree. Swami and Shah (2013) discussed the coordination of a manufacturer and a retailer in a vertical supply chain that put in efforts for ‘greening’ their operations. Some research has begun to emerge in the area how to coordinate the green supply chain. Li and Huan(2008) investigated that designing coordination schemes had been one of the core issues in the study of supply chain management. Veen (2005) and Koulamas (2006) studied that under a Revenue Sharing mechanism, the transactions between the supplier and the buyer were governed by a share of the Buyer's revenue that was received by the supplier. Rhee et al (2010) proposed a new type of revenue sharing (RS) contract mechanism for multi-echelon linear supply chains between the most downstream entity and all upstream entities. Zhang et al (2012) investigated how to coordinate a one-manufacturer-two-retailers supply chain with demand disruptions by revenue-sharing contracts. By building a revenue sharing mechanism, Zhang and Liu (2013) proposed supply chain members are motivated to respond positively to the cooperation in producing and marketing.
green products, and the income of each member is increased substantially. Palsule-Desai (2013) examined revenue-dependent and revenue-independent revenue sharing contracts and demonstrated how revenue-dependent contracts enhance supply chain coordination.

However, there are very few studies having addressed the issue of green degree for the green supply chain. Based on the above analysis, details on the established models of the decentralized decision, centralized decision and revenue-sharing contract under the n-echelon green supply chain are discussed in later sections, and we analyze some pertinent questions in this regard such as the effects of green degree for the equilibrium in green production. Further, their optimal policies are also proposed.

EXPERIMENTAL SECTION

The basic model

The basic model used below is similar to the one discussed in [3]. Consider a simple linear supply chain structure with a single enterprise at each of the $n$ ($n \geq 2$) echelons. From downstream to upstream in this model, denote Enterprise 1 as the most downstream and Enterprise $n$ as the most upstream entity. Enterprise 1 sells its products to the end customers and decides on the order quantity. Furthermore, Enterprise $i$ faces purchasing cost and operational costs per unit, and can decide the wholesale price of the Enterprise $i$-1.

The parameters related in the model are summarized as below.

- $\lambda$: Green degree (an abbreviation for the green satisfaction degree) which customer preferring to pay for green product
- $\lambda_a$: Green degree of non-green product
- $\lambda_b$: Green degree of green product
- $Q$: The order quantity
- $k$: Payment coefficient, said the additional amount that consumers are willing to pay for each unit increased in the green degree of their desired product; this also shows that different types of consumers.
- $c_i$: Operational cost of supply chain member $i$ per unit, $i = 2, 3, \cdots, n$
- $w_{i+1}$: Wholesale price (per unit) set by the enterprise $i+1$ to $i$, and $w_{n+1} = 0, i = 2, 3, \cdots, n$
- $p_0$: Retail price of non-green products per unit
- $\alpha$: Retailer’s green product markup rate based on $p_0$
- $\pi_i$: Profit of the green supply chain member $i$, $i = 1, 2, 3, \cdots, n$
- $\pi_{sc}$: Total channel profit in n-echelon green supply chain

We may assume that there exists a terminal consumer group in the market whose market capacity is 1 and the green degree of the product whose wants to buy is random variable $\lambda$, and the probability density of the green degree is $f(\lambda)$, which obey uniform distribution, we can get $\lambda \sim U[\lambda_a, \lambda_b]$, and $f(\lambda) = \frac{1}{\lambda_b - \lambda_a}$. As the non-green product has already been in its circulation maturity before R&D, its retail price is stabilized at $p_0$. However, for the green product with higher cost, the retailer decides on an appropriate price markup in order to ensure its own interest. The markup rate being $\alpha$, then the terminal selling price of the green product will be $p = p_0(1 + \alpha)$. Taking the consumers’ preference for product green degree into consideration, the unit acceptance level of the consumers for purchasing the product can be denoted by $p_0 + k(\lambda - \lambda_a) \geq p$, obviously, only when consumers’ acceptance level satisfies $\bar{\lambda} \geq \lambda_a + \frac{p - p_0}{k}$, consumers are willing to pay green product; otherwise, it will prefer non-green goods.

Meanwhile, the market demand of green products is as follows:

$$Q = \int_{\lambda_a}^{\bar{\lambda}} f(\lambda) d\lambda = \left[ 1 - \frac{\alpha p_0}{k(\lambda_b - \lambda_a)} \right]$$

The profit of Enterprise 1 is given as below:
\[ \pi_i = [(1 + \alpha)p_0 - w_2 - c_1] \left[ 1 - \frac{\alpha p_0}{k(\lambda_n - \lambda_i)} \right] \]  

(2)

The profit of Enterprise \( i \), \((i=2, \ldots, n)\) can be obtained as:

\[ \pi_i = (w_i - w_{i+1} - c_i) \left[ 1 - \frac{\alpha p_0}{k(\lambda_n - \lambda_i)} \right] \]  

(3)

It follows that the profit of the total GrSC is given by:

\[ \pi_{sc} = [(1 + \alpha)p_0 - \sum_{i=1}^{n} c_i] \left[ 1 - \frac{\alpha p_0}{k(\lambda_n - \lambda_i)} \right] \]  

(4)

### The n-echelon Green Supply Chain Model

**A. Decentralized decision under the n-echelon green supply chain**

Under the decentralized decision, in which it is assumed that there is no coordination among GSC, i.e., all enterprises act independently and take decisions that maximize their respective expected profits. It will be assumed that optimization takes place in a Stackelberg sequence, where in the time-line Entity \( n \) is the first decision maker, and the model is solved through backwards induction. Enterprise \( n \) determines its wholesale price \( w_n \), followed by Enterprise \( n-1 \), all the way to Enterprise 2 that determines its price \( w_2 \). Given \( w_2, c_1 \) and the demand distribution, Enterprise 1 decides to order \( Q \) units from Enterprise 2. The same order size is passed on from Enterprise 2 to Enterprise 3, all the way to Enterprise \( n \) [3, 6]. Solve the equations below:

Max\( \alpha \pi_i = [w_i - w_{i+1} - c_i] \left[ 1 - \frac{\alpha p_0}{k(\lambda_n - \lambda_i)} \right] \]  

(5)

S.t. Max\( \alpha \pi_i = [(1 + \alpha)p_0 - w_2 - c_1] \left[ 1 - \frac{\alpha p_0}{k(\lambda_n - \lambda_i)} \right] \]  

(6)

**Theorem1.** Under the decentralized decision, the optimal order quantity and the green product markup rate is given by:

\[ \alpha^* = \frac{k(\lambda_n - \lambda_i) - p_0 + w_2 + c_i}{2p_0} \]  

(7)

\[ Q^* = \frac{k(\lambda_n - \lambda_i) + p_0 - w_2 - c_1}{2k(\lambda_n - \lambda_i)} \]  

(8)

**Proof.** For the decision under decentralized, based on Eq. (6), we define \( \frac{d^2 \pi_i}{d\alpha^2} = -\frac{2p_0^2}{k(\lambda_n - \lambda_i)} < 0 \).

It is clear that the objective function \( \pi_i \) is the strict concave function on parameter \( \alpha \), and yield the following:

\[ \frac{d\pi_i}{d\alpha} = p_0 - \frac{(1 + 2\alpha)p_0 - p_0(w_2 + c_i)}{k(\lambda_n - \lambda_i)}, \quad \text{where} \quad \frac{d\pi_i}{d\alpha} = 0. \]

We get the optimal value of \( \alpha^* \), which is Eq. (7). Then inserting \( \alpha^* \) in Equation (1), also we can get the optimal quantity \( Q^* \) labeled as Eq. (8).

Thus, the optimal profits for each member in the green supply chain under the decentralized decision and the total channel profits are shown below:

\[ \pi_i^* = \frac{k(\lambda_n - \lambda_i) + p_0 - w_2 - c_1}{4k(\lambda_n - \lambda_i)} \]  

(9)
Theorem 1 implies a Proposition can be written as following:
Under the decentralized decision, the bigger difference in green degree between green product and non-green product, the higher markup rate of the green product has, and meanwhile, the optimal order quantity of retailer in the green supply chain will be less.

B. Centralized decision under the n-echelon green supply chain
Under the centralized decision environment, it is the cooperation relations among supply chain members; all members jointly determine the optimal quantity of products by adopting centralized decision, so as to maximize the total expected profits of the green supply chain system.

Subsequently, the GSC profit is:

\[
\pi_{sc}^* = \left[ w_1 - w_{n+1} - c_i \left( \frac{1}{2} - \frac{p_0 - w_i - c_i}{k(\lambda_h - \lambda_i)} \right) \right]
\]

(10)

\[
\pi_{sc}^* = \frac{k(\lambda_h - \lambda_i) + p_0 + w_i + c_i - 2 \sum_{i=1}^{n} c_i}{4k(\lambda_h - \lambda_i)} \left[ k(\lambda_h - \lambda_i) + p_0 - w_i - c_i \right]
\]

(11)

Theorem 2. Under the centralized scenario, the optimal decision to maximize the profits of GSC system is:

\[
\alpha^* = \frac{k(\lambda_h - \lambda_i) - p_0 + \sum c_i}{2p_0}
\]

(13)

\[
Q^* = \frac{k(\lambda_h - \lambda_i) + p_0 - \sum c_i}{2k(\lambda_h - \lambda_i)}
\]

(14)

Proof. For the decision under centralized, based on Eq. (12), we define,

\[
\frac{d^2 \pi_{sc}}{d \alpha^2} = \frac{2p_0^2}{k(\lambda_h - \lambda_i)} < 0
\]

Therefore, the objective function \( \pi_{sc} \) is strictly concave on the parameter \( \alpha \), and yields the following:

Where, \( \frac{d\pi_{sc}}{d\alpha} = 0 \), We get the optimal value of \( \alpha^* \), which is Eq. (13). Then inserting \( \alpha^* \) in Eq. (1), so we can get the optimal quantity \( Q^* \) labeled as Eq. (14).

It follows that the optimal profits of the green supply chain system can be written as below:

\[
\pi_{sc}^{**} = \frac{[k(\lambda_h - \lambda_i) + p_0 - \sum_{i=1}^{n} c_i]^2}{4k(\lambda_h - \lambda_i)}
\]

(15)

Theorem 3. \( Q^{**} > Q^* \), if and only if \( w_2 > c_2 + c_3 + \cdots + c_n \)

Proof. Sufficiency: Eq. (8) and (14) give the condition, \( Q^{**} - Q^* = \frac{w_2 + c_i - \sum_{i=4}^{n} c_i}{2k(\lambda_h - \lambda_i)} \), where \( w_2 > c_2 + c_3 + \cdots + c_n \).
therefore \( Q^* - Q^* > 0 \), namely \( Q^* > Q^* \).

**Necessity:** Where \( Q^* > Q^* \), so \( Q^* - Q^* = \frac{w_2 + c_1 - \sum_{i=1}^{n} c_i}{2k(\lambda_0 - \lambda_0)} > 0 \), we get \( w_2 + c_1 - \sum_{i=1}^{n} c_i > 0 \), namely, \( w_2 > c_2 + c_3 + \cdots + c_n \).

**Theorem 4.** The decentralized scenario results in a suboptimal green supply chain profit, i.e., \( \pi^{**}_{GC} > \pi^*_{GC} \)

**Proof.** Based on Eq. (11) and (15), we define

\[
\pi^{**}_{GC} - \pi^*_{GC} = (w_2 - \sum_{i=1}^{n} c_i)^2,
\]

therefore \( \pi^{**}_{GC} > \pi^*_{GC} \).

The Theorem 4 indicates that the total profits in the centralized decision will be significantly greater than that in the decentralized decision.

**C. Revenue-sharing contract under the n-echelon green supply chain**

To avoid sub-optimization contract mechanisms come into play. The revenue-sharing contract mechanism is governed by the wholesale prices and the percentages of the differences of the revenues of Enterprise 1 that are shared with Enterprise \( i = 2, 3, 4, \ldots, n \), and is denoted by \( \Phi_i \). Meanwhile, \( 0 < \sum_{i=2}^{n} \Phi_i < 1 \). The optimal wholesale prices which are many combinations of wholesale prices and revenue-sharing contract variables satisfy coordination and are able to create win-win outcomes.

Under the revenue-sharing contract mechanism, the purpose of the Enterprise 1 in green supply chain is to maximize its profit, and the expected profits are given by:

\[
\text{Max } \pi_i = (1 - \sum_{i=2}^{n} \Phi_i)(1 + \alpha)p_0 - w_2 - c_1 \left[ 1 - \frac{\alpha \Phi_i}{k(\lambda_0 - \lambda_0)} \right] = \left[ 1 - \sum_{k=1}^{n} c_k - c_1 \right] (16)
\]

And for Enterprise \( i = 2, 3, \ldots, n \):

\[
\text{Max } \pi_i = \left[ \Phi_i (1 + \alpha)p_0 + w_i - w_{i+1} - c_i \right] \left[ 1 - \frac{\alpha \Phi_i}{k(\lambda_0 - \lambda_0)} \right] (17)
\]

**Theorem 5.** Under the revenue-sharing contract mechanism, Enterprise \( i + 1 \) in the GSC provides per unit product for Enterprise \( i \) with the wholesale price \( w_{i+1} \), and \( w_{i+1} \) is fulfilled as follows:

\[
w_2 = (1 - \sum_{i=2}^{n} \Phi_i) \sum_{k=1}^{n} c_k - c_1 (18)
\]

\[
w_{i+1} = w_i - c_i + \Phi_i \sum_{k=1}^{n} c_k, \quad \forall w_{n+1} = 0 (19)
\]

**Proof.** Based on Eq. (16) and (17), we define

\[
\frac{d^2 \pi_i}{d \alpha^2} = -\frac{2p_0^2}{k(\lambda_0 - \lambda_0)} < 0, \quad \text{and} \quad \frac{d^2 \pi_i}{d \alpha^2} = -\frac{2p_0^2 \Phi_i}{k(\lambda_0 - \lambda_0)} < 0
\]

Therefore, the objective functions \( \pi_i \) and \( \pi_i \) are all strictly concave on the parameter \( \alpha \), and when \( \frac{d \pi_i}{d \alpha} = 0, \quad \frac{d \pi_i}{d \alpha} = 0 \).

We yield Eq. (20) and (21)
Theorem 6. Let $\pi_i$ be the profit of enterprise $i$ under the revenue-sharing contract mechanism, the necessary conditions for achieve a win-win outcome respectively satisfy as following:

$$\pi_{i}^{***} = (1 - \sum_{i=2}^{n} \Phi_i)\pi_{kc}^{**}$$

$$\pi_{i}^{***} = \Phi_i\pi_{kc}^{**}$$

**Proof.** Putting the values form $w_2$ (Eq. (18)) and $\alpha_{i}^{***}$ (Eq. (20)) into Eq. (16), we get

$$\pi_{i}^{***} = (1 - \sum_{i=2}^{n} \Phi_i)\pi_{kc}^{**}$$

From Eq. (15), we have $\pi_{kc}^{**} = \frac{k(\lambda_h - \lambda_i) + p_0 - \sum_{i=1}^{n} c_i}{4k(\lambda_h - \lambda_i)}.$

Therefore, $\pi_{i}^{***} = (1 - \sum_{i=2}^{n} \Phi_i)\pi_{kc}^{**}.$

Similarly, we get $\pi_{i}^{***} = \Phi_i\pi_{kc}^{**}.$

Theorem 7. Under the revenue-sharing contract mechanism environment, the profits distribution coefficient must satisfy as following:

$$\frac{\pi_{i}^{*}}{\pi_{sc}^{**}} < \Phi_i < 1 \cdot \Phi_2 + \Phi_3 + \cdots + \Phi_i < \min \left\{ \frac{c_2 + c_3 + \cdots + c_n}{c_1 + c_2 + \cdots + c_n} \frac{\pi_{sc}^{**} - \pi_{i}^{*}}{\pi_{sc}^{**}} \right\}.$$
\textbf{Proof.} The profits under the revenue-sharing contract mechanism environment for each member must be more than under Decentralized decision, therefore,

\[ \pi_1^{**} > \pi_1^*, \quad \pi_i^{**} > \pi_i^*, \quad i = 2, 3, \ldots, n. \]

Based on Theorem 6, we get

\[ \frac{\pi_i^*}{s} < \Phi_i < 1 \quad \Phi_2 + \Phi_3 + \cdots + \Phi_i < \frac{\pi_i^{**} - \pi_i^*}{s}, \]

Meanwhile, putting the value from \( w_2 = (1 - \sum_{i=2}^{n} \Phi_i) \sum_{k=1}^{n} c_k - c_1 \) > 0, \( \pi_1^*, \pi_i^*, \pi_i^{**} \).

This simplifies to \( \Phi_2 + \Phi_3 + \cdots + \Phi_i < \frac{c_2 + c_3 + \cdots + c_n}{c_1 + c_2 + \cdots + c_n} \).

\textbf{Numerical Analysis}

In this section we illustrate the above model under different decisions for a numerical example.

We set the parameter values as: \( p_0 = 80, \quad k = 5, \quad \lambda_1 = 2, \quad c_1 = 5, \quad c_2 = 10, \quad c_3 = 15, \quad c_4 = 20 \), The values of \( \lambda \)'s are so chosen the range of green degree varies from 3 to 11, which ensures that we operate within the region of feasibility. Under the revenue-sharing mechanism, the profits distribution coefficient are respectively \( \Phi_2 = 0.25, \quad \Phi_3 = 0.25, \quad \Phi_4 = 0.3 \).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\( \lambda_h \) & \( \alpha^* \) & \( \bar{Q}^* \) & \( W_2^* \) & \( W_3^* \) & \( W_4^* \) & \( \pi_1^* \) & \( \pi_2^* \) & \( \pi_3^* \) & \( \pi_4^* \) & \( \pi_{sc}^* \) \\
\hline
3 & 0.047 & 0.375 & 76.500 & 62.000 & 62.000 & 0.848 & 1.688 & 3.37 & 5.750 & 12.660 \\
5 & 0.188 & 0.167 & 87.000 & 71.000 & 56.000 & 0.508 & 1.002 & 2.004 & 4.008 & 7.522 \\
7 & 0.328 & 0.125 & 97.500 & 80.000 & 50.000 & 0.468 & 0.938 & 1.874 & 3.750 & 7.030 \\
9 & 0.469 & 0.107 & 108.000 & 89.000 & 56.000 & 0.484 & 0.963 & 1.926 & 3.852 & 7.225 \\
11 & 0.609 & 0.097 & 118.500 & 98.000 & 62.000 & 0.500 & 1.019 & 2.037 & 4.074 & 7.636 \\
\hline
\end{tabular}
\caption{Effect of changes in green degree on the markup rate, order quantity and the profits under the decentralized decision}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\( \lambda_h \) & \( \bar{Q}^{**} \) & \( W_2^{**} \) & \( W_3^{**} \) & \( W_4^{**} \) & \( \pi_1^{**} \) & \( \pi_2^{**} \) & \( \pi_3^{**} \) & \( \pi_4^{**} \) & \( \pi_{sc}^{**} \) \\
\hline
3 & 3.000 & 5.000 & 7.500 & 5.000 & 10.800 & 13.500 & 13.500 & 16.200 & 54.000 \\
5 & 1.333 & 5.000 & 7.500 & 5.000 & 6.400 & 8.000 & 8.000 & 9.600 & 32.000 \\
7 & 1.000 & 5.000 & 7.500 & 5.000 & 6.000 & 7.500 & 7.500 & 9.000 & 30.000 \\
9 & 0.857 & 5.000 & 7.500 & 5.000 & 6.171 & 7.714 & 7.714 & 9.257 & 30.857 \\
11 & 0.778 & 5.000 & 7.500 & 5.000 & 6.533 & 8.167 & 8.167 & 9.800 & 32.667 \\
\hline
\end{tabular}
\caption{Effect of changes in green degree on the markup rate, order quantity and the profits under the revenue-sharing mechanism}
\end{table}

In the numerical illustration, we attempted to look at cases involving identical as well as dissimilar values of green degree across various players in the green supply chain. The finding from analyzing the Table 1-2 and Fig1-5 are given below.

(i) Table 1 and Figure1 show that under the decentralized decision the markup rate will enhance with the increasing green degree, meanwhile, with the increasing markup rate, the optimal order quantity of the retailers will decrease. From Table 2, we can get the same conclusion under the revenue-sharing contract mechanism. From Fig3, so it clears that the order quantity of the retailers depends on \( \lambda \) (green degree) as an increment in the value of \( \lambda \) reduces the order quantity.

(ii) Table1 and Fig4 reveal that under the decentralized decision wholesale prices at the green products are highly sensitive with \( \lambda_h \). A higher green degree leads to the increment wholesale prices. On the contrary, Fig5 shows that wholesale prices only sensitive with \( \Phi \) under the revenue-sharing contract mechanism, having nothing with the green degree.

(iii) Comparing the Table 1 and 2, it indicates that the profits for each member in revenue-sharing contract can be improved significantly compared to in decentralized decision. In addition, Fig2 reveals that with the raising green degree the profits for each member will firstly decrease and then increase. In this case, the profits for each member attain
the minimum when $\lambda_h = 7$. Meanwhile, as the leader of the supply chain, the most upstream enterprise can obtain the more profits.

CONCLUSION

This paper considers three models for $n$-echelon green supply chain. Details on the established models of the decentralized decision, centralized decision and revenue-sharing contract are discussed in later sections, and their optimal policies are also proposed. Comparing the optimal equilibrium strategies, it is concluded that the green supply chain members can achieve win-win condition under the revenue-sharing contract mechanism model. Furthermore, a numerical case is given to validate the models and conclusions. It is shown that with the raising green degree the markup rate of the green product will increase, the optimal order quantity of retailer in the green supply chain will decrease, and meanwhile, the profits for each member will firstly decrease and then increase. In addition, the profits for each member in revenue-sharing contract can be improved signally compared to in decentralized decision. Sensitivity analysis has been performed to exhibit the stability of the optimal results.

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