# Retailer's optimal decision model when marketing cost affected by demand 

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#### Abstract

Retailers often play the role of co-brokers in the marketing channel, their decisions affect the optimal profit of the manufacturers at upstream and consumer surplus at downstream. So the research of retailer's optimal decision has a strong theoretical and practical significance. This paper assumes that the demand function is a function of the retail price and the marketing cost, and set up three stages model to solve the optimal strategy of retailer. We introduce marketing costs into demand rate, and get the optimal retail price, marketing costs and order quantity of retailer. The study shows that: wholesale price, order costs and demand elasticity of marketing costs have positive impact on marketing costs, while price elasticity of demand and order quantity have negative impact. Ordering costs, wholesale price and marketing cost flexibility of demand have positive impact on optimal retail price, but price elasticity of demand and order quantity have negative impact. Ordering costs, marketing costs and elasticity of marketing costs have positive impact on order quantity, while retail price, price elasticity of demand and inventory cost have negative impact. The innovation of this paper lies in multiple decision variables of the retailers, which are retail price, order quantity and marketing costs as three decision variables.This paper complement the existing research on the retailers' optimal decision.


Key words: Marketing Cost; Retailer; Optimal Decision

## INTRODUCTION

Retailers generally in the channel structure plays a common role of middlemen, the optimal decision affected the surplus profits size of the upstream manufacturers and downstream consumer. In the coexistence markets with a variety of different brand products, retailers need to spend a certain cost of marketing to promote the sale of their product. The study considered the retailer's optimal decision problem of marketing cost has very strong theoretical value and realistic significance.

Some of the domestic and foreign scholars have researched the retailer's optimal decision problem. Hwang and Shinn(1997)modeled an inventory system for retailer's pricing and lot sizing policy for exponential deteriorating products under the condition of permissible delay in payment[1]. Kumar Rajaram et al (2001) considered product substitution effect on retail purchases from two aspects of retailers ordering quantity and expected profit, the basic newsvendor model is extended to including the possibility of the excess inventory instead of "out of stock" product[2]. Chung and Huang(2003) extended this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy[3]. Shinn and Hwang(2003) determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price [4]. Arcelus et al. (2003) modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise [5]. Shankar and Bolton (2004) analyzed the various factors affecting the pricing strategy of the retailer, they found that competition factors are the most important factors of pricing[6].Zhong Deqiang and Zhong Weijun (2004)studied the benefits of retailer strategic alliance to obtain decision based on priority[7].Wang Xiangping (2004) considered from the angle of retailers' ordering quantity discount in market
demanding stable environment and got the maximization profit of the optimal order quantity[8]. Ye Fei and Li Yina (2006) considered the cooperation incentive mechanism of supply chain risk averse retailer[9].Wang Weijun, Tang Xiaowo and Ni Debing (2008) studied the relationship between the retailer and the bullwhip effect delayed demand information[10]. Liang Luo (2010) and Sheng Fangzheng studied on the option theory applied to the buyback contract, considering the existing real-time market situation in the supply chain. The expected profit function, that retailers option buyback contract under the joint concave function, is the optimal decision variables and the analytic solution[11].In recent years, scholars study the coordination issue of a supply chain consisting of one retailer and two suppliers, a main supplier and a backup supplier. They determine the retailers optimal ordering policy and the main supplier's production quantity that maximize expected profit of the centralized supply chain. And also analyze the decentralized scenario, and a combination of overproduction risk sharing and buy-back contracts with a side payment from/to the backup supplier is provided to coordinate the supply chain. Numerical examples are given to gain some qualitative insights[12-14].

But these studies did not consider the influence of marketing costs on the optimal decision of the retailer, and largely not assume that the concrete forms of demand function, but just assumed that the demand function for the deterministic demand and stochastic demand. Then built the model, designed the corresponding algorithm to solve the model. This paper assumes that the demand function is a function of the retail price and the marketing cost, This paper assumes that the demand function is a function of the retail price and the marketing cost, and set up three stages model to solve the optimal strategy of retailer. Another innovation of this paper lies in the retailers with multiple decision variables, and the study of general retailers optimal strategies are the decision variables. This paper selects the retail price, order quantity and marketing costs as three decision variables, complementing the existing research on the retailers' optimal decision.

## 2 Model Description

### 2.1 Model Assumptions

Assuming the retailer receives goods delivery from the supplier, it needs to spend a certain amount of marketing costs for product sales. Retailers need to make the optimal retail price, marketing costs and order quantity decision. Assumes that market demand is a function of the retail price and the cost of marketing, $\alpha$ and $\beta$ respectively stands for the price elasticity demand and the demand of marketing cost elasticity. Retailers can understand the market demand through $\alpha$ and $\beta$.

The retailer's decision variables include: order quantity Q , retail price P and retailer marketing cost M .

### 2.2 Input Variables

$V$ suppliers to retailers' wholesale price;
$k$ a demand function $(k>0)$;
$h_{b}$ unit inventory cost for the retailer;
$\alpha$ the price elasticity of demand, $\alpha>1$;
$\beta$ marketing cost elasticity of demand, meet $0<\beta<1, \beta+1<\alpha$;
$A_{b}$ retailers ordering cost ( $¥ /$ every order);
$D(P, M)$ is the market demand rate (per unit time), the same with the Kim and Lee (1998), and Jung and Klein
(2005) [15-16]. We assume that $D(P, M)=k P^{-\alpha} M^{\beta}, \mathrm{P}$ for retailers to sell to consumers' price, M is retailers marketing cost.

## 3 Analysis of Retailers'Decision-making Model

From the above analysis, the profits of retailers can be expressed as follows: the retailer's profit function $=$ sales product purchase cost -marketing cost-ordering cost-inventory cost, or expressed in mathematical symbols as:
$\pi_{b}(P)=P D-V D-M D-A_{b} \frac{D}{Q}-0.5 h_{b} Q$
$=k P^{-\alpha+1} M^{\beta}-k V P^{-\alpha} M^{\beta}-k P^{-\alpha} M^{\beta+1}-A_{b} k P^{-\alpha} M^{\beta} Q^{-1}-0.5 h_{b} Q$
The retailer's business objective is to select an optimal sales price to maximize the profit function. Because $\pi_{b}(P)$ is a concave function of products in the retail price $P$. So existence and uniqueness of the optimal $P^{*}$ make the profit function of retailers $\pi_{b}(P)$ maximum. The optimal $P^{*}$ can be determined by the first order conditions
retailers profit function. The first-order condition for solving the $\pi_{b}(P)$ :

$$
\begin{align*}
& \frac{\partial \prod_{b}(P, M)}{\partial P}=(-\alpha+1) k P^{-\alpha} M^{\beta}+\alpha k V P^{-\alpha-1} M^{\beta} \\
& \quad+\alpha k P^{-\alpha-1} M^{\beta+1}+\alpha A_{b} k P^{-\alpha-1} M^{\beta} Q^{-1}=0 \tag{2}
\end{align*}
$$

The retailer first-order conditions can be found on the retailer's optimal price $P^{*}$ :
$P^{*}=\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}$
Taking the equation (3) into the equation (2), the retailer's profit function can be turned into:
$\pi_{b}=\left(P-V-M-A_{b} \frac{1}{Q}\right) D-0.5 h_{b} Q$
$=\left(P-V-M-A_{b} \frac{1}{Q}\right) k P^{-\alpha} M^{\beta}-0.5 h_{b} Q$
$=\left(\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}-V-M-A_{b} \frac{1}{Q}\right) k\left[\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}\right]^{-\alpha} M^{\beta}-0.5 h_{b} Q$
Because the type is a concave function of marketing cost $M$, the retailers' optimal marketing costs $M^{*}$ can only decided by the first-order condition. The first-order conditions:

$$
\begin{align*}
& \left(\frac{\alpha}{(\alpha-1)}-1\right) k\left[\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}\right]^{-\alpha} M^{\beta}+\left(\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}-V-M-A_{b} \frac{1}{Q}\right) k \\
& *\left\{-\alpha\left[\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}\right]^{-\alpha-1} \frac{\alpha}{(\alpha-1)} M^{\beta}+\left[\frac{\alpha\left(V+M+A_{b} Q^{-1}\right)}{(\alpha-1)}\right]^{-\alpha} \beta M^{\beta-1}\right\}=0 \tag{5}
\end{align*}
$$

From the formula can be derived for optimum $M^{*}$ retailers marketing cost:
$M^{*}=\frac{\beta\left(V+A_{b} Q^{-1}\right)}{\alpha-\beta-1}$
Put the formula (6) into (3), we turn the $P^{*}$ into:
$P^{*}=\frac{\alpha\left(V+A_{b} Q^{-1}\right)}{\alpha-\beta-1}$
$P^{*}$ is the optimal retail price which retailers selling to customers, by the assumption that $\alpha>\beta+1$.
Put $M^{*}$ and $P^{*}$ into the profit function of retailers, available:
$\pi_{b}(Q)=P D-V D-M D-A_{b} \frac{D}{Q}-0.5 h_{b} Q=D\left(P-V-M-\frac{A_{b}}{Q}\right)-0.5 h_{b} Q$
$=k \alpha^{-\alpha} \beta^{\beta}\left[\frac{\left(V+A_{b} Q^{-1}\right)}{\alpha-\beta-1}\right]^{-\alpha+\beta}\left(\frac{(\alpha-\beta)\left(V+A_{b} Q^{-1}\right)}{\alpha-\beta-1}-V-\frac{A_{b}}{Q}\right)-0.5 h_{b} Q$
$\pi_{b}(Q)$ is the order quantity $Q$ 's concave function, so there exists an optimal $Q^{*}$, let the retailers' profit function $\pi_{b}(Q)$ get maximum value. The optimal $Q^{*}$ can be determined by the first order conditions the retailer's profit function $\pi_{b}(Q)$, The first-order conditions for solving $\pi_{b}(Q)$ is:
$k \alpha^{-\alpha} \beta^{\beta}\left[\frac{\left(V+A_{b} Q^{-1}\right)}{\alpha-\beta-1}\right]^{-\alpha+\beta} \frac{A_{b} Q^{-2}}{\alpha-\beta-1}-0.5 h_{b}=0$
By the type can find out optimal $Q^{*}$ :
$\operatorname{Root} O \mathrm{f}\left[-2\left(\frac{V_{-} Z+A_{b}}{{ }_{-}}\right)^{-\alpha+\beta} A_{b} k \alpha^{-\alpha} \beta^{\beta}+h_{b}(\alpha-\beta-1)^{-\alpha+\beta+1}-Z^{2}\right]$

## RESULTS AND DISCUSSION

Analysis optimization of the front part of the two-level echelon supply chain inventory transportation integrated, we can get the following conclusions.

Proposition 1: The retailer's optimal marketing costs is proportional to the cost elasticity of demand, wholesale price, purchase order cost, and it is inversely proportional to the price elasticity of demand, order batch.

Proof: because the other conclusions are obvious effect, here only proved cost elasticity of demand of marketing.
$\frac{\partial M^{*}}{\partial \beta}=\frac{(\alpha-\beta-1)\left(V+A_{b} Q^{-1}\right)+\beta\left(V+A_{b} Q^{-1}\right)}{(\alpha-\beta-1)^{2}}=\frac{(\alpha-1)\left(V+A_{b} Q^{-1}\right)}{(\alpha-\beta-1)^{2}}$
Because $\alpha>\beta+1$, so $\frac{(\alpha-1)\left(V+A_{b} Q^{-1}\right)}{(\alpha-\beta-1)^{2}}>0$

The cost of marketing and cost of marketing demand elasticity are positive correlation.
Proposition 2: The optimal retail price of retailers is proportional to retailers' ordering cost, the wholesale price of suppliers to retailers and marketing cost elasticity of demand, and it is inversely proportional to the price elasticity of demand and order quantity.

Proof: proposition conclusions are obvious, only the demand function of price elasticity and marketing cost elasticity is less evident, so we focused on analysis of their effects.

$$
\begin{align*}
& \frac{\partial P^{*}}{\partial \alpha}=-\frac{\left(V+A_{b} Q^{-1}\right)(\beta+1)}{(\alpha-\beta-1)^{2}}<0  \tag{12}\\
& \frac{\partial P^{*}}{\partial \beta}=\frac{\alpha\left(V+A_{b} Q^{-1}\right)}{(\alpha-\beta-1)^{2}}>0 \tag{13}
\end{align*}
$$

Proposition 3: the retailer's optimal order quantity and the retailers ordering costs, marketing costs and sale cost elasticity are directly proportional. The retail price, the price elasticity of demand and retailers is inversely proportional to the retailers' unit inventory costs.

## CONCLUSION

In this paper, we put the marketing costs into the demand rate, assuming that the demand rate is a function of price and marketing costs. Study on the retailers optimal decision model based on the retail price, marketing costs and order quantity in the situation of retail price and marketing costs affecting the demand of products, Studies have shown that: The retailer's optimal marketing costs is proportional to the cost elasticity of demand, wholesale price, purchase order cost, and it is inversely proportional to the price elasticity of demand, order batch. The optimal retail price of retailers is proportional to retailers' ordering cost, the wholesale price of suppliers to retailers and marketing cost elasticity of demand, and it is inversely proportional to the price elasticity of demand and order quantity. the retailer's optimal order quantity and the retailers ordering costs, marketing costs and sale cost elasticity is directly proportional, and the retail price, the price elasticity of demand and retailers is inversely proportional to the retailers' unit inventory costs.

The research on retailers develops the best marketing strategy has certain guiding function in a rapidly changing consumer market. What's more, expanding the demand rate form enriches the research of retailer's optimal decision.

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