Residual stress field and toughening mechanism for composite ceramic with eutectic interphases

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ABSTRACT

The four-phase-model method is used to study residual stress field of composite ceramic with eutectic interphases. Considering matrix, interphase and fiber inclusion are isotropy, the residual stress distribution of composite ceramic with eutectic interphase is obtained. The size dependence of residual stresses is analyzed. The residual-stress-field in the eutectic ceramic rod relates to the stiffness, thermal expansion coefficients, shape and volume content of inclusions and interphases in the eutectic ceramic rod. On basis of the residual-stress-field in the eutectic ceramic rod, the frictional force on the boundary of the eutectic ceramic rod can be computed. Then the frictional work is given. The toughening mechanism of residual stresses is gotten. The result shows that the toughening value is dependent on the size of fiber inclusion in the eutectic rod.

Key words: eutectic ceramic rod; fiber inclusion; residual stress field; toughening mechanism; size dependence.

INTRODUCTION

Over the last decade, melt-growth composite ceramic with eutectic interphases has been the rapid development. The material systems involved a variety of material, and such material has good mechanical properties at room and high temperature [1–3]. Particular the composite ceramic with eutectic interphases was mainly composed of eutectic ceramic rod, and performed with high fracture toughness and excellent performance [4,5]. The composite ceramics stiffness was studied by Li et al.[6]. According to the microstructure of composite ceramics, the micro mechanical stress field in eutectic ceramic rod and the stress concentration due to dislocation pileup on fiber-matrix interfaces were analyzed, and the fracture strength of the eutectic ceramic rod was gotten at once. Then, taking into account the probability theory, the macro mechanical strength model of eutectic ceramic composite was built by Ni et al.[7]. However, the research about the analysis on residual stress field and the toughening effect of ceramic matrix composite ceramic with eutectic interphases has not been reported at present. Experiments show that in the crack propagation process, the boundaries of the eutectic ceramic rod was weak-bonding interface and the crack opening displacement will increase followed the crack tip move. While the interface of the eutectic ceramic rod with large residual compressive stress liberated, the crack would propagate along the boundaries of the eutectic ceramic rod and result in the inter-granular fracture. On the basis of experimentations, according to microscopic structure of the eutectic ceramic rod and the four-phase-model method, size dependent residual stress field of the eutectic rod was researched in this article. Then the residual compressive stress was used to calculate the interface frictional shear stress. The friction work was given by the integral of the frictional force over the acting region, and the toughening mechanism of residual stresses was built.
SIZE DEPENDENT RESIDUAL STRESS FIELD OF EUTECTIC CERAMIC ROD

Based on four-phase-model method, residual stress field is researched. The fiber inclusion with eutectic interphase is called two phase cell. Firstly the two phase cell was embedded into a finite matrix materials region, which is called matrix-atmosphere, and the two phase cell together with the matrix-atmosphere were known as three phase cell. Then the three phase cell was embedded in an unknown infinite effective medium, the three phase cell had the same volume fraction as each component within the eutectic ceramic rod. The effective medium and the eutectic ceramic rod had the same effective elastic constants and thermal expansion coefficient, shown in Figure 1.

Assuming the stiffness tensor in matrix, interphase, fiber inclusion and effective medium respectively are \( C_0, C_a, C_b \) and \( C \), the thermal expansion coefficient tensor in matrix, interphase, fiber inclusion and effective medium respectively are \( \alpha_0, \alpha_a, \alpha_b \) and \( \alpha \). When the temperature of composite materials changed \( \Delta T \), the thermal stress will be produced due to the internal different thermal expansion coefficient in the eutectic ceramic rod. Assuming all the material is matrix in the three phase cell, according to the Eshelby theory, the average stress and average strain can be obtained

\[
\sigma^D = \Omega^D(I - \Omega^D H)^{-1}\varepsilon^T, \quad \varepsilon^D = S^0 \sigma^D
\]  

(1)

Where, \( \Omega^D = C^0(I - M) \), \( M \) is the Eshelby tensor of three phase cell, \( H \) is flexibility increment of eutectic ceramic rod, which can be calculated by the formula (7) in reference(Li B.F. et al. 2011). \( S^0 = (C^0)^{-1} \), is flexibility tensor in matrix. \( \varepsilon^D = (\alpha - \alpha^0)\Delta T \).

To make the original three phase cell have the same strain, the outer edges of interphase and inclusion will be respectively put on \( \sigma'n \) and \( \sigma''n \). Clearly the superimposed stress of interphase and inclusion respectively are

\[
\sigma' = (C^a - C^0)\varepsilon^D, \quad \sigma'' = (C^b - C^0)\varepsilon^D
\]  

(2)

Defining the thermal mismatch strain in interphase, \( \varepsilon^{\theta T} = (\alpha^a - \alpha^0)\Delta T \). It is a simple question with thermal mismatch strain, same as an infinite matrix containing inclusion with the interface phase materials. According to the effective self-consistent theory [8], the equivalent stress tensor of the interface phase is

\[
\sigma^\theta = -\Omega^\theta(I + \Omega^\theta H^\theta)^{-1}\varepsilon^{\theta T} + (I + \Omega^\theta H^\theta)^{-1}\sigma^D
\]  

(3)

Where, \( \Omega^\theta = C^\theta(I - M) \), \( H^\theta = S^a - S^0 \). \( S^\theta = (C^\theta)^{-1} \) is the flexibility tensor of interface phase. Definition of the thermal mismatch strain of fiber inclusion: \( \varepsilon^{\theta T} = (\alpha^b - \alpha^0)\Delta T \). Similarly, the equivalent stress tensor of fiber inclusion is
\[ \sigma^b = -\Omega^b (I + \Omega^b H^b)^{-1} \epsilon^b + (I + \Omega^b H^b)^{-1} \sigma^0 \]

(4)

Where, \( \Omega^b = C^b (I - M) \), \( H^b = S^b - S^0 \), \( S^b = (C^b)^{-1} \) is flexibility tensor of fiber inclusion.

The matrix, interphase and fiber inclusion are isotropic, the corresponding elastic constants can be expressed as follows

\[
\begin{align*}
C_{ijkl}^{0} & = \lambda_0 \delta_{ij} \delta_{kl} + \mu_0 \delta_{ik} \delta_{jl} + \mu_0 \delta_{il} \delta_{jk} \\
C_{ijkl}^{a} & = \lambda_0 \delta_{ij} \delta_{kl} + \mu_a \delta_{ik} \delta_{jl} + \mu_0 \delta_{il} \delta_{jk} \\
C_{ijkl}^{b} & = \lambda_b \delta_{ij} \delta_{kl} + \mu_b \delta_{ik} \delta_{jl} + \mu_0 \delta_{il} \delta_{jk}
\end{align*}
\]

(5)

Where, \( \lambda_0, \mu_0, \lambda_a, \mu_a, \lambda_b, \) and \( \mu_b \) respectively are the Lame constants in matrix, interphase and fiber inclusion. Let the thermal expansion coefficients in matrix, interphase and fiber inclusion respectively were \( \alpha_0, \alpha_a \) and \( \alpha_b \), the corresponding thermal expansion coefficient tensor are

\[
\begin{align*}
\alpha^0 & = (\alpha_0, \alpha_0, \alpha_0, 0, 0, 0)^T \\
\alpha^a & = (\alpha_a, \alpha_a, \alpha_a, 0, 0, 0)^T \\
\alpha^b & = (\alpha_b, \alpha_b, \alpha_b, 0, 0, 0)^T
\end{align*}
\]

(6)

Assuming coordinate axis 1 was along the longitudinal of fiber inclusion, substituting equation (5) and equation (6) into equation (1), the longitudinal and transverse residual stresses in matrix could be obtained

\[
\sigma_{11}^{(0)} = \frac{E_0}{1 - \nu_0^2} \left[ (A_{ia} + 1 + \nu_0 (A_{ia} + 1)) f_a (\alpha_0 - \alpha_a) \right. \\
\left. + [(A_{ib} + 1 + \nu_0 (A_{ib} + 1)) f_b (\alpha_0 - \alpha_b)] \Delta T \right]
\]

(7)

\[
\sigma_{22}^{(0)} = \frac{E_0}{2(1 - \nu_0)} \left[ (\nu_0 (A_{ia} + 1) + A_{ia} + 1) f_a (\alpha_0 - \alpha_a) \right. \\
\left. + [\nu_0 (A_{ib} + 1) + A_{ib} + 1] f_b (\alpha_0 - \alpha_b)] \Delta T \right]
\]

(8)

Where, \( E_0 \) and \( \nu_0 \) are respectively elastic modulus and poisson ratio in matrix, \( N_{11}^{0} \) and \( N_{22}^{0} \) are shown as follow:

\[
\begin{align*}
A_{ia} & = 2B_{ia}(C_{4a} + C_{6a} + C_{6a}) - (B_{ia} + B_{ia})(C_{ia} + C_{2a} + C_{3a}) \cdot A_{ib} = 2B_{ib}(C_{4b} + C_{5b} + C_{6b}) - (B_{ib} + B_{ib})(C_{ib} + C_{2b} + C_{3b}) \\
A_{2a} & = B_{4a}(C_{ia} + C_{2a} + C_{3a} - B_{4a}(C_{4a} + C_{5a} + C_{6a}) \cdot A_{2b} = B_{4b}(C_{ib} + C_{2b} + C_{3b} - B_{4b}(C_{4b} + C_{5b} + C_{6b}) \\
B_{ia} & = f_a D_{ia} + D_{ia} + (1 - f_a) (M_{2211} + M_{3311}) \cdot B_{ib} = B_{ia} = f_a + D_{ia} + (1 - f_a) (M_{2222} + M_{3322}) \\
B_{4a} & = B_{7a} = f_a + D_{3a} + (1 - f_a) (D_{7a} M_{2211} + M_{3311}) \cdot B_{3a} = B_{9a} = f_a D_{4a} + D_{4a} + (1 - f_a) (D_{9a} M_{2222} + M_{3322}) \\
B_{5a} & = B_{7a} = f_a + D_{3a} + (1 - f_a) (D_{7a} M_{2211} + M_{3311}) \cdot B_{3a} = B_{9a} = f_a D_{4a} + D_{4a} + (1 - f_a) (D_{9a} M_{2222} + M_{3322}) \\
B_{9a} & = B_{7a} = f_a + D_{3a} + (1 - f_a) (D_{7a} M_{2211} + M_{3311}) \cdot B_{3a} = B_{9a} = f_a D_{4a} + D_{4a} + (1 - f_a) (D_{9a} M_{2222} + M_{3322}) \\
C_{ia} & = C_{1a} = C_{7a} = C_{3a} = C_{5a} = C_{9a} = (1 - f_a) (M_{2222} + M_{3322}) \\
C_{4a} & = C_{7a} = (1 - f_a) (D_{1a} M_{2211} + M_{3311}) \cdot C_{5a} = C_{9a} = (1 - f_a) (D_{1a} M_{2222} + D_{1a} + M_{3322})
\end{align*}
\]
$C_{6a} = C_{6b} = (1 - f_a)(D_{6a}M_{2233} + M_{3333} - 1)$, $C_{1b} = (1 - f_b)(M_{2211} + M_{3311})$, $C_{2a} = C_{4a} = (1 - f_a)(M_{2222} - 1 + M_{3322})$, $C_{4b} = C_{7b} = (1 - f_b)(D_{2b}M_{2211} + M_{3311})$, $C_{5b} = C_{9b} = (1 - f_b)(D_{4b}M_{2222} - D_{6b} + M_{3322})$, $C_{6b} = C_{8b} = (1 - f_b)(D_{6b}M_{2233} + M_{3333} - 1)$, $D_{1a} = 1 + 2(\mu_a - \mu_0)/(\lambda_a - \lambda_0)$, $D_{2a} = (\lambda_a + 2\mu_0)/(\lambda_a - \lambda_0)$, $D_{3a} = \lambda_a/(\lambda_a - \lambda_0)$, $D_{4b} = 1 + 2(\mu_b - \mu_0)/(\lambda_b - \lambda_0)$, $D_{5b} = (\lambda_b + 2\mu_0)/(\lambda_b - \lambda_0)$, $D_{6b} = \lambda_b/(\lambda_b - \lambda_0)$.

Where, $f_a$ and $f_b$ respectively are volume fractions of interphase and fiber inclusion in eutectic rod. The volume fraction of matrix is $(1 - f_a - f_b)$.

For rod shaped $\text{Al}_2\text{O}_3$-$\text{ZrO}_2$ eutectic ceramics, $\nu_b = 0.233$, $E_0 = 402\text{GPa}$, $\alpha_0 = 8.3$, $E_a = 233\text{GPa}$, $f_a = 0.426$, $\alpha_a = 10.6$, $\nu_b = 0.233$, $E_b = 10E_{0b}$, $f_b = \frac{4A}{d + 2A}f_a$. For $\triangle = 1\text{nm}$ and $d = 10$~$300\text{nm}$, the residual stresses in matrix of eutectic composite can be gotten from equation (7) and (8), and the relation between the residual stresses in matrix and fiber diameter is shown in Fig.2. The results show that the residual stresses in matrix are compressive and dependent on the diameter of fiber. Their absolute value will increase when the diameter of nano-fiber increase. As the diameter of nano-fiber is bigger than 100nm, the residual stresses in matrix are approximately constants.

![Fig. 2 Residual stresses in matrix as a function of fiber diameter](image)

Substituting Eq.(5)~Eq.(8) into Eq.(4), we obtain average residual stresses in the interphase

$$\sigma_{11}^{(a)} = \frac{E_0}{1 - \nu_0} \left\{ [A_{1a} + 1 + \nu_0(A_{2a} + 1)](f_a - 1)(\alpha_a - \alpha_0) + [A_{1b} + 1 + \nu_0(A_{2b} + 1)]f_a(\alpha_a - \alpha_0) \right\} \Delta T \tag{9}$$

$$\sigma_{22}^{(a)} = \frac{E_0}{2(1 - \nu_0)} \left\{ [\nu_0(A_{1a} + 1) + A_{2a} + 1](f_a - 1)(\alpha_a - \alpha_0) + [\nu_0(A_{1b} + 1) + A_{2b} + 1]f_a(\alpha_a - \alpha_0) \right\} \Delta T \tag{10}$$

The relation between the residual stresses in interphase and fiber diameter is shown in Fig.3. It can be seen that the residual stresses in interphase are compressive and not almost dependent on the diameter of fiber.

Substituting Eq.(6)~Eq.(9) into Eq.(5), we get average residual stresses in the fiber.
\[
\sigma_{11}^{(b)} = \frac{E_0}{1 - \nu_0^2} \left[ [A_{\alpha_{\alpha}} + 1 + \nu_0(A_{\beta_{\beta}} + 1)] f_{\alpha_{\alpha}} (\alpha_{\alpha} - \alpha_0) \\
+ [A_{\alpha_{\beta}} + 1 + \nu_0(A_{\beta_{\beta}} + 1)] (f_{\beta_{\beta}} - 1) (\alpha_{\beta} - \alpha_0) \right] \Delta T
\]

\[
\sigma_{22}^{(b)} = \frac{E_0}{1 - \nu_0^2} \left[ [\nu_0 (A_{\alpha_{\alpha}} + 1) + A_{\beta_{\beta}} + 1] f_{\alpha_{\alpha}} (\alpha_{\alpha} - \alpha_0) \\
+ [\nu_0 (A_{\alpha_{\beta}} + 1) + A_{\beta_{\beta}} + 1] (f_{\beta_{\beta}} - 1) (\alpha_{\beta} - \alpha_0) \right] \Delta T
\]  

Fig. 3 Residual stresses in interphase as a function of fiber diameter

Fig. 4 shows the residual stresses in fiber as a function of fiber diameter. It can be seen that the residual stresses in fiber are tensile and dependent on the diameter of fiber. Their absolute value will increase when the diameter of nano-fiber increase. When the diameter of nano-fiber is bigger than 50nm, the residual stresses in fiber are approximately constants.

Fig. 4 Residual stresses in fiber as a function of fiber diameter

TOUGHENING MECHANISM FOR RESIDUAL STRESS
Composite ceramic with eutectic interphase is mainly composed of eutectic ceramic rod, and a small amount of particles are distributed in the eutectic ceramic rod around. Because of the boundaries of eutectic ceramic rod is weak-bonding, the crack opening displacement will increase followed the crack tip move. There is large residual compressive stress on the boundaries of eutectic ceramic rod, the crack propagation follow the boundaries of the eutectic ceramic rod resulting in the intergranular cracking. Energy need to be consumed with the interface dissociation of eutectic ceramic rod, and the composite materials were strengthened. Plastic deformation of eutectic ceramic rod is very small. New fracture surface is found in the dissociation process. The residual compressive stress can produce the frictional force in the fracture surface of eutectic ceramic rod. The friction force will work during the separation process, and this work was called additional work of fracture.
Assuming the diameter of eutectic ceramic rod is \( D \), and the frictional force is \( \tau \), the additional fracture work can be calculated through the integral of the frictional force over the acting region. If \( l \) is the length that the frictional force acts along the eutectic ceramic rod, which will change in the dissociation process. Then the frictional force acting area is \( \pi D l \). If the initial length (slip length) is \( l_0 \), the length will decrease in the dissociation process. If the length of any time is \( x \), the length of frictional force acted will be \( l = l_0 - x \). Therefore, the work of frictional force is

\[
W = \int_0^{l_0} \tau \pi D (l_0 - x) \, dx = \frac{1}{2} \tau \pi D l_0^2
\]

(13)

If the boundary frictional shear stress of eutectic ceramic rod is \( \tau = \mu \sigma_{22}^{(0)}, \sigma_{22}^{(0)} \) is the transverse residual stress of the matrix in eutectic ceramic rod determined by formula (8). The dissociated length is also associated with the position of the main crack. If the main crack is near the point of eutectic ceramic rod, the dissociated length of boundary will be clearly close to zero. If the main crack is near the middle, the dissociated length will be close to \( l_0/2 \). So the average length of dissociation is \( l_0/4 \), then the work of frictional force can be computed

\[
W = \frac{1}{32} \mu \pi D l_0^2 \sigma_{22}^{(0)}
\]

(14)

The cross-sectional area of eutectic ceramic rod is \((\pi D^2)/4\). Considering the volume fraction of eutectic ceramic rod is \( \phi \), according to the formula (14), the dissociated work of per unit area can be obtained

\[
\Delta J = \frac{1}{8D} \phi \mu l_0^2 \sigma_{22}^{(0)}
\]

(15)

The formula (15) showed that the larger the slip length, the greater the dissociated work of eutectic ceramic rod. So it need a greater slip length to produce a larder additional fracture work for eutectic ceramic rod; but the slip length was limited in a certain conditions, and only a relatively weak connection interface can have a large slip length. Experimental results showed that micro-slip would be found in the weak interface of the eutectic ceramic rod with loading. When the slip was up to a certain value, the micro-interface debonding, and then as the load increased, it was dissociated. Obviously, the additional work of fracture with the slip length and interface frictional shear stress increased.

Further, analyze the slip length of eutectic ceramic rod. The slip length should be associated with the critical length. Both the slip length and critical length were all dependent on the interfacial bonding strength. When eutectic ceramic rod is long, that \( L > L_c \), and order \( l_0 \approx L_c \), then substituting equations \( L_c = \frac{D \sigma_{fu}}{2\tau} = \frac{D \sigma_{fu}}{2\mu \sigma_{22}^{(0)}} \) into equation (15), we can see that

\[
\Delta J = \frac{1}{32} \phi D \sigma_{fu}^2 \mu \sigma_{22}^{(0)}
\]

(16)

We knew that: when the eutectic ceramic rod was longer, the maximum fracture work can be produced by increasing the volume fraction of eutectic ceramic rod, reducing the interfacial friction coefficient and residual compressive stress, and increasing the fracture strength and radius of eutectic ceramic rod.

If the eutectic ceramic rod was shorter, and its length was less than the critical length, the average length rod of interface dissociation of eutectic ceramic rod would be \( L/4 \). In this case, \( l_0 \) could be replaced by \( L \) in formula (15). The fracture work of interface dissociation is
\[ \Delta J = \frac{1}{8D} f_f \mu_f L^2 \sigma_{22}^{(0)} \]  

(17)

The formula (17) showed that: when the eutectic ceramic rod was shorter, to get a larger fracture work of eutectic ceramic rod in interface dissociation, it was required to increase the volume fraction and length of eutectic ceramic rod, interfacial friction coefficient and residual compressive stress, and reduce the radius of eutectic ceramic rod.

According to the view of energy dissipation, the relation between the fracture toughness and energy dissipation is \( \Delta K_c = (E \Delta J)^{1/2} \). The residual stress toughening effect is

\[
\Delta K_c = \begin{cases} 
\frac{L}{4} \left[ \frac{2 f_f \mu_f E \sigma_{22}^{(0)}}{D} \right] & (L < L_c) \\
\frac{\sigma_{fu}}{4} \left[ \frac{2 f_f E}{D \mu_f \sigma_{22}^{(0)}} \right] & (L > L_c)
\end{cases}
\]

(18)

According to formula (14), we know that: the toughening mechanism is different for the different microstructure. When the eutectic rod is shorter and the fracture strength of eutectic rods is larger, the toughening effect will increase as interfacial friction coefficient and residual compressive stress increase. When the eutectic rod is longer, the effect of toughening will decrease as interfacial friction coefficient and residual compressive stress increase. The elastic modulus \( E \) of composite ceramic and fracture strength \( \sigma_{fu} \) of eutectic rod can be determined by Li, B.F. et al. (2011) and Ni, X.H., Sun, T. et al. (2011). Same as residual stress, they are associated with the size of fiber inclusion in eutectic rod, so the toughening values of residual stress are also associated with the diameter of fiber inclusion.

For \( \text{Al}_2\text{O}_3\)-\( \text{ZrO}_2 \) composite ceramic with eutectic interphase, the volume fraction \( f_f=0.9 \), the interfacial friction coefficient \( \mu_f=0.2 \), the radius of eutectic rod \( R=20\mu m \), and the length \( L=200\mu m \). Figure 5 shows the toughening value of residual stress as a function of the diameter of fiber inclusion. It can be seen that the toughening value is the minimum at \( d=0.8\mu m \). The decreasing rate of the toughening value is significant when \( d<0.8\mu m \), especially when the diameter is close to the nano-scale, the toughening value changes more obvious; the toughness value increases slowly, when \( d>0.8\mu m \).

![Fig.5 The relation between toughening effect and fiber diameter](image)

CONCLUSION
1) Based on the four-phase-model theory, micro-mechanics model of residual stress field for composite ceramic with eutectic interphase is built. According to this model, the residual stress field of matrix in eutectic rod is gotten at once. The residual stress field in eutectic ceramic rod is associated with the stiffness, thermal expansion coefficient, the shape of inclusions and interphase and the volume contents in eutectic rod.

2) Based on the analysis of the residual stress field, according to the work produced by the frictional force during the interface dissociation process of eutectic ceramic rod, the toughening mechanism of residual stress is built. The results show that the toughening mechanism is associated with microstructure for composite ceramic. When the eutectic rod is shorter and the fracture strength of eutectic rods is larger, the toughening effect will increase as interfacial friction coefficient and residual compressive stress increase. When the eutectic rod is longer, the effect of toughening will decrease as interfacial friction coefficient and residual compressive stress increase.

3) The toughening value of residual stress for Al₂O₃-ZrO₂ composite ceramic with eutectic interphase is analyzed quantitatively. The results show that size dependence of toughening value is obvious: when d<0.8μm, the toughening value of residual stress will decrease as fiber inclusion diameter increase, especially the toughening value changes more obvious when the diameter is close to nano-scale; after d>0.8μm, the toughness value will increase as fiber inclusion diameter increase slowly. The toughness value is the minimum at d=0.8μm.

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