Research on inventory and transportation integrated optimization model of supply chain on online shopping based on the revenue sharing contract

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ABSTRACT

When the online stores and the logistics service providers make decision alone, as trade off phenomenon exists, inventory costs achieve the minimum while transportation costs are not the minimum or the opposite situation occurs. When implementing inventory and transportation integrated optimization, the profit of the whole supply chain and the stores increases, while the profit of the logistics service providers declines, thus the logistics service providers will not cooperate with the stores and the paper aims to solve these problems that how to minimize the costs of inventory and transportation and promote cooperation between the online stores and the logistics service providers. Solving such problems can not only minimize inventory costs and transportation costs, but also increase the profit of the logistics service providers and the stores. The core innovation of the paper is that both revenue sharing contract and ITIO are introduced into supply chain on online shopping which previous studies have not involved in. Compared with previous work, the paper takes both the online stores and the logistics service providers as the object of study and designs a kind of profit distribution mechanism which can raise both the profit of the logistics service providers and the stores in the process of ITIO. The paper builds revenue model of the individual decisions firstly, revenue model of ITIO secondly, revenue sharing model of ITIO based on the revenue sharing contract thirdly, and a numerical example verified the feasibility of the model finally.

Keywords: Supply chain on online shopping; Inventory and transportation integrated optimization; Contract parameter.

INTRODUCTION

With the development of e-commerce, online shopping is not only increasingly becoming a favorite way to shop, but also a new growth point of the modern economy. As indicated in the “Chinese online retail market data monitoring report on 2013(a)”, Chinese online shopping users reached 277 million people, an increase of 29.4%; personal online stores’ number 12460000; Chinese online retail market’ transactions reached 754.2 billion RMB, an increase of 47.3% in the year to June 2013. Online shopping is playing an increasingly important role in the economy. Inventory and transportation costs are a major component of the total cost in the online shopping process, therefore, the article constructs a inventory and transportation integrated optimization (inventory and transportation integrated optimization, hereinafter referred to as ITIO) model of supply chain on online shopping based on revenue sharing contract, in which the online stores and the logistics service providers have to achieve revenue sharing contract on the basis of ITIO, namely the stores will transfer part of its earnings to the logistics service providers, so that the stores and the logistics service providers can realize higher profit than that of individual decisions, so as to realize the win-win of the stores and the logistics service providers.

Since revenue sharing contract is proposed, a lot of attention had been paid for it at home and abroad. Anderson E and Coughlan A (1987) [1] pointed out that the distribution of benefits directly related to whether the construction,
operation and management of the supply chain can be smoothly conducted; Mortimer (2000) [2] and IAA van der Veen et al (2005) [3] introduced revenue sharing contract in the video rental industry, and pointed out that revenue sharing contract can enhance overall profits in supply chain; Dana et al (2001) [4] noted that revenue sharing contract which retailers provided can reduce conflicts with upstream suppliers; Cachon et al (2005) [5] studied the wholesale price contract and revenue sharing contract, pointing out in both cases with a fixed retail price and that retailers set prices, the parties in the supply chain can achieve the optimal profit. A large number of attentions had also been paid to revenue sharing contract at home. For example Shoufeng Ji, Mingjia Liu, Wei Ding and Xiaoyuan Huang (2008) [6], Yali Hou, Dequn Zhou and Beiyi Tian (2009) [7], Chundong Wan and Liquan Gu (2009) [8], Qinghua Pang and Zhanbin He (2011) [9] and Meiping Huang, Xianyu Wang and Ziyang Geng (2012) [10] when studied revenue sharing contract, have pointed out that revenue sharing contract can maximize the profits of the whole supply chain, to coordinate the supply chain. Qinghua Pang, Hui Jiang, Yueming Hou and Yang Luo[11] analyzed the impact of effort level on revenue sharing contract on coordinating the supply chain, and remarked that the constraint of the optimal order quantity and the optimal effort is determined with the channel coordinating the supply chain.

What another related documents to the article are the literature about ITIO. Daganzo and Newell (1986) [12] studied the problems of distribution system in the supply chain, and designed the most cost-effective inventory location and vehicle scheduling scheme, which can be seen as the prototype of ITIO; Speranza and Ukovich (1994)[13] studied how to minimize inventory and transportation costs of multi-product distribution problems. Domestic scholars also conducted some research on ITIO, such as Ruxiu Zhang and Tianfang Xu (2005) [14], Guiqing Liu and Yongwu Zhou (2009) [15], and Cong Wu and Dongyuan Yang (2009) [16]. Fuchang Li pointed out the obstacles in ITIO, and put forward the corresponding countermeasures [17], and then pointed out in the study of profit distribution in ITIO, the profit distribution is related to whether ITIO can carry out, and the supply chain can develop new profit growth point or not [18], finally considering the condition of the price discount and flexible transport and found that the total cost of ITIO is less than that of the decentralized optimization [19].Although a lot of research on revenue sharing contract and ITIO have done, neither will revenue sharing contract be applied to online shopping, nor will ITIO applied to online shopping contract.

2. Brief descriptions
Assuming in the supply chain of online shopping, the online stores and the logistics providers are risk neutral and completely rational, that is, they tend to adopt decisions to maximize their own gains. The online stores are of dominant statues in the supply chain, facing the market demand as $X$, and the suppliers can always provide the required quantity of goods to the stores in constant prices. $X$ is a function of both price $P$ and effort level of the logistics service providers $e$ ($0 < e < 1$), where $P$ and $e$ are the decision variables, and demand information is completely known to the stores and the logistics service providers. $c_1$ is unit purchase cost of the stores. $c_2$ is unit cost of service. $c_3$ is unit inventory costs. $w_1$ is unit distribution price. $w_2$ is unit distribution cost. $g(e)$ is effort cost. $c_1, c_2, c_3, w_1$ and $w_2$ are all constants.

3. Research on ITIO model of supply chain on online shopping based on the revenue sharing contract
A. Revenue model of decision-making bodies of the individual decisions
In order to compare the individual decision-making with ITIO, the article builds a separate decision model firstly, and solves the revenue function of the stores and the logistics service providers. The article assumed that the market demand function which the stores faced as follows:

$$X(P, e) = \alpha - \beta P + \lambda e,$$  \hspace{1cm} (1)

where $\alpha > 0, \beta > 0, \lambda > 0$ and $\alpha, \beta, \lambda$ are the constants. $\alpha$ is the maximum market demand which is determined by the sales price $P$; $\beta$ is a sensitivity coefficient of sales price $P$, as $\beta$ increases, the impact of the sales price $P$ on the market demand also increases; $\lambda$ is a sensitivity coefficient of the logistics service providers, as $\lambda$ increases, the impact of effort level $e$ on the market demand also increases.

Effort cost function of the logistics service providers can be expressed as:

$$g(e) = \delta e^2,$$  \hspace{1cm} (2)

where $\delta > 0$ and $\delta$ is a constant. $\delta$ is a sensitivity coefficient of effort level of the logistics service providers.
\(e\), as \(\delta\) increases, the impact of effort level of the logistics service providers \(e\) on effort cost also increases.

The profit function of the stores after which is simplified can be expressed as:

\[
\Pi_1(P, e) = (P - k_1) (\alpha - \beta P + \lambda e).
\]

(3)

where \(k_1 = c_1 + c_2 + \frac{c_3}{2} + w_1\).

The profit function of the logistics service providers after which is simplified can be expressed as:

\[
\Pi_2(P, e) = (w_1 - w_2) (\alpha - \beta P + \lambda e) - \sigma e^2.
\]

(4)

Since the stores and the logistics service providers are two independent decision-making bodies, under the completely rational conditions, they decided their own decisions according to their own profit maximization criterion. According Stackelberg dynamic game ideas, the stores adjust the sales price and the logistics service providers adjust the level of effort in order to achieve an equilibrium state, where the stores are the makers of sales and the logistics service providers decided their own effort level according to a fixed price that the stores provided in order to maximize profits. Under the conditions of the sale price \(P\) given, if the logistics service providers want to achieve their own profit maximization, we need to solve the maximum value of the formula (4), thus the formula (4) should satisfy the condition that the decision variables \(e\) of bivalent derivative is less than zero \((d^2\Pi_2(P, e) / de^2 < 0)\) and first order derivative is equal to zero \((d\Pi_2(P, e) / de = 0)\). We are very easy to get \(d^2\Pi_2(P, e) / de^2 = -2\delta < 0\). When \(e = e_1\), the logistics service providers achieve the maximum profit. Seeking the first order derivative of \(e\) in the formula (4), we conclude:

\[
e_1 = \frac{\lambda(w_1 - w_2)}{2\delta}.
\]

(5)

If the stores want to achieve their own profit maximization, we need to solve the maximum value of the formula (3), thus formula (3) should satisfy the condition that the decision variables \(P\) of bivalent derivative is less than zero \((d^2\Pi_1(P, e) / dP^2 < 0)\) and first order derivative is equal to zero \((d\Pi_1(P, e) / dP = 0)\). We are very easy to get \(d^2\Pi_1(P, e) / dP^2 = -2\beta < 0\), namely the stores can achieve the maximum profit. Seeking the first order derivative of \(P\) in the formula (3), we conclude:

\[
P_1 = \frac{1}{2} \left( \frac{\alpha}{\beta} - k_1 \right) + \frac{\lambda^2(w_1 - w_2)}{4\beta\delta}.
\]

(6)

Taking formula (8) and (6) into the formula (1), we conclude:

\[
X_1 = \frac{\alpha - \beta k_1}{2} + \frac{\lambda^2(w_1 - w_2)}{4\delta}.
\]

(7)

Taking formula (8) and (6) into the formula (3) and (4), we conclude:

\[
\Pi_1^* = \frac{1}{8\delta} \left( \frac{\alpha}{\beta} - k_1 \right) + \frac{\lambda^2(w_1 - w_2)}{16\beta\delta^2}.
\]

(8)

and \(\Pi_2^* = \frac{(w_1 - w_2)(\alpha - \beta k_1)}{2}\).

(9)

Total profit function after which is simplified can be expressed as:

\[
\Pi(P, e) = \left[ P - k_2 \right] (\alpha - \beta P + \lambda e) - \delta e^2.
\]

(10)
where \( k_2 = c_1 + c_2 + \frac{c_1}{2} + w_2 \) and \( k_2 < k_1 \).

Then total profit \( \Pi^* \) can be expressed as:

\[
\Pi^* = \Pi_1^* + \Pi_2^*. \tag{11}
\]

When the stores and the logistics service providers make decision separately, we solve the two most important unknown variables that are effort level of logistics service providers \( e_1 \) and sales prices \( P_1 \), then taking \( e_1 \) and \( P_1 \) into relative formulas, and solve optimal order quantity \( X_1 \), profit of the stores \( \Pi_1^* \) and profit of the logistics service providers \( \Pi_2^* \).

**B. Revenue model of decision-making bodies of ITIO**

When the stores and logistics service providers make decisions alone, they can achieve their maximum profit, but it apparently did not realize the maximum profit of the whole supply chain. To achieve the maximum profit of the whole chain of the stores and logistics service providers, namely to seek maximum of formula (10), we need to seek the first derivative, second derivative and mixed second derivative of \( P \) and \( e \) in formula (10). Such conditions of \( \frac{\partial \Pi(P, e)}{\partial P} = 0 \), \( \frac{\partial^2 \Pi(P, e)}{\partial e^2} = 0 \), \( AC - B^2 > 0 \) and \( A < 0 \) should be met in formula (10), where

\[
A = \frac{\partial^2 \Pi(P, e)}{\partial P^2}, \quad C = \frac{\partial^2 \Pi(P, e)}{\partial e^2}, \quad \text{and} \quad B = \frac{\partial^2 \Pi(P, e)}{\partial P \partial e}. \]

We can conclude:

\[
P_2 = 2\delta h + k_2, \tag{12}
\]

and \( e_2 = \lambda h. \tag{13} \)

Taking the formula (19) and (13) into the formula (1), we conclude:

\[
X_2 = \alpha - \beta \left(2\delta h + k_2 \right) + \lambda^2 h, \tag{14}
\]

where \( h = \frac{\alpha - \beta k_2}{4\beta - \lambda^2} \) and \( \alpha - \beta k_2 > 0 \) apparently. Because of \( A = -2\beta < 0 \), \( C = -2\delta < 0 \) and \( B = \lambda > 0 \), we get \( AC - B^2 = 4\beta \delta - \lambda^2 > 0 \) and then \( h > 0 \) and we also conclude \( e_2 = \lambda h > 0 \) and \( P_2 = 2\delta h + k_2 > 0 \), namely the optimal solution of the formula (19) and (13) is meaningful. So conditions of supply chain to achieve maximum profit are met, let \( \Pi_{ITIO}^* \) be the maximum of formula (10), namely:

\[
\Pi_{ITIO}^* > \Pi^*. \tag{15}
\]

Let \( \Pi_{ITIO1}^* \) is the maximum profit of the stores and \( \Pi_{ITIO2}^* \) is the maximum profit of the logistics service providers, when \( P = P_2 \), \( e = e_2 \) and \( X = X_2 \). We get:

\[
\Pi_{ITIO1}^* = (P_2 - k_1)(\alpha - \beta P_2 + \lambda e_2), \tag{16}
\]

and \( \Pi_{ITIO2}^* = (w_1 - w_2)(\alpha - \beta P_2 + \lambda e_2) - \delta e_2^2, \tag{17} \)

There are two reasonable possibilities. Case a is \( \Pi_{ITIO1}^* > \Pi_1^* \) and \( \Pi_{ITIO2}^* < \Pi_2^* \), namely the profit of the stores increase while the profit of the logistics service providers decrease, and case b is \( \Pi_{ITIO1}^* > \Pi_1^* \) and \( \Pi_{ITIO2}^* > \Pi_2^* \), namely the profit of both the stores and the logistics service providers increase. When the case b occurs, the stores and the logistics service providers will choose to cooperate, and the article will only study the case a.
C. Revenue sharing model of ITIO based on the revenue sharing contract

Assuming the case a situation occurs and the stores do not take any incentives for logistics service providers, cooperation can not be carried out smoothly. Thus the stores provide revenue sharing contract and transfer their own profit of \( \tau \) times to the logistics service providers. Let \( \Pi_{\text{ITIO}}^* \) be the profit of the stores and \( \Pi_{\text{ITIO2}}^* \) the profit of the logistics service providers. The value of \( \tau \) is related to cooperation conditions with the logistics service providers. From the assumptions above we obtain:

\[
\Pi_{\text{ITIO}}^* = (1-\tau)\Pi_{\text{ITIO1}},
\]
\[
\Pi_{\text{ITIO2}}^* = \Pi_{\text{ITIO1}}^* + \Pi_{\text{ITIO2}},
\]
\[
\Pi_{\text{ITIO1}}^* > \Pi_1^*,
\]
\[
\Pi_{\text{ITIO2}}^* > \Pi_2^*,
\]
and \( \Pi_{\text{ITIO}} = \Pi_{\text{ITIO1}}^* + \Pi_{\text{ITIO2}}^* \).

From the relative formulas above, we obtain:

\[
(w_1 - w_2)\left[ x \left( e_1 - e_2 + \beta \left( P_2 - P_1 \right) \right) \right] / (P_2 - k) < \tau < 1 - \left( P_2 - k \right) \left( \alpha - \beta P_2 - e_2 \right). \tag{23}
\]

As long as the value of \( \tau \) satisfies the formula (23), cooperation can be continued, namely under the conditions of revenue sharing contract, ITIO can achieve their maximum profit individually.

From the analysis above, we get: a) with respect to revenue model of the individual decisions, the model of ITIO can obtain greater profit of the whole supply chain, and in fact only the case a can occur; b) to facilitate cooperation of ITIO, the stores transfer their own earnings of \( \tau \) times to logistics service providers, and only retain earnings of \( 1 - \tau \) times, which makes the profit of the stores and the logistics service providers enhanced; c) the value of \( \tau \) indicates that the profit can be coordinated by revenue sharing contract, and also indicates that profit distribution has a characteristic of flexibility; d) the value of \( \tau \) is determined by cooperative power and status in the game, and the impacts on the value of \( \tau \) are different among decision-making bodies, and as the dominant of the stores, the value of \( \tau \) is determined more by the stores than the logistics service providers.

4. An example

In order to verify the changes of the profit and strategy selection in the supply chain, the article will give a specific example. Assuming \( \alpha = 1100, \beta = 12, \lambda = 15 \) and \( \delta = 100 \), we get \( X(P, e)=1100-12P+15e \) and \( g(e)=100e^2 \). Assuming \( c_1=40, c_2=20, c_3=20, w_1=10 \) and \( w_2=6 \), we get \( k_1=75, k_2=71 \) and \( h=0.0542 \). Taking these figures above into the formulas above, we get \( e_1=0.3, P_1=79.35, X_1=152.3, \Pi_1^*=662.5, \Pi_2^*=600.2, \Pi^*=1262.7 \) and \( e_2=0.813, P_2=81.84, X_2=130.12, \Pi_{\text{ITIO1}}=900.0, \Pi_{\text{ITIO2}}=453.9, \Pi_{\text{ITIO}}=1353.9 \), then taking these figures obtained above into from the formula (18) to the formula (23), we get \( 0.163 < \tau < 0.264 \).

We conclude from the example above, when the stores and the logistics service providers make decisions alone, the effort level is 0.3, the sales price 79.35, the market demand 152.3, the maximum profit of the stores and the logistics service providers and the whole supply chain respectively 662.5, 600.2 and 1262.7; while implementing the tactic of ITIO, the effort level is 0.813, the sales price 81.84, the market demand 130.12, the maximum profit of the stores and the logistics service providers and the whole supply chain respectively 900.0, 453.9 and 1353.9. Compared the two cases above, to realize the maximum profit of the whole supply chain in the process of ITIO, the logistics service providers need work harder than before, and now the profit of the stores increase while the profit of the logistics service providers decreases. Thus the stores have to transfer their own earnings of \( \tau \) times to the logistics service providers so as to facilitate cooperation and retain earnings of \( 1 - \tau \) times. Through the analysis above, we get \( 0.163 < \tau < 0.263 \), which meets the conditions and verifies the feasibility of the model of ITIO on online shopping supply chain which based on the revenue sharing contract.

Compared with the decentralized decision-making, the decision-making based on revenue sharing contract improve the profit of both the stores and the logistics service providers. When the values of the parameter \( \tau \) are different, the
profit of the stores and the logistics service providers will also change. The profit of the stores and the logistics service providers is shown in figure 1 and 2, when the parameter $\tau$ takes different values.

CONCLUSION

The study found that the stores provide revenue sharing contract to the logistics service providers and transfer their own earnings of $\tau$ times to the logistics service providers, and only retain earnings of $(1 - \tau)$ times by determining the contract parameter $\tau$, thus the profit of the stores and the logistics service providers have been enhanced. The article also concludes that when the parameter $\tau$ takes different values, the profit of the stores and the logistics service providers will also change, and the value of $\tau$ is determined more by the stores than the logistics service providers because of the dominant of the online stores.

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