Prediction of CNG automobile ownership by using the combined model

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ABSTRACT

In order to accurately predict the ownership of CNG automobile, Grey System Theory GM Model and Elastic Coefficient Method were investigated and compared in terms of their prediction errors. A new Combined Model was established based on the Weight Factor, which is calculated by using Suboptimal Combination Method and the Linear Combination Model. Compared to the Single Model, the prediction accuracy of the new Combined Model is greatly improved when it was applied to predict the CNG automobile ownership in S province and the errors achieved are no more than 10% than before. Thus the study demonstrated that the new Combined Model was effective and practical in its application on the rational layout of the gas stations.

Key words: grey theory; elastic coefficient method; combined model; CNG automobile

INTRODUCTION

In recent years, with the increase of the vehicle population in China, a large amount of automobile exhaust has been discharging which leads to the serious atmosphere pollution. The development of compressed natural gas (CNG) automobile is the important way to solve the problem. Compressed natural gas automobile has advantages of safety and environmental friendliness. After years of development, the technologies of CNG automobile and gas station have become increasingly mature, but the theory of the layout of the CNG stations is still in the exploration, it brings adverse effect to the development of CNG automobile industry in our nation.

Taking Sichuan province as an example, during the Eleventh Five-Year period, the number of CNG station only increased by 57 which is far less than that of CNG automobiles [1]. The shortage of gas station not only poses an obstacle to the CNG vehicles’ refueling, but also has an unfavorable impact on the development of CNG automobile industry. We should focus on the study of the reasonable layout of gas station so as to effectively promote the use of natural gas resource, the prediction of CNG automobile population is a prerequisite for the planning of CNG stations. According to that prediction, the more precise projection of the needs of gas station can be made to keep the balance between developments of both gas station and CNG automobile and avoid overbuilding and deficient building as well.

EXPERIMENTAL SECTION

The combined model

There are many prediction methods, some of them are used commonly in many fields, including: grey theory GM (1, 1) model, regression analysis, elasticity coefficient method and etc. Among those theories, the grey theory model which requires a small sample size, has been used widely in data forecast, such as prediction of incidence, electricity demand forecasting and traffic volume forecast. But the prediction accuracy is acceptable only when the sample data conform to the index under the condition of increasing and decreasing, it is different from the actual situation of the growth of the natural gas car ownership. Secondly, economic development is an important factor for the development of CNG car ownership, the grey theory model can’t establish direct contact between CNG automobile ownership and the speed of economic development. Elastic coefficient method can indicate the relationship between the economy...
and prediction result. Although elastic coefficient method has been applied in many fields, it can't reflect the impact of other factors and the prediction accuracy is not high.

Combined model can integrate the advantages of single models, it improves the prediction precision. We predicted the CNG automobile ownership through adopting liner combination model to establish new model on the basis of those two methods.

The grey system GM (1, 1) model
The "Grey" in the grey forecasting method means incomplete information. grey prediction method is established based on the uncertainty of information, the establishment of the grey prediction model does not require lots of sample data, so the application range is very wide [2].

The GM (1, 1) model is used usually, Here is how it is constructed:
Assuming a time series $X^{(0)}$ have $n$ observations:

$$ X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(n)\} $$

(1)

We can get a new series through accumulation:

$$ X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(k) $$

(2)

$$ X^{(1)} = \{X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(n)\} $$

(3)

The corresponding differential equation of grey GM model is:

$$ \frac{dX^{(1)}}{dt} + \alpha X^{(1)} = \mu $$

(4)

$\alpha$ ——developing gray scale; $\mu$ ——The endogenous control gray scale

$\hat{\alpha}$ is estimated parameter vector, we can got it by the least square method:

$$ \hat{\alpha} = [\alpha, \mu] = (B^T B)^{-1} B^T y_n $$

(5)

$y_n$ is a column vector, $y_n = [X^{(0)}(2), X^{(0)}(3), \cdots, X^{(0)}(n)]^T$

$$ B = \begin{bmatrix}
-\frac{1}{2} [X^{(1)}(1) + X^{(1)}(2)] & 1 \\
-\frac{1}{2} [X^{(1)}(2) + X^{(1)}(3)] & 1 \\
\vdots & \vdots \\
-\frac{1}{2} [X^{(1)}(n-1) + X^{(1)}(n)] & 1
\end{bmatrix} $$

(6)

Solve the differential equation, got the GM(1,1) predict model:

$$ \hat{X}^{(1)}(k+1) = X^{(0)}(1) - \frac{\mu}{\alpha} e^{-\alpha k} + \frac{\mu}{\alpha} \hat{\alpha} \quad (k = 0, 1, 2, \cdots, n - 1) $$

(7)

Restored the model to get:

$$ \hat{x}^{(0)} = x^{(0)}(k+1) - \hat{x}^{(1)} $$

(8)

$\hat{x}^{(1)}(k+1)$ is the predict result of CNG automobile ownership.

Elastic coefficient method
CNG automobile ownership and economic development have close relations. With the development of economy, the CNG automobile ownership is increasing. economic development is characterized by GDP. Elastic coefficient is the
key of elastic coefficient method [3, 4]. Elastic coefficient is the ratio of CNG automobile ownership rate and GDP annual growth rate.

Elastic coefficient $e$ can be calculated by the formula below:

$$e = \frac{\beta}{\alpha}$$

(9)

$\beta$ is the annual growth rate of CNG automobile, $\alpha$ is the annual growth rate of GDP, usually we get the Average value as sample data.

To predict model is:

$$M = M_0 (1 + e \times \alpha)^{t-t_0}$$

(10)

$M$ is the predict result of CNG automobile ownership, $M_0$ is the CNG automobile ownership in base year, $e$ is Elastic coefficient, $\alpha$ is predicted annual growth rate of GDP, $t$ is the predicted year , $t_0$ is base year.

The new combined model

Bates and Granger combination forecast theory put forward for the first time in the 1960s [5-7], a combined model can improve the prediction accuracy, it has been widely studied and applied. According to the function relationship between combination prediction and single prediction, it can be divided into linear combination model and the nonlinear combination model, the linear combination model is more suitable for the application.

Linear combination described as:

$$f(t) = \sum_{i=1}^{m} w_i f_i(t)$$

(11)

$f(t)$ As the combination model prediction, $f_i(t)$ is the first I forecast model in t time, $w_i$ is the weight of the model $i$, the scope of $w_i$ is 0 to 1, and the sum is 1.

Got the weight factor

The key of combined method is determining the weight, there are two methods to determine the weight: optimal combination method and the suboptimal combination method, the calculating process of optimal combination method is relatively complex, the suboptimal combination method is simple, and the prediction accuracy of the two methods are almost same[8], the suboptimal combination method is more convenient. The optimal combination weights calculation formula is:

$$\sigma_i = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\chi_i - \mu)^2}$$

(12)

$$\sigma = \sum_{i=1}^{m} \sigma_i$$

(13)

$$w_i = \frac{\sigma_i - \sigma}{\sigma} \times \frac{1}{m-1}$$

(14)

$\sigma_i$ is the standard deviation of the error of model $i$, $\sigma$ is the sum of the standard deviation of the error of models, $w_i$ is the weight of model $i$. 
RESULTS AND DISCUSSION

Examples of application
Predict the CNG automobile ownership in S province by the grey system model, the elastic coefficient model and the new combination model. Select the CNG automobile ownership in the S province from 2003 to 2007 as samples. The CNG automobile ownership and economic developments in the S province from 2003 to 2007 such as table 1:

<table>
<thead>
<tr>
<th>time</th>
<th>GDP(10^8 Yuan)</th>
<th>GDP growth (Annual %)</th>
<th>CNG automobile population(10^4)</th>
<th>Growth rate of CNG automobile (annual %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>03</td>
<td>5456</td>
<td>11.8</td>
<td>4.71</td>
<td>18.64</td>
</tr>
<tr>
<td>04</td>
<td>6379</td>
<td>12.7</td>
<td>5.55</td>
<td>17.83</td>
</tr>
<tr>
<td>05</td>
<td>7385</td>
<td>12.6</td>
<td>5.95</td>
<td>7.2</td>
</tr>
<tr>
<td>06</td>
<td>8690</td>
<td>13.5</td>
<td>8.58</td>
<td>44.2</td>
</tr>
<tr>
<td>07</td>
<td>10562</td>
<td>14.5</td>
<td>12.98</td>
<td>51.28</td>
</tr>
<tr>
<td>average</td>
<td>13.02</td>
<td></td>
<td></td>
<td>27.83</td>
</tr>
</tbody>
</table>

Grey system method
First, we used sample from table 1 to make the original time series \( X^{(0)} = \{4.71, 5.55, 5.95, 8.58, 12.98\} \), translated it to a new time series \( X^{(1)} = \{4.71, 10.26, 15.85, 24.43, 37.41\} \) established matrix \( B \) and \( y_n \):

\[
B = \begin{bmatrix}
-7.485 & 1 \\
-13.055 & 1 \\
-20.14 & 1 \\
-30.92 & 1
\end{bmatrix}
\]

\[
y_n = [5.55, 5.95, 8.58, 12.98]^T
\]

We got \( \hat{\alpha} = [-0.3315, 2.3318]^T \) by MATLAB and established the model:

\[
\hat{X}^{(1)}(k + 1) = 11.74e^{0.3315k} - 7.034 (k = 0, 1, 2, \cdots, n - 1)
\]

\[
M_1 = X^{(0)}(k + 1) = X^{(1)}(k + 1) - X^{(1)}(k)
\]

\( M_1 \) is the predicted result of CNG automobile ownership by grey system method

According to the model, we got the predicted result of S province CNG automobile ownership from 2008 to 2011 by grey system method as shown in table 2:

<table>
<thead>
<tr>
<th>time</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>The predict result(10^4)</td>
<td>17.15</td>
<td>24.21</td>
<td>36.89</td>
<td>46.98</td>
</tr>
<tr>
<td>Actual value(10^4)</td>
<td>17.8</td>
<td>20.6</td>
<td>25.2</td>
<td>31.4</td>
</tr>
<tr>
<td>error</td>
<td>-3.8%</td>
<td>17.5%</td>
<td>46.39%</td>
<td>49.61%</td>
</tr>
</tbody>
</table>

As table 2, in the first year the result is very accurate, the error is within 5%, but in the second year, it is close to 20%, with the passage of time, error is bigger and bigger, the grey system model is not suitable for medium and long term prediction of CNG automobile.

Elastic coefficient method
Got the elastic coefficient from the sample in the table 1:

\[
e = \frac{\beta}{\alpha} = \frac{27.83\%}{13.02\%} = 2.14
\]
\( \beta \) is average annual growth rate of CNG automobile ownership of the sample, \( \alpha \) is average annual growth rate of GDP of the sample. We got the elastic coefficient \( e \). Set 2007 as base year, according to data released form the government of S province in 2007, the growth rate of expected GDP is 9%, established the new combined model:

\[
M_2 = 12.98 \times (1.1926)^{t-2007} \quad (t=2008, 2009\ldots)
\]

\( M_2 \) is the predict result of CNG automobile ownership by elastic coefficient method

To research the relationship between the growth rate of expected GDP and the accuracy of the predicted result, we established another model \( M_2^* \) by using a more accurate the growth rate of expected GDP, the value is 13%:

\[
M_2^* = 12.98 \times (1.2782)^{t-2007} \quad (t=2008, 2009\ldots)
\]

The predicted result of S province CNG automobile ownership from 2008 to 2011 by elastic coefficient method as shown in table 3:

### Table 3 The predicted results of elastic coefficient method model

<table>
<thead>
<tr>
<th>time</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>The predict result when the growth rate of expected GDP is 9%( (10^4) )</td>
<td>15.48</td>
<td>18.46</td>
<td>22.02</td>
<td>26.26</td>
</tr>
<tr>
<td>The predict result when the growth rate of expected GDP is 13%( (10^4) )</td>
<td>16.59</td>
<td>21.21</td>
<td>27.11</td>
<td>34.65</td>
</tr>
<tr>
<td>Actual value( (10^4) )</td>
<td>17.8</td>
<td>20.6</td>
<td>25.2</td>
<td>31.4</td>
</tr>
<tr>
<td>Error (GDP9%)</td>
<td>-13.03%</td>
<td>-10.39%</td>
<td>12.62%</td>
<td>16.37%</td>
</tr>
<tr>
<td>Error (GDP13%)</td>
<td>-6.8%</td>
<td>2.96%</td>
<td>7.58%</td>
<td>10.35%</td>
</tr>
</tbody>
</table>

We could see that the error of elastic coefficient method is smaller than the error of the grey system method in long-time prediction from table 3. Usually the error of elastic coefficient method is less than 15%, if we can improve the accuracy of the growth rate of expected GDP, the error will less than 10%. The error of elastic coefficient method become bigger as the predicted time become longer.

**The predicted result of combined model**

Got the weight factor by suboptimal combined forecasting method

Standard deviation of grey system model \( \sigma_1 = 21.94 \)

Standard deviation of elastic coefficient method \( \sigma_2 = 13.49 \)

The weighting factor of grey system model:

\[
w_1 = \frac{\sigma - \sigma_1}{\sigma} \times \frac{1}{m-1} = 0.38
\]

The weighting factor of elastic coefficient method:

\[
w_2 = \frac{\sigma - \sigma_2}{\sigma} \times \frac{1}{m-1} = 0.62
\]

New combined model: \( M_3 = 0.38M_1 + 0.62M_2 \)

\( M_1 \) is the predict result of grey theory model; \( M_2 \) is predict result of elastic coefficient method. \( M_1 \) is the predict result of new combined model. The predict result of S province CNG automobile ownership from 2008 to 2011 by new combined model as shown in Table 4 (the growth rate of expected GDP is 9%):

### Table 4 The prediction results of combined model

<table>
<thead>
<tr>
<th>time</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>The predict result( (10^4) )</td>
<td>16.11</td>
<td>20.65</td>
<td>27.67</td>
<td>34.13</td>
</tr>
<tr>
<td>Actual value( (10^4) )</td>
<td>17.8</td>
<td>20.6</td>
<td>25.2</td>
<td>31.4</td>
</tr>
<tr>
<td>error</td>
<td>-9.49%</td>
<td>0.24%</td>
<td>9.80%</td>
<td>8.69%</td>
</tr>
</tbody>
</table>
CONCLUSION

(1) Although grey system model can be used conveniently, it does not apply to the prediction of CNG automobile ownership, especially for medium and long-term prediction. The longer time we predicted, the more errors we will get. It just has certain accuracy when it is used in short-term forecast such as 1-2 years.

(2) Elastic coefficient method can accurately reflect the relationship between the economy development and CNG automobile ownership, if economy development trend can be predicted accurately, it will improve the prediction accuracy greatly, especially in short-term prediction.

(3) Based on the predicted results of combined model, we know the new combined model can synthesize the advantages of the single model, improve the prediction accuracy. And the new combined model does not need a quite big sample size, so that the new model can be used simply and easily.

(4) We should control the prediction time in a small range as far as possible the accuracy is bad in long prediction time, and we suggest that it is advisable to predict time within five years.

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