Oscillating gear transmission with cosine shockwave push rod

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ABSTRACT

Transmission principle is the basis of design theory. This work proposed new oscillating gear transmission of cosine shockwave push rod with arbitrary tooth difference. Tooth profile curve of the shockwave gear is cosine curve or its equidistant curve; profiles at both ends of push-rod oscillating tooth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group: shockwave cam keeps fixed ratio of transmission with oscillating tooth carrier, and oscillating tooth moves along the radial hole of oscillating tooth carrier with circumferential movement. The basic structure and transmission principle are described to prove that the transmission has fixed ratio. Besides, different ways of installation and roller numbers are utilized to obtain twelve transmission ratios, thus establishing theoretical, actual equation of tooth profile for design of such transmission.

Key words: Cosine curve; push-rod oscillating gear transmission; tooth profile; equation; transmission ratio

INTRODUCTION

Oscillating gear transmission with few tooth difference is an active branch of mechanical transmission research. It utilizes a set of oscillating teeth to pass gyratory motion and power between two axes, with compact structure, high transmission efficiency, large transmission ratio and carrying capacity. Due to the innovation in transmission principle and structure, such transmission has good overall performance and prospects. Especially with the emergence of new metal and digital processing technology, oscillating gear transmission has gained rapid development and become an active field in planetary gear transmission. It has a variety of structural forms, such as swinging oscillating teeth transmission [1], push-rod oscillating teeth transmission [2, 3], oscillating rolling teeth transmission [4], oscillating sleeve tooth, etc. [1]. Push-rod oscillating teeth transmission is an important form of oscillating gear transmission. The input mechanism of one-tooth differential oscillating teeth transmission is two symmetrically-arranged eccentrics with complicated structure. The eccentrics always keep certain axial distance, so the subversion moment on the eccentric gear cannot be balanced, thereby affecting dynamic characteristics of the transmission. Two-teeth-differential oscillating teeth transmission has a compact structure, high transmission efficiency, large transmission ratio and carrying capacity. In addition, due to the axis symmetry structure of transmission, static and dynamic self-balance can be achieved in the whole transmission process, avoiding vibration excitation of the machine by theory. It includes cycloidal-cam oscillating gear transmission [5], spatial-cam oscillating gear transmission [6], rolling oscillating gear transmission with cam shockwave [7], push-rod oscillating gear transmission with two teeth differences [9] and swing oscillating gear transmission with two teeth differences [8, 10]. Oscillating gear transmission of two teeth differences was developed from that of one tooth difference, with improved performance but little option in transmission ratio. Oscillating gear transmission with multi teeth differences has more optional transmission ratio and research value. Literature [11] proposed pure rolling oscillating-teeth transmission with arbitrary tooth difference, and profile of shockwave gear and fixed gear were equidistance lines of cosine curve, thus achieving isokinetic conjugate transmission with arbitrary tooth difference. This transmission was simple in design and optimization, and its oscillating teeth utilized rolling bearing components to achieve pure rolling contact transmission. Literature [12] studied the meshing curve of double-cosine oscillating-teeth transmission. Literature [13] studied the three-shockwave roller gear transmission which was actually a special case of cosine shockwave. Literatures [11, 12, and 13] were oscillating gear transmission based on
the structure of cosine shockwave roller. Literatures [14] researched the oscillating gear transmission of hypocycloidal shockwave push rod with arbitrary tooth difference. Literatures [15] researched the shockwave swing oscillating gear transmission—swing oscillating gear transmission of shockwave isometric polygonal profile. Literatures [16] researched swing oscillating gear Transmission with hypocycloid shockwave. Oscillating gear transmission of cosine shockwave push rod has not been reported. In this work, the oscillating gear transmission of cosine shockwave push rod was researched, with a brief introduction of its transmission principles and structures. Gear profile equation of engagement member was derived based on speed transformation and envelope principle, laying the foundation for the design of such transmission. In the new transmission, moving parts such as the input axis are self-balancing speed variator with arbitrary tooth difference. With easy dismounting and large torque, the mechanical transmission is suitable for heavy duty.

TRANSMISSION PRINCIPLE AND STRUCTURE
With the same rotation centre, shockwave gear, internal gear and oscillating tooth group can all be regarded as fixed member, input member or output member, thus achieving different transmission effects. The uniform rotation of input axis will drive shockwave to slew uniformly. Restricted by shockwave gear, internal gear and oscillating tooth group, oscillating tooth gear will also slew with constant speed ratio. Shockwave periodically drives oscillating teeth gear, oscillating teeth gear and internal gear. Fig.1 shows the principle of oscillating tooth transmission based on cosine shockwave push rod. Due to the driving torque, shockwave cam maintain counterclockwise rotation and drive push-rod oscillating teeth (No. 4, 5, 6, 9, 10) to move along the radial holes of tooth carrier in the working area. Meshing force between oscillating teeth and the working segment of internal gear will rotate the oscillating tooth carrier, achieving speed conversion and power output. Meanwhile, under the repulsion of oscillating tooth carrier, oscillating teeth (No. 1, 2, 3, 7, 8) at non-working area will rotate along internal gear and return to the working position in order.

Fig. 1: Oscillating tooth transmission diagram of cosine shockwave push rod

Fig.2 shows the structure of oscillating tooth transmission of cosine shockwave push rod, with two cosine curve waves. The main components include input axis, shockwave cam, push-rod oscillating tooth, internal gear, output axis and body frame.

Fig. 2: Transmission structure
(1. Output axis 2. oscillating tooth carrier 3. internal gear 4. push-rod oscillating tooth 5. shockwave cam 6. input axis)

TRANSMISSION RATIO CALCULATION
In oscillating gear transmission with cosine shockwave push rod, fixed shockwave gear can be regarded as
conversion mechanism without planetary gear. Then an angular velocity is added to the shockwave oscillating gear system, which is equal to the angular velocity of shockwave gear but with contrary direction. According to transmission ratio equation of fixed-axis gear,

$$i_{GK}^H = \frac{\omega_G}{\omega_K^H} = \frac{\omega_G - \omega_H}{\omega_K - \omega_H} = \frac{Z_K}{Z_G}$$

Then we can be obtained from Formula (1):

$$\omega_G = \omega_K i_{GK}^H + \omega_H (1 - i_{GK}^H)$$

where H is the gear shockwave; G the oscillating gear; K the internal gear; i the transmission ratio.

The superscript letters indicate the corresponding fixed member, while subscript letters indicate the relative state of left member to the right member. For example, $i_{GK}^H$ is the specific value between the relative angular velocity of oscillating gear G (compared with shockwave gear H) and that of internal gear K. Superscript letter of $\omega$ indicates corresponding fixed member, and subscript letter is the corresponding member; $Z_H$ is the shockwave number of shockwave gear; $Z_G$ the number of pin rollers in oscillating gear; $Z_K$ the wave number of internal gear. Tab.1 shows the transmission ratio in different forms of installing.

<table>
<thead>
<tr>
<th>Transmission program</th>
<th>Transmission ratio</th>
<th>Rotation direction of master-slave member</th>
<th>Number of oscillating gear roller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal gear fixed</td>
<td>$i_{loc}^k = \frac{Z_G}{Z_K - Z_G}$</td>
<td>$Z_G &gt; Z_K, \text{syntropy}$</td>
<td>$Z_G = Z_K + Z_h$</td>
</tr>
<tr>
<td></td>
<td>$i_{GH}^k = \frac{Z_G - Z_K}{Z_G}$</td>
<td>$Z_G &lt; Z_K, \text{reverse}$</td>
<td>$Z_G = Z_K - Z_h$</td>
</tr>
<tr>
<td>Oscillating gear fixed</td>
<td>$i_{gsk}^k = \frac{Z_G}{Z_K - Z_G}$</td>
<td>$Z_G &gt; Z_K, \text{syntropy}$</td>
<td>$Z_G = Z_K + Z_h$</td>
</tr>
<tr>
<td></td>
<td>$i_{MK}^k = \frac{Z_G - Z_K}{Z_K}$</td>
<td>$Z_G &lt; Z_K, \text{reverse}$</td>
<td>$Z_K = Z_K - Z_h$</td>
</tr>
<tr>
<td>Shockwave gear fixed</td>
<td>$i_{GK}^H = \frac{Z_K}{Z_G}$</td>
<td>$Z_G &gt; Z_K, \text{syntropy}$</td>
<td>$Z_G = Z_K + Z_h$</td>
</tr>
<tr>
<td></td>
<td>$i_{KG}^H = \frac{Z_G}{Z_K}$</td>
<td>$Z_G &lt; Z_K, \text{syntropy}$</td>
<td>$Z_G = Z_K - Z_h$</td>
</tr>
</tbody>
</table>

GEAR PROFILE EQUATION

In cosine oscillating gear transmission with shockwave push rod, the profile of shockwave cam is the equidistant curve of cosine shockwave curve; profiles at two end of push-rod oscillating teeth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group: shockwave cam keeps fixed ratio of transmission with oscillating tooth carrier, and oscillating tooth moves along the radial hole of oscillating tooth carrier with circumferential movement.

Fig.3 shows the profile generation principle of cosine oscillating gear transmission with shockwave push rod, and in the figure, shockwave gear has two cosine curve waves. $xOy$ is regarded as the body-fixed coordinate system of internal gear, coordinate origin O as the geometric center of internal gear. $x'Oy'$ and $x''Oy''$ are fixed coordinate systems of shockwave cam and oscillating tooth carrier, respectively. Three coordinate systems coincide at the initial position of transmission. The upper, lower end centers of push-rod oscillating teeth locate on axis $Ox''$. During the transmission, the track of lower end center of push-rod oscillating teeth is the theoretical profile of shockwave cam (curve 3) in $xOy'$. Regarding oscillating teeth radius $r$ as offset distance, the inner equidistant line is the working profile of shockwave cam (curve 4). In addition, the track of upper end center of push-rod oscillating teeth (curve 2) is the theoretical tooth profile of internal gear in $xOy'. Working tooth profile of the internal gear is outer

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equidistance line (curve 1) of its theoretical tooth profile regarding oscillating teeth radius $r_2$ as the offset distance. $r_1$ and $r_2$ are not necessarily equal to each other.

![Diagram of forming principle diagram of inner gear profile](image)

**Fig. 3:** Forming principle diagram of inner gear profile
(1. Actual internal gear profile 2. Theoretical internal gear profile curve 3. Theoretical profile curve of shockwave gear 4. Actual gear profile of shockwave gear)

### SHOCKWAVE GEAR PROFILE EQUATION

Cosine curve equation of shockwave gear is

$$r = R + h \cos(Z_H \theta)$$

(3)

where $R$ is the radius of base circle; $A$ the amplitude; $Z_H$ the wave number of cosine curve; $\theta \in [0, 2\pi]$ the angular parametric variable.

Rectangular coordinate equation of theoretical shockwave gear profile curve can be obtained by formula (3).

$$\begin{align*}
x_1 &= [R + h \cos(Z_H \theta)] \cos \theta \\
y_1 &= [R + h \cos(Z_H \theta)] \sin \theta
\end{align*}$$

(4)

Shockwave gear profile (curve 3) is the inner equidistant offset of theoretical gear profile, so the actual shockwave gear profile equation [17] can be obtained through mechanical principles.

$$\begin{align*}
x_2 &= x_1 - \frac{r_1}{\sqrt{\left(\frac{dx_1}{d \theta}\right)^2 + \left(\frac{dy_1}{d \theta}\right)^2}} \\
y_2 &= y_1 + \frac{r_1}{\sqrt{\left(\frac{dx_1}{d \theta}\right)^2 + \left(\frac{dy_1}{d \theta}\right)^2}}
\end{align*}$$

(5)

where

$$\begin{align*}
\frac{dx_1}{d \theta} &= -\sin(\theta)[R + h \cos(Z_H \theta)] - hZ_H \sin(Z_H \theta) \cos \theta \\
\frac{dy_1}{d \theta} &= \cos(\theta)[R + h \cos(Z_H \theta)] - hZ_H \sin(Z_H \theta) \sin \theta
\end{align*}$$

### INTERNAL GEAR PROFILE EQUATION

At any moment, the body-fixed rectangular coordinates of shockwave gear $x'Oy'$ and oscillating teeth carrier $x''Oy''$ have turned the angle of $\alpha$ and $\theta$ (relative to the fixed rectangular coordinate system), respectively. Meanwhile, shockwave gear has the same direction of rotation with transmission ring, so $\alpha$ and $\theta$ should meet

$$i_{HG}^K = \frac{\alpha}{\theta} = \frac{Z_G}{Z_H}.$$  From Fig.3, the origin $O$, as well as the upper, lower end center of push-rod oscillating teeth $O_1$
and $O'$ have turned the angle of $\alpha - \theta$ relative to axis $Ox'$. Setting $L = |O'O_1|$, then

$$|OO| = R + h \cos(Z_H(\alpha - \theta))$$

(6)

and

$$|OO_1| = R + h \cos(Z_H(\alpha - \theta)) + L$$

(7)

Theoretical gear profile of internal gear is the track of $O_1'$ in coordinate system $xOy$, and the theoretical profile equation is:

$$x_3 = (R + h \cos(Z_H(\alpha - \theta)) + L) \cos \theta$$

$$y_3 = (R + h \cos(Z_H(\alpha - \theta)) + L) \sin \theta$$

When roller number $Z_G = Z_K + Z_H$, the theoretical profile curve of internal gear in $xOy$ is:

$$x_3 = (R + h \cos(Z_K \theta) + L) \cos \theta$$

$$y_3 = (R + h \cos(Z_K \theta) + L) \sin \theta$$

(8)

When the shockwave gear and transmission ring have the opposite rotation, $\alpha$ and $\theta$ should meet $i_{KG} = \frac{\alpha}{\theta} = \frac{Z_G}{Z_H}$. And the roller number $Z_G = Z_K - Z_H$, thus obtaining the theoretical gear profile of internal gear in $xOy$ as Formula (8).

Therefore, theoretical profile curve of internal gear in $xOy$ is:

$$x_3 = (R + h \cos(Z_K \theta) + L) \cos \theta$$

$$y_3 = (R + h \cos(Z_K \theta) + L) \sin \theta$$

(9)

Actual profile of internal gear (curve 1) is the outer equidistant curve with a theoretical profile offset distance of $r_2$, obtaining the actual gear profile of internal gear [17].

$$x_4 = x_3 + \frac{r_2 \frac{dy_3}{d\theta}}{\sqrt{\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2}}$$

$$y_4 = y_3 - \frac{r_2 \frac{dx_3}{d\theta}}{\sqrt{\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2}}$$

(10)

where

$$\frac{dx_3}{d\theta} = -\sin \theta (R + L + h \cos(Z_K \theta)) - hZ_K \sin(Z_K \theta) \cos(\theta)$$

$$\frac{dy_3}{d\theta} = \cos \theta (R + L + h \cos(Z_K \theta)) - hZ_K \sin(Z_K \theta) \sin(\theta)$$

The MATLAB program of calculation is:
CONCLUSION

In oscillating gear transmission of cosine shockwave push rod with arbitrary tooth difference, tooth profile curve of the shockwave gear is cosine curve or its equidistant curve; profiles at both ends of push-rod oscillating tooth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group; shockwave cam keeps fixed transmission ratio with oscillating tooth carrier, and oscillating tooth moves along the radial hole of oscillating tooth carrier with circumferential movement. It can achieve isokinetic conjugate transmission of arbitrary tooth
difference, with simple design method and optimization. Through calculation in Matlab platform and Solidworks parametric modeling, gear profile equation can create the model for oscillating gear transmission with cosine shockwave push rod. Then Solidworks motion simulation shows no interference, theoretically verifying the above deduction. These deduced formulas can work in any oscillating gear transmission of shockwave push rod. As a new practical gear transmission with application prospects, the transmission is superior to other gear transmissions in the transmission efficiency, lubrication and manufacturing processes.

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REFERENCES