



## Numerical study on dynamic properties of concrete

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### ABSTRACT

*A new mesoscale numerical model is presented adopting finite element method to simulate the dynamic properties of concrete. A new constitutive law based on Kelvin viscoelastic model is utilized to characterize the evolutionary processes of damage under dynamic load. The finite element method is utilized for discretization of components of concrete block, which are aggregate, cement and interfacial transition zone between the aggregate and the mortar. The strengths of these components and viscous parameters are adopted according to Weibull distribution. The dynamic analysis of concrete block is performed by using this model and the calculated result is discussed. The result shows the proposed model can characterize the dynamic properties of concrete well.*

**Key words:** Dynamic properties; Concrete Material; Kelvin viscoelastic model; Weibull distribution

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### INTRODUCTION

The problem of dynamic properties of concrete is of central importance in the research of concrete. In the field of structural mechanics of concrete, the study of dynamic effects is acquiring a prominent role, due to the need for a proper prediction of the mechanical properties in structures, based on the available test data of concrete under dynamic load. The theory and models in dynamic properties for concrete have been developed significantly during the last thirty years and have been documented in an increasing number of publications. Although from a theoretical point of view the field has reached a stage where the developed methodologies are becoming widespread, the numerical analysis for the dynamic properties of concrete is still a complex and difficult task.

However, as is well known, concrete is a very complex material, consisting of aggregates bounded by cement paste, which makes the material behavior of concrete very complicated. Many authors make the analysis for the model to simulate concrete<sup>[1]</sup>. Zhou<sup>[2]</sup> makes a study on mesoscale modeling of concrete tensile failure mechanism at high strain rates. HAO<sup>[3]</sup> proposes numerical model to simulate concrete material properties at high strain rate under direct tension. Haußler-Combe<sup>[4]</sup> proposes a novel approach to model the influence of high strain rates on the behavior of concrete. It is based on gradient continuum damage, where the gradient part is extended with inertia of damage. Gianluca Cusatis<sup>[5]</sup> extends a previously developed meso-scale model for concrete, called the confinement shear lattice model, in order to include the effect of loading rate on concrete strength and fracturing behavior. The rate dependence of concrete behavior is assumed to be caused by two different physical mechanisms. The first is a dependence of the fracture process on the rate of crack opening, and the second is the viscoelastic deformation of the intact (unfractured) cement paste. Discrete microstructural models for concrete, such as random lattice models or random particle models, have established themselves as a powerful and realistic alternative to the no local continuum models for softening damage and fracturing. Because the grade and aggregate content of concrete have a direct influence on its trait of macro-dynamics, a suitable model of the aggregate concrete with high aggregate content is a precondition to successfully execute concrete meso-mechanical simulation. Li et al.<sup>[6,7]</sup> executes a parallel numerical computation of the wet-sieved concrete specimen to verify the model of random convex polyhedron aggregate. Alberto et al.<sup>[8]</sup> collect the most recent developments of the fractal approach by Carpinteri and co-workers to damage and fracture of quasi-brittle materials. Gianluca et al.<sup>[9]</sup> simulate the mechanical behavior of

the mesostructure of concrete by a three-dimensional lattice connecting the centers of aggregate particles. The model can describe not only tensile cracking and continuous fracture but also the nonlinear uniaxial, biaxial, and triaxial response in compression, including the post peak softening and strain localization. Zdenek<sup>[10,11]</sup> presents a new improved microplane constitutive model for concrete, representing the fourth version in the line of microplane models. The proposed constitutive law is characterized as a relation between the normal, volumetric, deviatoric, and shear stresses and strains on planes of various orientations. The strain components on the microplanes are the projections of the continuum strain tensor, and the continuum stresses are obtained from the microplane stress components according to the principle of virtual work. Wei<sup>[12]</sup> establishes a fractal model to simulate the spatial structure of concrete slurry and deduces a formula that describes the relationship among the fractal dimension, the porosity and the pore distribution. Other models, which described the concrete material, mainly contain: grid model<sup>[13,14]</sup>, particle model<sup>[15]</sup>, beam-particle model<sup>[16]</sup>, multi-dimensional virtual internal bond model<sup>[17,18,19]</sup> and radio frequency power amplifier model (RFPA)<sup>[20]</sup>. The basic view of these methods<sup>[21]</sup> is that microcracks and microvoids obeying certain random distribution will affect the propagation of cracks of concrete material, consequently have a significantly effect on the mechanical characteristics of concrete material. These models described the components of concrete in numerical approach, respectively. And through simulating the mechanical characteristics of these components of concrete material, these models could describe the mechanical behaviors of concrete. However, these models could not describe dynamic properties of concrete well in low velocity.

In this paper, a new constitutive law based on Kelvin viscoelastic model is presented adopting FEM to characterize the dynamic properties of concrete. A constitutive relation for concrete is presented adopting the strengths of these components are adopted according to Weibull distribution. The finite element method is utilized for discretization of components of concrete block, which are aggregate, cement and interfacial transition zone between the aggregate and the mortar. Two dimensional dynamic analysis of concrete block is performed by using this model and the calculated result is discussed.

#### THE ESTABLISHMENT OF THE CONSTITUTIVE RELATIONS FOR CONCRETE

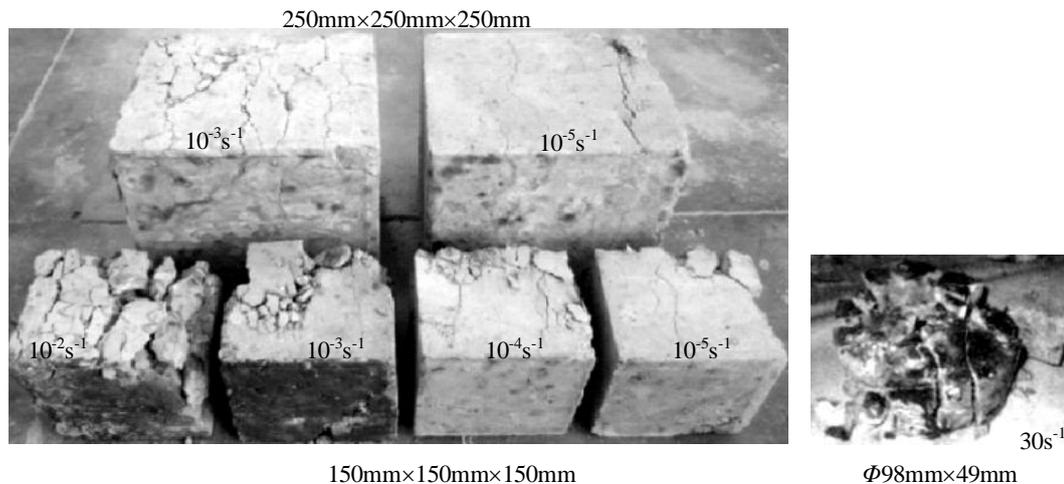


Fig.1: Typical failure modes of concrete specimen by different strain rates

As shown in Fig.1, the similarity between the results of experiments of concrete specimen under static and dynamic compressive loads is columnar damage. And the difference is that more and more dispersed cracks appear with the increase of strain rate. When strain rate is  $30\text{s}^{-1}$ , concrete specimen is full of cracks. When strain rate is less than  $0.02\text{s}^{-1}$ , failure modes are similar between static and dynamic compressive loads. Based on the results of experiments of concrete specimen, the calculated models simulating on the failure of concrete under dynamic loads are established as follow.

The control equation of concrete under dynamic load contains the equations of motion, the relationship between strain and displacement, the stress-strain relationship, the boundary condition and the initial condition. These equations are shown as follow.

The equations of motion is

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \quad (1)$$

The relationship between strain and displacement is

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

The boundary condition is

$$\sigma_{ij}n_j = T_i \tag{3}$$

$$u_i = u_{i0} \tag{4}$$

The initial condition is

$$u_i|_{t=0} = u_{i0}^0 \tag{5}$$

$$\dot{u}_i|_{t=0} = \dot{u}_{i0}^0 \tag{6}$$

where  $\sigma_{ij}$ ,  $\varepsilon_{ij}$  and  $u_i$  are the stress tensor, strain tensor and displacement vector, respectively.  $f_i$ ,  $T_i$  and  $u_{i0}$  are the body force, surface force and definite displacement on the boundary, respectively.  $u_{i0}^0$  and  $\dot{u}_{i0}^0$  are the initial displacement and velocity vector, respectively.  $\rho$  is the density of concrete.

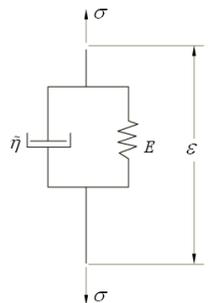


Fig. 2: The behavior of the one-dimensional model for concrete

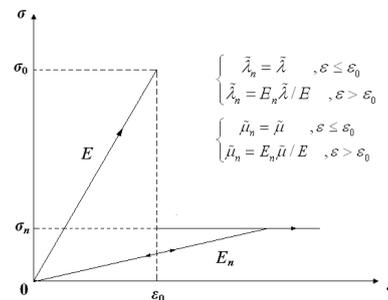


Fig. 3: The relation for  $E$ ,  $\tilde{\lambda}$  and  $\tilde{\mu}$

Because concrete shows the characteristic of rate dependent, it can be represented as Kelvin viscoelastic model. And the behavior of the one-dimensional model for concrete shown in Fig.2 is

$$\sigma = \eta_n \dot{\varepsilon} + E_n \varepsilon \tag{7}$$

where  $E_n$  and  $\eta_n$  are the elastic modulus and the viscous constant for concrete, respectively.  $\sigma$ ,  $\varepsilon$  and  $\dot{\varepsilon}$  are stress, strain and the time rates of change of stress and strain respectively. In fact, it is Kelvin viscoelastic model.

This can be generalized for small straining of an isotropic solid for the two-dimensional model as

$$\sigma_x = \lambda_n \varepsilon_V + 2\mu_n \varepsilon_x + \tilde{\lambda}_n \dot{\varepsilon}_V + 2\tilde{\mu}_n \dot{\varepsilon}_x \tag{8}$$

$$\sigma_y = \lambda_n \varepsilon_V + 2\mu_n \varepsilon_y + \tilde{\lambda}_n \dot{\varepsilon}_V + 2\tilde{\mu}_n \dot{\varepsilon}_y \tag{9}$$

and

$$\sigma_{xy} = \mu_n \gamma_{xy} + \tilde{\mu}_n \dot{\gamma}_{xy} \tag{10}$$

in which

$$\varepsilon_V = \varepsilon_x + \varepsilon_y \tag{11}$$

where  $\lambda$  and  $\mu$  are the Lamé constants,  $\tilde{\lambda}_n$  and  $\tilde{\mu}_n$  are viscous constants for the two-dimensional model of concrete.

Because concrete shows the characteristic of brittle failure, the elastic modulus and viscous constants are reduced when concrete achieves strength. Thus the constitutive relations in different scales for concrete are established as

The constitutive relation of concrete is

$$\begin{cases} E_n = E, \quad \tilde{\lambda}_n = \tilde{\lambda}, \quad \tilde{\mu}_n = \tilde{\mu}, \quad \varepsilon \leq \varepsilon_0 \\ E_n = \sigma_n / \varepsilon, \quad \tilde{\lambda}_n = E_n \tilde{\lambda} / E, \quad \tilde{\mu}_n = E_n \tilde{\mu} / E, \quad \varepsilon > \varepsilon_0 \end{cases} \quad (12)$$

where  $E$ ,  $\tilde{\lambda}$  and  $\tilde{\mu}$  are the initial elastic modulus and viscous parameters for the concrete. And the constitutive relation for concrete is shown in Fig.3.

**MECHANICAL PARAMETERS OF EACH COMPONENT OF CONCRETE BLOCK FOR CONCRETE**

According to certain aggregate gradation of concrete, the random distribution of the locations for aggregates generated are adopted conforming to Fuller curve and the sizes of aggregates are also adopted according to Fuller curve. Concrete block is obtained through the method of randomly dropping aggregates [22]. The randomness of strength is much higher than elastic modulus of concrete. Thus, the strengths of each component of concrete are generated conforming to Weibull distribution but the elastic modulus of each component of concrete are certain. Mechanical parameters of each component of concrete are shown in Tab.1. and Tab.2.

The strengths of each component and viscous parameters of concrete are generated conforming to Weibull distribution on the whole model as

$$f(u) = \frac{m}{u_0} \left( \frac{u}{u_0} \right)^{m-1} \exp\left(-\frac{u}{u_0}\right) \quad (13)$$

where  $u$  is the parameter of Weibull distribution.  $u_0$  is the parameter for the mean value of the strengths of all meshes of concrete model.  $m$  is the shape parameter of the density function of Weibull distribution.

$$E(u) = u_0 \Gamma\left(1 + \frac{1}{m}\right) \quad (14)$$

where  $E(u)$  is the expected value of Weibull distribution. And the relation between the value of  $m$  and  $\Gamma()$  is shown in Tab.3.

**Tab.1. Mechanical parameters of each component of concrete block**

Component	Mean value of compressive strength of strain $\varepsilon_0$	Mean value of tensile strength of strain $\varepsilon_0$	The shape parameter $m$ of Weibull distribution	Elastic modulus /GPa	Poisson's ratio
Aggregate	0.008	0.00042	6	60	0.2
Mortar	0.005	0.00075	3	20	0.25
Interfacial transition zone	0.001	0.0008	1.5	10	0.18

**Tab.2. Mechanical parameters of each component of concrete block**

Component	Mean value of $\tilde{\lambda}$ /MPa s	Mean value of $\tilde{\mu}$ /MPa s	The shape parameter $m$ of Weibull distribution
Mortar	179	156	6
Interfacial transition zone	1.81	1.43	6

**Tab.3. The relation between  $m$  and function  $\Gamma()$**

$m$	1.5	3	6
$\Gamma\left(1 + \frac{1}{m}\right)$	0.9026	0.8930	0.9279

**THE NUMERICAL EXAMPLES FOR CONCRETE TEST BLOCK**

Concrete specimen is adopted according to 150mm×150mm( $L=150$ mm) and the mesh are adopted according to

75×75. And the residual strength  $\sigma_n=0.2\sigma_0$ . The components of concrete are aggregate, mortar and interfacial transition zone between the aggregate and the mortar. And the mechanical parameters of each component of concrete block and affected region of node are shown in Tab.1-3. The dynamic loading strain rate is shown in Fig.4.

Each components of concrete block are shown in Fig.5 to Fig.8. Concrete specimen's constitutive relationship under uniaxial compression and tension are shown in Fig.9. The propagation of cracks of concrete block 1 to 4 respectively under dynamic load ( $\dot{\epsilon}=1\times 10^{-5}$  and  $\dot{\epsilon}=0.01$ ) are shown in Fig.10 to Fig.13. The experiment results refer to literature<sup>[23]</sup>.

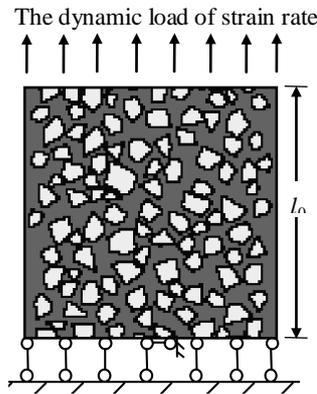


Fig. 4: Meso-concrete geometric model

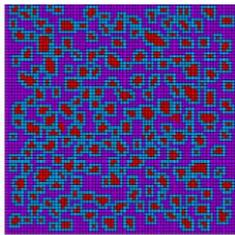


Fig.5: Each component of concrete specimen 1

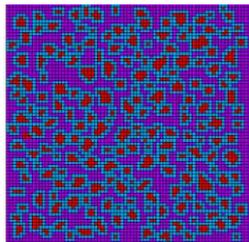


Fig.6: Each component of concrete specimen 2

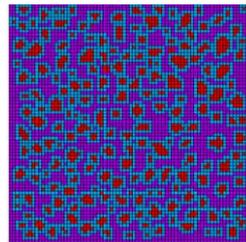


Fig.7: Each component of concrete specimen 3

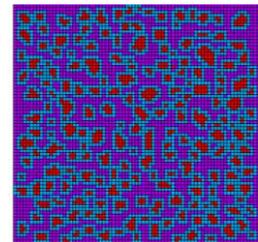


Fig.8: Each component of concrete specimen 4

### RESULTS AND DISCUSSION

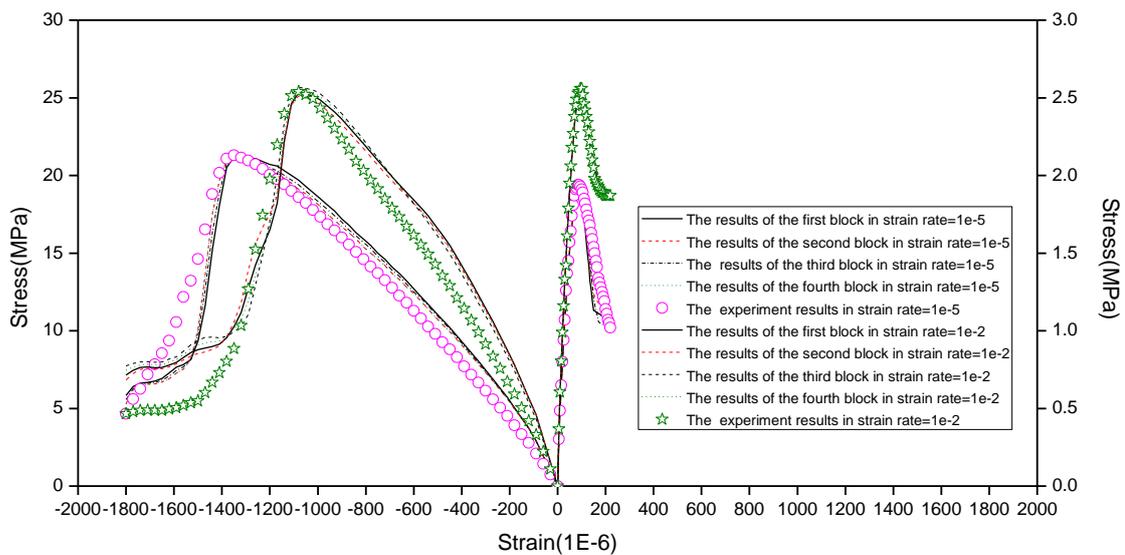


Fig. 9: Concrete specimen's constitutive relationship

Fig.9 shows that the dynamic properties influence significantly both the peak stress and the peak strain. The compressive strength of concrete is increased as the strain rate increases, and the growth range is about 17%. The tensile strength of concrete is increased as the strain rate increases, and the growth range is about 25%. With the increase of strain rate, the compressive peak strain of concrete is decreased but the tensile peak strain of concrete is increased. However, the changes of strength of strain are smaller than strength of stress.

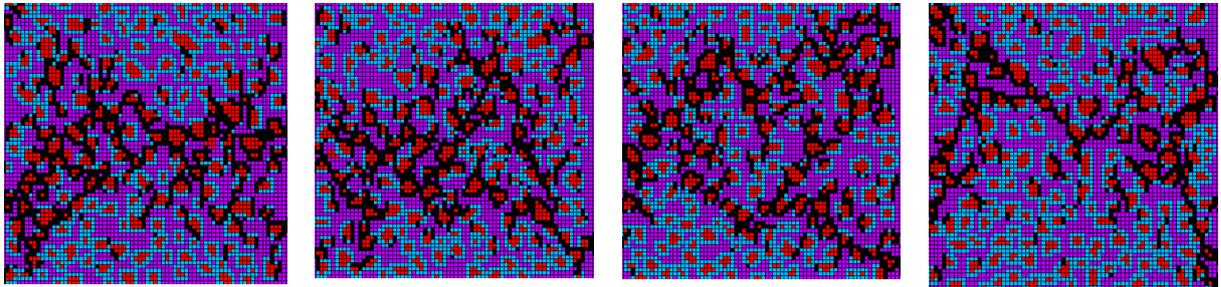


Fig. 10 :The cracks of the concrete specimens under uniaxial compression in  $\dot{\epsilon}=1\times 10^{-5}$

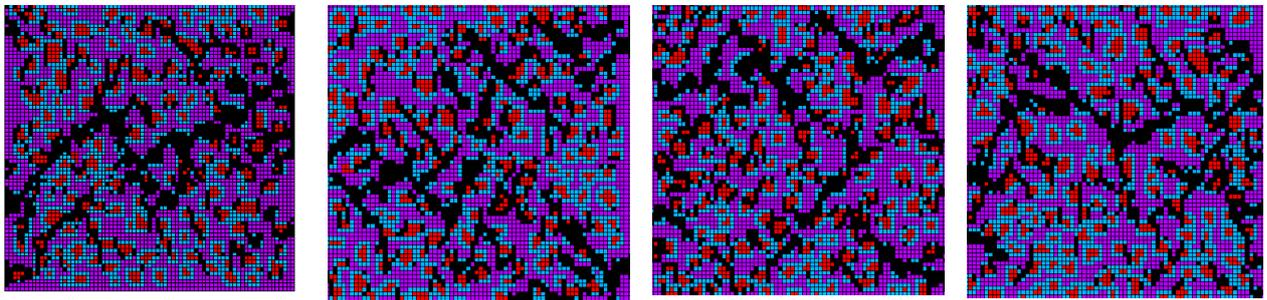


Fig. 11: The cracks of the concrete specimens under uniaxial compression in  $\dot{\epsilon}=0.01$

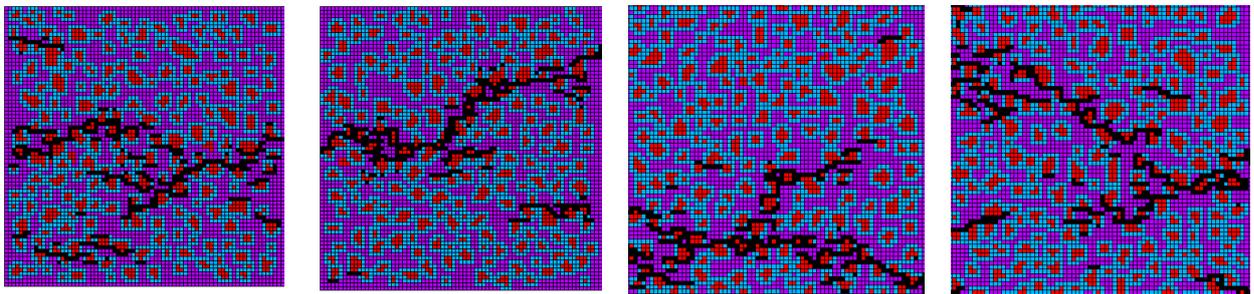


Fig. 12: The cracks of the concrete specimens under uniaxial tension in  $\dot{\epsilon}=1\times 10^{-5}$

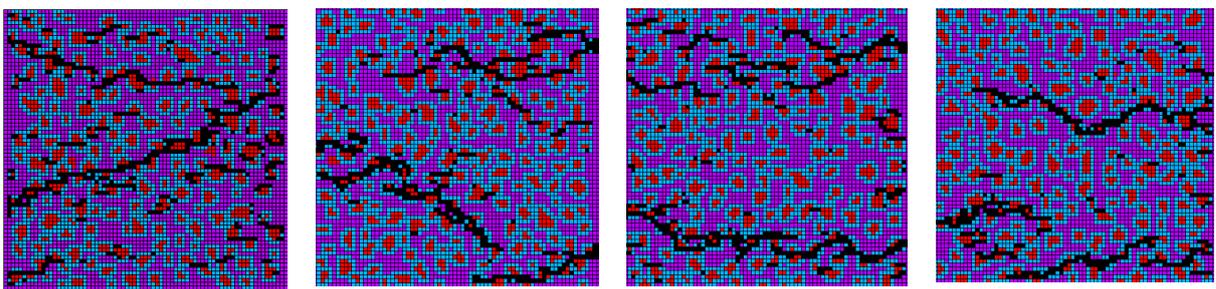


Fig. 13: The cracks of the concrete specimens under uniaxial tension in  $\dot{\epsilon}=0.01$

Figs.9-13 show that the crack patterns are similar in different strain rate. But the more and more cracks generate through aggregate with the increase of strain rate. This is because, with the increase of strain rate, the sticky and viscous properties of mortar and interfacial transition zone between the aggregate and the mortar can bear part of stress. It leads mortar and interfacial transition zone between the aggregate and the mortar is more difficult to destroy. Thus the tensile and compressive strength of concrete are increased as the strain rate increases and the more and more cracks generate through aggregate with the increase of strain rate.

## CONCLUSION

This paper proposed a numerical model adopting finite element method to simulate the dynamic properties of concrete. Firstly, the constitutive law is established based on Kelvin viscoelastic model to characterize the evolutionary processes of damage under dynamic load. The strengths of components of concrete and viscous parameters are adopted according to Weibull distribution. The dynamic analysis for concrete block is performed by using this model. From the analysis of the results, the proposed model can simulate the dynamic properties of concrete well.

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