



Research Article

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Multi-attribute lattice order decision-making model based on set pair reasoning

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ABSTRACT

Defines the basic concept of fuzzy set pair logic with contact number as the true value. Considering the true degree and false degree of set pair true value, respectively we gives calculation rules of fuzzy set pair logic, established the fuzzy set pair reasoning forms in single domain and dual domain, and the inference rules are given. For the reasoning relationship between optional plan and each attribute, we establishment the set of true value contact function, and analyzing its order relation and lattice structure, establishment the lattice order decision-making model based on set pair reasoning, the application example is given in the end.

Key words: Set pair reasoning; partial ordering relation; lattice order decision-making

INTRODUCTION

Professor Guo Yaohuang, using algebra lattice theory, promoted the total order depict of Von Neumann-Morgenstern's rational behavior axiom system to the lattice ordered depict, and then he proposed a new lattice order decision-making theory. In this way, Professor Guo build a lattice order decision-making system of rational behavior. He points out that the lattice order is a kind of suitable partial order structure, it does not require each pair of elements in partially ordered set are comparable, meaning that it does not require meet the conditions of connectivity, it only require that any two elements exist themselves supremum and infimum in partially ordered set. Lattice order is the order structure between total order and partial order. It can reflect the preference of decision maker really, and can meet the transitivity, independence and weak continuity, and it adapt to the condition which the traditional connectivity is missing. The creation of lattice order decision-making theory, fills the blank of lattice theory applied in the decision sciences, at the same time opened a research direction of the decision science. Currently, there is no specific practical method and technology of lattice order decision. Therefore, how to make full use of the relevant mathematical theory, and use for reference other effective calculation method, generalization and practicability research of lattice order decision model become one of the problem that can not to be ignore.

For study uncertain problem, fuzzy mathematic theory provides an effective method. From two-valued logic to fuzzy logic, true value of the proposition is a certain number. It didn't describe the things of continuous development changes and the uncertainty in the process of things change. Therefore, we use the form of connection degree in set pair analysis to description the truth value of proposition in logical relationship. From positive and negative and uncertain three aspects described the characteristics of the proposition, is a reasoning form of more conform the objective reality. So, how to apply it to the lattice order decision-making is one of important research problem.

SET PAIR REASONING

In the traditional two-valued logic, there is only two possible of the true value of proposition, right and wrong, and description of things in the objective world has never been so sure. For the same proposition different people have different views, different people have different views, but there is no absolute right and wrong. Generate of fuzzy

mathematics provides a tool for solve this problem, fuzzy propositions mapped to the closed interval $[0,1]$, it greatly expanded the scope of proposition's true value, and it more objectively reflect the characteristics of objective things. When given proposition's truth value, fuzzy logic just considered true or false of the proposition. In some cases, we are not easy to determine whether a proposition is true or false. In order to more objectively, comprehensively and systematically describe things, we give the basic knowledge of set pair logic.

Definition 1: For a fuzzy proposition, if you can get true, false and uncertainty degrees of the propositions relative to its domain of discourse, you can say this fuzzy proposition is fuzzy set pair proposition, for short FSP proposition, with capital letters A,B,C….

Definition 2: Using numerical value $\mu(A) = a + bi + cj$ measures the true and false and uncertainty of a FSP proposition, so called $\mu(A) = a + bi + cj$ is the proposition's true value. a represents its true degree, b represents its pseudo degree, c represents its uncertainty degree. Obvious, $0 < a, b, c < 1$, it satisfy normalization conditions $a + b + c = 1$.

The true value of FSP proposition is the contact number form, $\mu(A) = a + bi + cj$, a, b, c respectively express the degree of true and false and uncertainty of the proposition.

Definition 3: For a FSP proposition A , if its true degree is $a(A)$, pseudo degree is $c(A)j$, uncertainty degree is $b(A) = 1 - a(A) - c(A)$, and the true value of proposition A is $\mu(A) = a(A) + b(A)i + c(A)j$.

Here, $0 \leq a(A), b(A), c(A) \leq 1$, and it satisfy $a(A) + b(A) + c(A) = 1$.

If A, B is two propositions, relative to the determinate set P , there is $\mu(A) = a(A) + b(A)i + c(A)j$ and $\mu(B) = a(B) + b(B)i + c(B)j$, so we can get the calculation rules of fuzzy set pair logic.

1) Disjunction

$$\begin{aligned} \text{True degree: } a(A \vee B) &= a(A) \vee a(B) = \max\{a(A), a(B)\} \\ c(A \vee B)j &= c(A)j \vee c(B)j \end{aligned}$$

$$\begin{aligned} \text{Pseudo degree: } &= \max\{c(A)j, c(B)j\} \\ &= \min\{c(A), c(B)\}j \end{aligned}$$

$$\text{Uncertainty degree: } b(A \vee B) = 1 - \max\{a(A), a(B)\} - \min\{c(A), c(B)\}$$

Then

$$\begin{aligned} \mu(A \vee B) &= \mu(A) \vee \mu(B) \\ &= \max\{a(A), a(B)\} + (1 - \max\{a(A), a(B)\}) \\ &\quad - \min\{c(A), c(B)\}i + \min\{c(A), c(B)\}j \end{aligned}$$

2) Conjunction

$$\text{True degree: } a(A \wedge B) = a(A) \wedge a(B) = \min\{a(A), a(B)\}$$

Pseudo degree:

$$\begin{aligned} c(A \wedge B)j &= c(A)j \wedge c(B)j \\ &= \min\{c(A)j, c(B)j\} \\ &= \max\{c(A), c(B)\}j \end{aligned}$$

$$\text{Uncertainty degree: } b(A \wedge B) = 1 - \min\{a(A), a(B)\} - \max\{c(A), c(B)\}$$

Then

$$\begin{aligned} \mu(A \wedge B) &= \mu(A) \wedge \mu(B) \\ &= \min\{a(A), a(B)\} + (1 - \min\{a(A), a(B)\}) \\ &\quad - \max\{c(A), c(B)\}i + \max\{c(A), c(B)\}j \end{aligned}$$

3) Negative

$$\text{True degree: } a(\bar{A}) = -a(A)$$

Pseudo degree: $c(\bar{A})j = -c(A)j$

Uncertainty degree: $b(\bar{A}) = 1 - a(A) - c(A) = b(A)$

Then

$$\mu(\bar{A}) = \overline{\mu(A)} = c(A) + b(A)i + a(A)j$$

U is a set of language object, T is a set of conceptual word.

Definition 4: we called the declarative sentence like “ x is a ” is judgment sentence with $x \in U, a \in T$. Denoted by (a) , called $A = \{x \in U | (a) \text{ for } x \text{ is true}\}$ is the true domain of judgment sentence (a) .

Obvious, sometimes we can use the true and false two truth values to describe some sentences, sometimes we can't. At this moment we need to use fuzzy set theory.

Definition 5: In the judgment sentence (a) , if a shows the concept is clear, we called (a) is an ordinary judgment sentence. If a shows the concept is fuzzy, we called (a) is a fuzzy judgment sentence.

Definition 6: For a fuzzy judgment sentence (a) , $\forall x \in U$, “ A : x is a ” is a fuzzy proposition, if we can get true, false and uncertainty degrees of the propositions relative to its domain of discourse, so called this fuzzy judgment sentence is a FSP judgment sentence, meaning FSP proposition, its true value is $\mu(A) = a(A) + b(A)i + c(A)j$

Definition 7: For judgment sentences “ A : x is a ” and “ B : x is b ”, so called the declarative sentence like “if A, B ” is a reasoning sentence, denoted by $(a) \rightarrow (b)$. If $(a), (b)$ is common judgment sentences, and called $(a) \rightarrow (b)$ is a common reasoning sentence. If $(a), (b)$ are fuzzy judgment sentences, and called $(a) \rightarrow (b)$ is a fuzzy reasoning sentence. If $(a), (b)$ are FSP judgment sentences, and called $(a) \rightarrow (b)$ is a FSP reasoning sentence.

For a FSP reasoning sentence $(a) \rightarrow (b)$, if give the variable x a particular object x_0 , “if x_0 is a , x_0 is b ” is a FSP proposition, denoted by $[(a) \rightarrow (b)](x_0)$ (or $[(a) \rightarrow (b)](x_0)$). The proposition's true value is

$$\begin{aligned} \mu(A \rightarrow B) &= \overline{\mu(A)} \vee \mu(B) \\ &= \max\{c(A), a(B)\} + (1 - \max\{c(A), a(B)\}) \\ &\quad - \min\{a(A), c(B)\}i + \min\{a(A), c(B)\}j \end{aligned}$$

FSP proposition respectively from the three aspects of affirmation and negation and uncertain describes the characteristic of proposition, it is a more objective form of reasoning, it is a supplement and improvement for fuzzy reasoning.

Definition 8: For a FSP proposition, there is “if A, B ” (“ A : x is a ” and “ B : x is b ”), if a, b are two different concepts discussed in the same discourse domain $x \in U$, called FSP proposition is FSP proposition in single discourse domain.

Definition 9: For a FSP proposition $A \rightarrow B$, there is “if A, B ” (“ A : x is a ” and “ B : y is b ”), if a, b are two different concepts discussed in two different discourse domain $x \in U, y \in V$, called FSP proposition is FSP proposition in the double discourse domain, denoted by $A_x \rightarrow B_y$.

Definition 10: If the true value of A_x and B_y respectively are $\mu(A_x) = a(A_x) + b(A_x)i + c(A_x)j$ and $\mu(A_y) = a(B_y) + b(B_y)i + c(B_y)j$, in the double discourse domain the true value of FSP proposition is defined as

$$\begin{aligned}
\mu(A_x \rightarrow B_y) &= \mu(\overline{A_x}) \vee (\mu(A_x) \wedge \mu(B_y)) \\
&= \max\{c(A_x), \min\{a(A_x), a(B_y)\}\} \\
&\quad + (1 - \max\{c(A_x), \min\{a(A_x), a(B_y)\}\}) \\
&\quad - \min\{a(A_x), \max\{c(A_x), c(B_y)\}\}i \\
&\quad + \min\{a(A_x), \max\{c(A_x), c(B_y)\}\}j
\end{aligned}$$

LATTICE ORDER DECISION-MAKING METHOD OF SET PAIR REASONING

For the problem of multi-attribute lattice order decision-making, between scheme set $P = \{P_1, P_2, \dots, P_m\}$ (P_1, P_2, \dots, P_m representative m independent scheme) and attribute set $Q = \{Q_1, Q_2, \dots, Q_n\}$ (Q_1, Q_2, \dots, Q_n representative n independent attributes or goals) established the reasoning relation, and using the form of set pair contact number denote the relationship between them, established the lattice structure, and in the end find out the optimal solution.

For the problem of multi-attribute lattice order decision-making, there are scheme set $P = \{P_1, P_2, \dots, P_m\}$ and attribute set $Q = \{Q_1, Q_2, \dots, Q_n\}$, $\forall P_i \in P, \forall Q_j \in Q$, ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$), established the reasoning relation $P_i \rightarrow Q_j$ between P_i and Q_j , and its true value is

$$\begin{aligned}
\mu(P_i \rightarrow Q_j) &= \mu(\overline{P_i}) \vee (\mu(P_i) \wedge \mu(Q_j)) \\
&= \max\{c(P_i), \min\{a(P_i), a(Q_j)\}\} \\
&\quad + (1 - \max\{c(P_i), \min\{a(P_i), a(Q_j)\}\}) \\
&\quad - \min\{a(P_i), \max\{c(P_i), c(Q_j)\}\}i \\
&\quad + \min\{a(P_i), \max\{c(P_i), c(Q_j)\}\}j
\end{aligned}$$

In the decision-making system, there is a certain relationship between each plan and the considering attributes set, expressed in true value contact number: $\mu(P_i \rightarrow Q_1, P_i \rightarrow Q_2, \dots, P_i \rightarrow Q_n)$, ($i = 1, 2, \dots, m$), denote by μ_{P_i} , so

$$\begin{aligned}
\mu_{P_i} &= \mu(P_i \rightarrow Q_1, P_i \rightarrow Q_2, \dots, P_i \rightarrow Q_n) \\
&= (\mu(P_i) \wedge \mu(Q_1)) \vee (\mu(P_i) \wedge \mu(Q_2)) \\
&\quad \vee \dots \vee (\mu(P_i) \wedge \mu(Q_n)) \\
&= \min\{a(P_i), \max\{a(Q_1), a(Q_2), \dots, a(Q_n)\}\} \\
&\quad + (1 - \min\{a(P_i), \max\{a(Q_1), a(Q_2), \dots, a(Q_n)\}\}) \\
&\quad - \max\{c(P_i), \min\{c(Q_1), c(Q_2), \dots, c(Q_n)\}\}i \\
&\quad + \max\{c(P_i), \min\{c(Q_1), c(Q_2), \dots, c(Q_n)\}\}j
\end{aligned}$$

Then we called $U(P) = \{\mu_{P_1}, \mu_{P_2}, \dots, \mu_{P_m}\}$ is true value contact function set.

As a general rule, the true value contact function set $U_P = \{\mu_{P_1}, \mu_{P_2}, \dots, \mu_{P_m}\}$ can be represented as $U_P = \{(a_{P_1}, b_{P_1}, c_{P_1}), (a_{P_2}, b_{P_2}, c_{P_2}), \dots, (a_{P_m}, b_{P_m}, c_{P_m})\}$ among $a_{P_i} + b_{P_i} + c_{P_i} = 1$, ($i = 1, 2, \dots, m$).

Definition 11: U_P is a true value contact function set in the some problem of multi-attribute lattice order decision-making, $\mu_{P_1}, \mu_{P_2} \in U$, and $\mu_{P_1} = a_{P_1} + b_{P_1}i + c_{P_1}j$, $\mu_{P_2} = a_{P_2} + b_{P_2}i + c_{P_2}j$, then:

(1) If $a_{P_1} = a_{P_2}, b_{P_1} = b_{P_2}, c_{P_1} = c_{P_2}$, μ_{P_1} and μ_{P_2} are equivalence, marked as $\mu_{P_1} = \mu_{P_2}$;

- (2) If $a_{P_1} \leq a_{P_2}$, $a_{P_1} + b_{P_1} \leq a_{P_2} + b_{P_2}$, μ_{P_2} takes priority to μ_{P_1} , marked as $\mu_{P_1} \leq \mu_{P_2}$;
 (3) If $a_{P_1} < a_{P_2}$, $a_{P_1} + b_{P_1} < a_{P_2} + b_{P_2}$, μ_{P_2} takes absolute priority to μ_{P_1} , marked as $\mu_{P_1} < \mu_{P_2}$.

Definition 11 shows that precedence relation of the set of true value contact function's has the following characters:

Character 1: reflexivity: for any μ_{P_i} , then $\mu_{P_i} = \mu_{P_i}$;

Character 2: antisymmetry: if $\mu_{P_1} \leq \mu_{P_2}$, $\mu_{P_2} \geq \mu_{P_1}$, then $\mu_{P_1} = \mu_{P_2}$;

Character 3: transitivity: if $\mu_{P_1} \leq \mu_{P_2}$, $\mu_{P_2} \leq \mu_{P_3}$, then $\mu_{P_1} \leq \mu_{P_3}$.

Obviously, set $U(P)$ under the precedence relationship constitutes a poset (U_P, \leq) .

In a poset (U_P, \leq) , suppose μ_{P_1} and μ_{P_2} are poset elements in $U(P)$. If $\mu_{P_1} \leq \mu_{P_2}$ or $\mu_{P_2} \leq \mu_{P_1}$, μ_{P_1} and μ_{P_2} is comparable; otherwise μ_{P_1} and μ_{P_2} are not comparable, marked as $\mu_{P_1} \parallel \mu_{P_2}$.

U is a set of true value contact function, let $\forall \mu_{P_1}, \mu_{P_2} \in U_P$, $\mu_{P_i} = a_{P_i} + b_{P_i}i + c_{P_i}j$, and $\mu_{P_2} = a_{P_2} + b_{P_2}i + c_{P_2}j$, suppose:

(1) Supremum of the set $\{\mu_{P_1}, \mu_{P_2}\}$ is a connection degree expression of $\mu_{P_i} = a_{P_i} + b_{P_i}i + c_{P_i}j$, as $\mu_{P_i} = \sup(\{\mu_{P_1}, \mu_{P_2}\})$, then:

$$\begin{aligned} a_{P_i} &= [\max(a_{P_1} + b_{P_1}, a_{P_2} + b_{P_2}) \\ &\quad + \max(a_{P_1} - b_{P_1}, a_{P_2} - b_{P_2})]/2 \\ b_{P_i} &= [\max(a_{P_1} + b_{P_1}, a_{P_2} + b_{P_2}) \\ &\quad - \max(a_{P_1} - b_{P_1}, a_{P_2} - b_{P_2})]/2 \\ c_{P_i} &= 1 - a_{P_i} - b_{P_i} \end{aligned} \tag{1}$$

(2) Infimum of the set $\{\mu_{P_1}, \mu_{P_2}\}$ is a connection degree expression of $\mu_{P_j} = a_{P_j} + b_{P_j}i + c_{P_j}j$, as $\mu_{P_j} = \inf(\{\mu_{P_1}, \mu_{P_2}\})$, then:

$$\begin{aligned} a_{P_j} &= [\max(a_{P_1} + b_{P_1}, a_{P_2} + b_{P_2}) \\ &\quad + \max(a_{P_1} - b_{P_1}, a_{P_2} - b_{P_2})]/2 \\ b_{P_j} &= [\max(a_{P_1} + b_{P_1}, a_{P_2} + b_{P_2}) \\ &\quad - \max(a_{P_1} - b_{P_1}, a_{P_2} - b_{P_2})]/2 \\ c_{P_j} &= 1 - a_{P_j} - b_{P_j} \end{aligned} \tag{2}$$

Supremum and Infimum of true value set pair connection degree can be calculated with the following rules:

1) Idempotent rate: $\mu_{P_1} = \sup(\{\mu_{P_1}, \mu_{P_1}\})$, $\mu_{P_1} = \inf(\{\mu_{P_1}, \mu_{P_1}\})$

2) Commutation rate: $\sup(\{\mu_{P_1}, \mu_{P_2}\}) = \sup(\{\mu_{P_2}, \mu_{P_1}\})$, $\inf(\{\mu_{P_1}, \mu_{P_2}\}) = \inf(\{\mu_{P_2}, \mu_{P_1}\})$

3) Combine rate:

$$\sup(\{\mu_{P_1}, \mu_{P_2}\}, \{\mu_{P_3}\}) = \sup(\{\mu_{P_1}\}, \{\mu_{P_2}, \mu_{P_3}\})$$

$$\inf(\{\mu_{P_1}, \mu_{P_2}\}, \{\mu_{P_3}\}) = \inf(\{\mu_{P_1}\}, \{\mu_{P_2}, \mu_{P_3}\})$$

Definition 12: (U_P, \leq) is a poset of true value contact function, if any of the two elements have a supremum and an infimum, U_P on partial order " \leq " constitutes a lattice of true value contact function, and " \leq " is a lattice order of set pair true value contact in U_P .

Definition 13: In any lattice of set pair true value contact (U_P, \leq) , $\forall \mu_{P_i}, \mu_{P_j} \in U_P$, $\mu_{P_i} = a_{P_i} + b_{P_i} i + c_{P_i} j$, $\mu_{P_j} = a_{P_j} + b_{P_j} i + c_{P_j} j$, the distance function can be defined as $d(\mu_{P_i}, \mu_{P_j}) = \sqrt{(a_{P_i} - a_{P_j})^2 + (b_{P_i} - b_{P_j})^2}$.

In any lattice of set pair true value contact (U_P, \leq) , $\forall u_i, u_j, u_k \in U$, distance function has the following characters:

Character 4: $d(\mu_{P_i}, \mu_{P_i}) = d(\mu_{P_i}, \mu_{P_i})$

Character 5: $d(\mu_{P_i}, \mu_{P_j}) = d(\mu_{P_j}, \mu_{P_i})$

Character 6: $d(\mu_{P_i}, \mu_{P_j}) + d(\mu_{P_j}, \mu_{P_k}) \leq d(\mu_{P_i}, \mu_{P_k})$

For a decision problem, there is only a limited kind of incompatible possible state, marked as state set or attribute set, then $Q = \{Q_1, Q_2, Q_3, \dots, Q_n\}$. At the same time, there is only a limited possibility of action, then $P = \{P_1, P_2, P_3, \dots, P_m\}$, marked as action set or alternative plan set. Using $\mu(P_i \rightarrow Q_j)$ for the reasoning relationship between alternative plan set and each attribute. Decision-makers make the decision according to certain decision rules or choosing appropriate decision-making model, then selecting the most appropriate decision.

First step: establishing the reasoning relationship of decision-making system. According to the expert decision-making system to establish the reasoning relationship $\mu(P_i \rightarrow Q_j)$ between alternative plan set and each attribute.

Second Step: according to decision matrix to structure the set of true value contact number.

Firstly, according to the decision matrix, we can get the reasoning relationship $\mu(P_i \rightarrow Q_j), i = 1, 2, \dots, m; j = 1, 2, \dots, n$ between each alternative plan and each attribute. Secondly, according to the logic operation formula, we can determine the reasoning relationship $\mu(P_i \rightarrow Q_1, P_i \rightarrow Q_2, \dots, P_i \rightarrow Q_n), (i = 1, 2, \dots, m)$ between each plan and the existing attributes. Finally, it is output the set of true value contact number $U_P = \{\mu_{P_1}, \mu_{P_2}, \dots, \mu_{P_m}\}$ of the decision problem.

The third step: structuring the lattice structure.

We compared with any two elements in the set of true value contact number. According to the definition 11:

If any two elements are comparison, that is to say there is precedence relationship. We can select the optimal plan set.

If there are two elements are not comparison, according to the formula (1) and (2) calculate supremum and infimum of the two elements, and continue turn its supremum and infimum compared with other elements, until we find out the top element and bottom element of the partially ordered set.

If any two elements are not comparison, according to the formula (1) and (2) calculate supremum and infimum of the two elements, and continue compare the precedence relation of these supremum and infimum, until we find out the top element and bottom element of the partially ordered set.

The fourth step: drawing the Hasse diagram.

The fifth step: determine the optimal solution.

If Hasse diagram is a straight chain, the largest is the optimal solution of the model.

If Hasse diagram is not a straight chain:

If the top element is the element in the set of true value contact number, this element represents the alternative plan

is the optimal plan.

If the top element is not the element in the set of true value contact number, we select the next layer element. If there is only one element belongs to the set of true value contact number, then this element represents the alternative plan is the optimal plan. If there are two or more than two elements belongs to the set of true value contact number, then according to the definition 3.2.2 calculate the distance between these elements and their up layer elements, the smallest distance is the optimal plan. If there is no elements belongs to the set of true value contact number, looking for the next under layer elements, methods and so on, finding the optimal plan in the end.

SAMPLE ANALYSIS

A factory prepared to produce a batch of new products. Planners have five different alternative plans. Considering four decision attribute: 1) the large production, 2) high quality, 3) the low production cost, 4) less raw material loss. There is the set of decision plan $P = \{P_1, P_2, P_3, P_4, P_5\}$ and the set of decision attribute $Q = \{Q_1, Q_2, Q_3, Q_4\}$, then Q_1 :the large output of products, Q_2 :the high quality of products, Q_3 :the low production cost of products, Q_4 :the less raw material loss of products. On the basis of the evaluate analysis of panel, the set pair contact number of each plan and each attribute is given, it is shown in the table 1.

According to the above table:

$$\begin{aligned}\mu(P_1 \rightarrow Q_1) &= 0.66 + 0.19i + 0.15j, \mu(P_1 \rightarrow Q_2) = 0.42 + 0.31i + 0.27j, \\ \mu(P_1 \rightarrow Q_3) &= 0.48 + 0.15i + 0.37j, \mu(P_1 \rightarrow Q_4) = 0.5 + 0.32i + 0.18j \\ \mu(P_2 \rightarrow Q_1) &= 0.57 + 0.3i + 0.13j, \mu(P_2 \rightarrow Q_2) = 0.42 + 0.28i + 0.3j \\ \mu(P_2 \rightarrow Q_3) &= 0.61 + 0.17i + 0.22j, \mu(P_2 \rightarrow Q_4) = 0.45 + 0.33i + 0.22j \\ \mu(P_3 \rightarrow Q_1) &= 0.65 + 0.2i + 0.15j, \mu(P_3 \rightarrow Q_2) = 0.4 + 0.32i + 0.28j \\ \mu(P_3 \rightarrow Q_3) &= 0.56 + 0.13i + 0.31j, \mu(P_3 \rightarrow Q_4) = 0.43 + 0.34i + 0.23j \\ \mu(P_4 \rightarrow Q_1) &= 0.49 + 0.23i + 0.28j, \mu(P_4 \rightarrow Q_2) = 0.53 + 0.22i + 0.25j \\ \mu(P_4 \rightarrow Q_3) &= 0.65 + 0.2i + 0.15j, \mu(P_4 \rightarrow Q_4) = 0.53 + 0.28i + 0.19j \\ \mu(P_5 \rightarrow Q_1) &= 0.63 + 0.13i + 0.24j, \mu(P_5 \rightarrow Q_2) = 0.55 + 0.22i + 0.23j \\ \mu(P_5 \rightarrow Q_3) &= 0.47 + 0.22i + 0.31j, \mu(P_5 \rightarrow Q_4) = 0.54 + 0.32i + 0.14j\end{aligned}$$

TABLE 1. Known data

	Q_1	Q_2	Q_3	Q_4
P_1	$0.66+0.19i+0.15j$	$0.42+0.31i+0.27j$	$0.48+0.15i+0.37j$	$0.5+0.32i+0.18j$
P_2	$0.57+0.3i+0.13j$	$0.42+0.28i+0.3j$	$0.61+0.17i+0.22j$	$0.45+0.33i+0.22j$
P_3	$0.65+0.2i+0.15j$	$0.4+0.32i+0.28j$	$0.56+0.13i+0.31j$	$0.43+0.34i+0.23j$
P_4	$0.49+0.23i+0.28j$	$0.53+0.22i+0.25j$	$0.65+0.2i+0.15j$	$0.53+0.28i+0.19j$
P_5	$0.63+0.13i+0.24j$	$0.55+0.22i+0.23j$	$0.47+0.22i+0.31j$	$0.54+0.32i+0.14j$

According to the formula:

$$\begin{aligned}\mu_{P_i} &= \mu(P_i \rightarrow Q_1, P_i \rightarrow Q_2, \dots, P_i \rightarrow Q_n) \\ &= (\mu(P_i) \wedge \mu(Q_1)) \vee (\mu(P_i) \wedge \mu(Q_2)) \\ &\quad \vee \dots \vee (\mu(P_i) \wedge \mu(Q_n)) \\ &= \mu(A) \wedge (\mu(B_1) \vee \mu(B_2) \vee \dots \vee \mu(B_l)) \\ &= \min\{a(P_i), \max\{a(Q_1), a(Q_2), \dots, a(Q_n)\}\} \\ &\quad + (1 - \min\{a(P_i), \max\{a(Q_1), a(Q_2), \dots, a(Q_n)\}\})i \\ &\quad - \max\{c(P_i), \min\{c(Q_1), c(Q_2), \dots, c(Q_n)\}\}j \\ &\quad + \max\{c(P_i), \min\{c(Q_1), c(Q_2), \dots, c(Q_n)\}\}j\end{aligned}$$

We can get the true value contact number of each plan.

$$\mu_{P_1} = 0.66 + 0.19i + 0.15j$$

$$\mu_{P_2} = 0.61 + 0.26i + 0.13j$$

$$\mu_{P_3} = 0.65 + 0.2i + 0.15j$$

$$\mu_{P_4} = 0.65 + 0.2i + 0.15j$$

$$\mu_{P_5} = 0.63 + 0.23i + 0.14j$$

Then we can get the set of true value contact function $U_P = \{\mu_{P_1}, \mu_{P_2}, \mu_{P_3}, \mu_{P_4}, \mu_{P_5}\}$ of the decision problem.

According to the above decision theory:

- 1) $\mu_{P_3} = \mu_{P_4}$, μ_{P_3} and μ_{P_4} are equivalence;
- 2) $\mu_{P_3} \leq \mu_{P_1}$, μ_{P_1} takes priority to μ_{P_3} ;
- 3) The supremum of μ_{P_1} and μ_{P_5} is $\mu_{S_1} = \sup\{\mu_{P_1}, \mu_{P_5}\} = 0.665 + 0.195i + 0.14j$;
- 4) The supremum of μ_{P_2} and μ_{P_5} is $\mu_{S_2} = \sup\{\mu_{P_2}, \mu_{P_5}\} = 0.635 + 0.235i + 0.13j$;
- 5) The supremum of μ_{S_1} and μ_{S_2} is $\mu_{S_0} = \sup\{\mu_{S_1}, \mu_{S_2}\} = 0.67 + 0.2i + 0.13j$;
- 6) The infimum of μ_{P_3} and μ_{P_5} is $\mu_{T_1} = \inf\{\mu_{P_3}, \mu_{P_5}\} = 0.625 + 0.225i + 0.15j$;
- 7) The infimum of μ_{P_2} and μ_{P_5} is $\mu_{T_2} = \inf\{\mu_{P_2}, \mu_{P_5}\} = 0.605 + 0.255i + 0.14j$;
- 8) The infimum of μ_{T_1} and μ_{T_2} is $\mu_{T_0} = \inf\{\mu_{T_1}, \mu_{T_2}\} = 0.6 + 0.25i + 0.15j$;

Then partial order structure of the set of true value contact function $U_P = \{\mu_{P_1}, \mu_{P_2}, \mu_{P_3}, \mu_{P_4}, \mu_{P_5}\}$ of the Hasse diagram is figure 1.

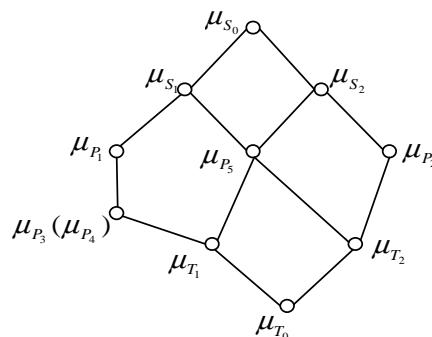


Fig1. Hasse diagram of decisions results

Finally, the analysis result is as follows:

- 1) $d(\mu_{P_1}, \mu_{S_1}) \leq d(\mu_{P_3}, \mu_{S_1})$, μ_{P_1} takes priority to μ_{P_3} ;
- 2) $d(\mu_{P_1}, \mu_{S_0}) \leq d(\mu_{P_2}, \mu_{S_0})$, μ_{P_1} takes priority to μ_{P_2} .

Through the analysis above, P_1 is the optimal solution of the model.

CONCLUSION

Considering the reasoning relations under the fuzzy conditions, set pair coefficients form of real-domain expressions and inference rules of operation are given, and it applied the decision-making system, constructed the truth value linking function set and give the principles and methods of constructing lattice structure, also build model based on set pair reasoning of lattice order decision-making. In the decision-making system, the research of the set pair coefficients of lattice ordered provides a new method for solving the problem of multi-attribute decision-making, also for the development and improvement of lattice order decision theory.

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