Modeling and simulation of competition-cooperation and neutrality relations in SDN enterprises

Deyi Tai¹², Fuyuan Xu¹ and Wei Hu¹

¹School of Management, University of Shanghai for Science & Technology, China
²Key Laboratory of Machine Vision and Intelligence Control Technology, Hefei University, China

ABSTRACT

Besides competition and cooperation, there also have neutrality relations, which is noncompetition and noncooperation, exists in Supply and Demand Network of Enterprises with Multi-function and Opening Characteristics (SDN). To reveal the evolution of cooperation, competition and neutrality relations among enterprises in SDN, an evolutionary model on node relations in SDN was build to solve the problem of real cooperative strategy selection. Base on evolutionary game theory, an enterprises relations evolutionary game model was established, and analyzed with numerical simulations in MATLAB. The results show that transferable and non-transferable incomes determined the choice of cooperation, competition and neutrality strategy. Opportunistic behavior and malicious competition exited in SDN, which reduced the overall revenue and destroyed the stability of SDN. Enhanced supervision and punishment in SDN will improve enterprises’ relations from neutrality to cooperation.

Key words: SDN; enterprises relations; cooperation; competition; neutrality; evolutionary game

INTRODUCTION

Along with economic globalization and demands diversification, enterprises are required to provide personalized, short cycle, lower cost and high quality product and service, which make many enterprises survived difficultly by virtue of their own resource. For this reason, academic community put forward many business ope ration pattern shifting from adversarial competition mode into cooperation mode. The introduction of supply and demand network of enterprise with multifunctional and opening characteristics (SDN) is committed to promot the formation of that shift.

SDN[1] refers to a multifunction and full-open supply and demand network in a global scope, formed among relevant enterprises by their interactive "Supply and Demand Flow", with the aim of access to global resource, manufacture and marketing. The supply and demand flows refer to information, material, funds, technology, human resource, management, etc. Breaking through the cooperation mode in traditional Supply Chain, which mainly focuses on "products", SDN emphasizes interactive cooperation of products, information, technology (knowledge), funds, management, corporate culture, facilities and other resources. Driven by supply and demand information, the dynamic cooperation of enterprises with equal position is established between any nodes in the network. The federal concept of "collaborate inside chain, compete outside chain" been discarded. Contrastingly, the concept of "full-open and win-win cooperation" is proposed. It is encouraged to improve the competitiveness through cooperation within enterprises, and to gain cooperation through competition. The overall robustness of SDN and the system's benefits will thereby be maximized. The comprehensive cooperative partners of SDN involve enterprises, business alliances, natural person, and even the competitors as the collaborative partner.

Generally, there have three relations exist in enterprises of SDN, cooperation, neutrality and competition. SDN emphasize expanding cooperation and decreasing competition to realize the system integration effect of 1+1 greater
than 2. Compare to cooperative relation, the neutral relation is far greater in SDN, since most of the enterprises in SDN neither competition nor cooperation. Cooperation and competition are short-term dynamic relations in SDN. How to scientifically define neutral relation besides competitive and cooperative relations, effectively promote the transform of neutrality to cooperation not to competition, will tremendously increase the efficiency and stability of SDN.

At present, most researches of business relations focused on the interior and outside cooperation and competition of Supply Chain [2-4], Virtual Enterprise [5], Cooperative R&D [6,7], Industry Alliance[8,9], which all ignored the neutral relation.


This paper takes advantage of the evolutionary theory of bounded rationality, to research the relations’ evolutionary mechanism of cooperation, competition and neutrality in SDN. Numerical simulation shows the impact of incomes on the relationships among cooperation, competition and neutrality. Also the impact of supervision and punishment, which promote the neutrality to cooperation, was discussed.

DESCRIPTION AND HYPOTHESIS OF THE PROBLEM
To simplify the problem, we divided the notes of SDN into Resource Supplier (RS) and Resource Demander (RD), which can exchange in transactions. Assuming that there only two strategies can be select for both RS and RD, the one is positive cooperation and the other is passive wait. Positive cooperation indicate to improve the comprehensive capability of SDN nodes, partners obtained incomes by investing cooperative cost. Passive wait indicate that the partner want share the cooperative incomes by taking opportunistic behavior, not investing cooperative cost. Both RD and RS select the strategy according to the income, including transferable and can’t transferable profit, getting from the cooperation. In terms of s elected strategy, there formed three relations of cooperation, competition and neutrality in SDN.

(1) Both partners take positive cooperative strategy formed cooperation relation in RS and RD. Beside the incomes obtained in neutrality, they will get the cooperative spill incomes when pay the cooperative cost.
(2) Both partners take passive wait strategy formed neutrality relationships in RS and RD. Since nobody will invest the cooperative cost, the cooperation relation can’t be formed, partners only get neutrality revenue.
(3) One partner take positive strategy and the other take passive wait strategy formed competitive relations. The passive wait partner share the spill incomes which was created by the other partner’s investment.

Assuming both RS and RD bounded rationality, status equal, the choice of cooperation, competition or neutrality strategy only according to the partner’s income. Due to asymmetric information, market uncertainties and other factors, strategic choice is not a one-time game. The optimal strategy can not been found initially, need to learn continuously and observe the behavior of the process, then determine their strategy.

Consistent with the actual situation and simplify the analysis, Table 1 gives the payoffs of RS and RD. A1, B1, refer to the cooperative payoff of RS and RD when they take cooperative strategy. A2, B2 refer to the positive payoff when the partners take passive strategy. A3, B3 refer to the passive strategy payoff when the partners take positive strategy. A4, B4 refer the payoffs both the partners take passive strategy.

Usually A1> A2, B1> B2, A3>A3, B3> B4. Competitive strategy will always lead to low the incomes of the partners’. The game matrix payoff of supplier and demander is shown in Table 1.
ANALYSIS OF EVOLUTIONARY MODE

3.1 Modeling

Assuming at time t, the ratio taken active cooperative strategy in RS is \( x \), where \( x \in [0,1] \), taken passive waiting strategy is \( 1-x \), 1-\( x \in [0,1] \), the RD taken active cooperative strategy is \( y \), \( y \in [0,1] \), taken passive waiting strategy is \( 1-y \), \( 1-y \in [0,1] \). The average incomes of RS take active cooperative strategy is \( u_x \).

\[
u_x = yA_1 + (1-y)A_2 \quad (1)
\]

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Supplier} & \textbf{Demande} & \\
\hline
\textbf{Positive (x)} & \textbf{Cooperative Strategy} & \textbf{Positive (y)} & \textbf{Passive (1-y)} \\
\hline
\textbf{Positive (y)} & A1, B1 & A2, B3 \\
\hline
\textbf{Passive (1-x)} & A3, B2 & A4, B4 \\
\hline
\end{tabular}
\caption{Payoffs Matrix of RD and RS}
\end{table}

The average incomes of RS taken passive wait strategy is \( u_{1-x} \).

\[
u_{1-x} = yA_3 + (1-y)A_4 \quad (2)
\]

The average incomes of both strategies is \( u_A \).

\[
u_A = xu_x + (1-x)u_{1-x} \quad (3)
\]

The replicator dynamic equation of RS is:

\[
dx = x(u_x - u_A) \quad (4)
\]

Substitute (1), (2), (3) into (4), we can get:

\[
dx = x(1-x)(u_x - u_{1-x}) \]
\[
= x(1-x)\left[ A_2 - A_4 + y(A_1 + A_4 - A_2 - A_3) \right] \quad (5)
\]

Similarly, the replicator dynamic equation of RD is:

\[
dy = y(1-y)(u_y - u_{1-y}) \]
\[
= y(1-y)\left[ B_2 - B_4 + x(B_1 + B_4 - B_2 - B_3) \right] \quad (6)
\]

Combine (5) and (6), we can get the differential kinetic equation of SDN relations as follow:

\[
\begin{align*}
\frac{dx}{dt} &= x(1-x)[A_2 - A_4 + y(A_1 + A_4 - A_2 - A_3)] \\
\frac{dy}{dt} &= y(1-y)[B_2 - B_4 + x(B_1 + B_4 - B_2 - B_3)]
\end{align*} \quad (7)
\]

3.2 Model analysis

Set \( dx = 0, dy = 0 \), solving differential (7), we can get 5 equilibrium points. \((0,0), (0,1), (1,0), (1,1)\) and \((x_0, y_0)\).

\[
(x_0, y_0) = \left( \frac{B_4 - B_2}{B_1 + B_4 - B_2 - B_3}, \frac{A_1 - A_2}{A_1 + A_4 - A_2 - A_3} \right)
\]

According to the theory of differential equation, the stability of the equilibrium point is determined by the signs of determinant and the trace of the system’s JACOBIAN Matrix. The JACOBIAN Matrix of (7) is:

\[
J = \begin{bmatrix}
\frac{\partial (dx)}{\partial x} & \frac{\partial (dx)}{\partial y} \\
\frac{\partial (dy)}{\partial x} & \frac{\partial (dy)}{\partial y}
\end{bmatrix} \quad (8)
\]

\[
\frac{\partial (dx)}{\partial x} = (1-2x)[A_2 - A_4 + y(A_1 + A_4 - A_2 - A_3)] \quad (9)
\]

\[
\frac{\partial (dx)}{\partial y} = x(1-x)(A_1 + A_4 - A_2 - A_3) \quad (10)
\]

\[
\frac{\partial (dy)}{\partial x} = y(1-y)(B_1 + B_4 - B_2 - B_3) \quad (11)
\]
\[
\frac{\partial(dy)}{\partial y} = (1 - 2y)[B_2 - B_4 + x(B_1 + B_4 - B_2 - B_3)]
\] (12)

According to (8), both the determinant and the trace of 5 equilibrium points show in table 2. \(\text{Det} J\) is the value of Determinant \(J\) and \(\text{Tr} J\) is the trace of the Matrix.

<table>
<thead>
<tr>
<th>(x_0, y_0)</th>
<th>(\text{Det} J)</th>
<th>(\text{Tr} J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>((A_2 - A_4)(B_2 - B_4))</td>
<td>((A_2 - A_4)(B_2 - B_4))</td>
</tr>
<tr>
<td>(0,1)</td>
<td>(-(A_1 - A_4)(B_2 - B_4))</td>
<td>((A_1 - A_4)(B_2 - B_4))</td>
</tr>
<tr>
<td>(1,0)</td>
<td>(-(A_2 - A_4)(B_1 - B_4))</td>
<td>((B_1 - B_4)(A_2 - A_4))</td>
</tr>
<tr>
<td>(1,1)</td>
<td>((A_1 - A_4)(B_1 - B_3))</td>
<td>(-(A_1 - A_4)(B_1 - B_3))</td>
</tr>
</tbody>
</table>

Analysis of the 5 possible equilibrium points’ stability as follow:
(1) When \(A_1 - A_3 > 0, B_1 - B_3 > 0, B_2 - B_4 < 0, A_2 - A_4 < 0\).

Substitute the setting parameter into table2, the value and trace of equilibrium points shown in table3. At point (0,1) and (1,0) \(\text{Det} J > 0\) and \(\text{Tr} J > 0\), so they are instabiliy points. At point \((x_0, y_0)\) \(\text{Tr} J = 0\), so this point is saddle point.

At points (0,0) and (1,1) \(\text{Det} J > 0, \text{Tr} J < 0\), the two points is the equilibrium points of system.

<table>
<thead>
<tr>
<th>(x_0, y_0)</th>
<th>(\text{Det} J)</th>
<th>(\text{Tr} J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(0,1)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(1,0)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(1,1)</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Active strategy can earn more income when the partners take active strategy. When the partners take passive strategy, taking passive can earn more incomes by avoiding invest cost. Evolutionary phase Figure of the system shows in Fig.1. Under strict supervision opportunistic behavior punishment, there have evolutionary stability strategies, cooperation and neutrality, which determined not only by the initial strategy ratio but also by cooperative incomes.
(2) $A_{1}-A_{3}>0$, $B_{1}-B_{3}>0$, $A_{2}-A_{4}>0$, $B_{2}-B_{4}>0$

Under the setting, $\frac{B_{1} - B_{2}}{B_{1} + B_{4} - B_{2} - B_{3}} > 1$ and $\frac{A_{4} - A_{2}}{A_{1} + A_{4} - A_{2} - A_{3}} > 1$, points $(x_{0}, y_{0})$ is insignificance. The only evolutionary stability points is $(1,1)$, other three points are instability points.

Fig. 2 indicate cooperation is the only stability strategy under this setting. No matter what strategy the other side taken, take active strategy can always obtain greater incomes, which is an ideal SDN enterprise collaboration can induce the neutrality and competition partners transform to cooperation.

(3) When $A_{1}-A_{3}<0$, $B_{1}-B_{3}<0$, $B_{2}-B_{4}<0$, $A_{2}-A_{4}<0$

Under the setting, points $(x_{0}, y_{0})$ is insignificance. The only evolutionary stability strategy (ESS) is $(0,0)$, passive wait, other three points are instability points.

Fig. 3 indicate that no matter what strategy the other taken, passive strategy can always get better returns. When the market significant uncertainty, passively wait will be the best choice at this time, actively cooperate disappeared, there is no relations of cooperation and competition, SDN turn into hibernation.
(4) When $A_1-A_3<0$, $B_1-B_3<0$, $B_2-B_4>0$, $A_2-A_4>0$

Ditto analysis, under this setting, $(0,0)$ and $(1,1)$ are instability points, points $(x_0, y_0)$ is saddle point, points $(0,1)$ and $(1,0)$ are the evolutionary stability points.

Fig. 4 indicates that when the one taken active strategy, the best choice of the partners is take passive strategy. Both passive wait strategy and active cooperative strategy coexist in the market, which full of competition. No cooperation exists in SND which will led to SDN dismiss. This equilibrium is harmful to the stability of SDN, which should be avoided.

3.3 Analysis of influencing Factors

Full cooperation and complete neutral relations in the real world is just an ideal model, unfair competition is a bad status which reduced overall system revenue which should be avoid by adjusting and regulating. The more condition is between neutrality and cooperation in SDN, thus establishment of effective supervision and punishment mechanism which change the incomes expectations, will conducive to neutral relation convert to cooperative relation.
Assume in the process of cooperation, one side takes passive waiting strategy and make profit by taking opportunistic behavior, which will pay the penalty $C$, $C > 0$. Then the value of $A3$ and $B3$ should subtract a positive punishment $C$. The value of $D$ changes to $(A4 - A2) / (B4 - B2 - B3 + C)$. Because $A1-A3>0$, so $B1-B3>0$, $B2-B4<0$, $A2-A4<0$, at this moment, point $D$ will approach to point $O$, just as Fig.5 show. Compare Fig.1 to Fig.5 we can get that the possibility of neutral relations convert to cooperation increased, which indicate that the supervision and punishment in SDN promote the neutral relation convert to cooperative relation.

NUMERICAL SIMULATIONS AND ANALYSIS

4.1 Numerical simulation

To analysis of the evolution of cooperative, neutral and competitive relations, we make numerical simulation in MATLAB under the assumptions of symmetry game and dissymmetry game. Symmetry game indicates that RS and RD have same payoff. Dissymmetry game indicates that RS and RD have different payoff.

(1) When $A1-A3>0$, $B1-B3>0$, $B2-B4<0$, $A2-A4<0$, neutrality and cooperation coexist in SDN.

1) Symmetry game

Set $A1-A3=B1-B3$, $A2-A4=B2-B4$, assume $A1=B1=6$, $A2=B2=2$, $A3=B3=4$, $A4=B4=3$, set multi initial value of $x$ as 0.1, 0.3, 0.5, 0.7, 0.9. The simulation result show as Fig.6.

2) Dissymmetrical game

Set $A1-A3 \neq B1-B3$, $A2-A4 \neq B2-B4$. Assume $A1=6$, $A2=2$, $A3=4$, $A4=3$, $B1=10$, $B2=4$, $B3=6$, $B4=8$, multi initial value of $(x,y)$ as $[0.2 \ 0.8]$ , $[0.4 \ 0.6]$, $[0.3 \ 0.3]$, $[0.7 \ 0.4]$, $[0.9 \ 0.2]$, $[0.1 \ 0.7]$, $[0.1 \ 0.3]$, $[0.1 \ 0.4]$, $[0.1 \ 0.8]$, $[0.1 \ 0.6]$. The simulation result show as Fig.7.
(2) Evolution of Ideal cooperation
When A1-A3>0, B1-B3>0, A2-A4>0, B2-B4>0, cooperation is the only ESS.
1) Symmetry game
Assume A1=B1=8, A2=B2=7, A3=B3=6, A4=B4=3, set multi initial value of x as 0.1, 0.3, 0.5, 0.7, 0.9. The simulation result show as Fig.8.

2) Dissymmetrical game.
Assume A1=8, A2=7, A3=6, A4=3, B1=6, B2=6, B3=4, B4=5, multi initial value of (x,y) are [0.2 0.8], [0.4 0.6], [0.3 0.3], [0.7 0.4], [0.9 0.2], [0.1 0.7], [0.1 0.3], [0.1 0.4], [0.1 0.8], [0.1 0.6]. The simulation result show as Fig.9.

(3) Evolution of neutral relation
When A1-A3<0, B1-B3<0, B2-B4<0, A2-A4<0, neutrality is the only ESS.
1) Symmetry game
Assume A1=B1=8, A2=B2=7, A3=B3=6, A4=B4=3, set multi initial value of x as 0.1, 0.3, 0.5, 0.7, 0.9. The simulation result show as Fig.10.
2) Dissymmetrical game.
Assume A1=8, A2=7, A3=6, A4=3, B1=6, B2=6, B3=4, B4=5, set multi initial value of (x,y) as [0.2 0.8], [0.4 0.6], [0.3 0.3], [0.7 0.4], [0.9 0.2], [0.1 0.7], [0.1 0.3], [0.1 0.4], [0.1 0.8], [0.1 0.6]. The simulation result show as Fig.11.
(4) Evolution of competitive relation
When $A_1 - A_3 > 0$, $B_1 - B_3 > 0$, $B_2 - B_4 < 0$, $A_2 - A_4 < 0$, competition is the only ESS.

1) Symmetry game
Assume $A_1 = B_1 = 7$, $A_2 = B_2 = 6$, $A_3 = B_3 = 10$, $A_4 = B_4 = 4$, set multi initial value of $x$ as 0.1, 0.3, 0.5, 0.7, 0.9. The simulation result show as Fig.12.

2) Dissymmetry game
Assume $A_1 = 4$, $A_2 = 3$, $A_3 = 6$, $A_4 = 2$, $B_1 = 8$, $B_2 = 6$, $B_3 = 10$, $B_4 = 4$, set multi initial value of $(x, y)$ as: [0.2, 0.8], [0.4, 0.6], [0.3, 0.3], [0.7, 0.4], [0.9, 0.2], [0.1, 0.7], [0.1, 0.3], [0.1, 0.4], [0.1, 0.8], [0.1, 0.6]. The simulation result show as Fig.13.
4.2 Simulation analysis

It can be observed from Fig.6 and Fig.7, whether symmetry or dissymmetry games, there have 2 ESS, cooperation and neutrality. The final ESS depends on not only comprehensive incomes but also initial strategy selection ratio. From Fig.8 and Fig.9, we can get that the only ESS in SDN is cooperation regardless of the select ratio, when cooperation is the best selection. Fig.10 and Fig.11 indicate that neutrality is the best selection, when there has not cooperative chance regardless of the initial ratio. Fig.12 and Fig.13 indicate that the competition is ESS regardless of initial strategy selection ratio. In that case SDN will be dismissed.

The evolution of SDN enterprises’ relations indicate that cooperation, neutrality or competition relation was decided by total incomes earned in games.

(1) Cooperation relation will be the ESS when cooperative incomes greater than neutral incomes. The evolutionary speed from no-cooperation to cooperation is decided by incomes difference of cooperation and no-cooperation.

(2) Neutrality relation will be the final ESS when no-cooperation incomes greater than cooperation incomes, which indicated that market uncertainty with high-risk, any pay out cannot obtain returns.

(3) Competition is the final ESS, when competitive strategy can earn greater income regardless of the partners’ strategy. The one’s incomes obtain result in other’s lost, which is harm to the stability of SDN, and should be avoid by adjusting the payoff in games.

(4) When both partners all take cooperative strategy gain maximum incomes, neutral incomes greater than cooperative incomes under partners taken competition strategy, SDN system have 2 ESS, cooperation and neutrality. In this case supervision and punishment can improve the ratio of cooperation.

CONCLUSION

Since Axelrod R. research cooperation relation evolution taken advantage of iterative prisoner's dilemma game, evolutionary game theory provides a convenient mathematical framework for the study of the evolution of cooperation. Bulks of the works have focused on the evolution of the relationship between cooperation and competition, lacking of systematic research on the neutrality relation. This paper initially established a competition, cooperation and neutrality evolutionary game model and given the MATLAB simulation. As a complex system, the dynamic evolutionary mechanism of SDN are uncertainty, some unknown factors are at work besides the consolidated incomes. In this paper, only the consolidated incomes as the sole basis for business cooperation choice, which did not consider the interaction and cooperation of people working, the individual risk preferences, external economic, political and cultural change, and other non-economic factors’ disturbance.

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