Gap element method of longitudinal vibration of drill string

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ABSTRACT

Due to the lower tool as the research object to analyze the vibration of the drill string, causes the loss of the low order natural frequency of the drill string and the inherent vibration mode, give rise to the distortion and error of the result from the transient analysis of the drill string. For this purpose, the whole drill string of the deep wells is studied in the article, build the mechanical model of axial vibration for the deep wells. Analyze and calculate the dynamic response of the drill string by adopting the method of three-dimensional dynamic finite element theory improved the Ritz vector method at the same time, present the fatigue limit stress formula of the drill string, and the software has been compiled based on the above model and the theory. Convenient for drilling construction, contemporary, give the effective action of the vibration reduction based on the theory and calculation.

Keywords: Drill string; Vibration; Gap Element; Fatigue; Vibration Reduction;

INTRODUCTION

In the drilling operation, the failure the drill string often occurs, and destructive, so we should avoid the drill string failure actively. Fatigue fracture failure is the main failure mode of the drill string, the fatigue fracture failure of the drill string includes three basic types: The first is the pure fatigue failure; the second is scars of fatigue failure; the third is corrosion fatigue damage. These damage and the drill string vibration have a direct relationship, especially with the longitudinal vibration of the drill string. The fatigue failure the drill string is mainly caused by the longitudinal vibration of the drill string. Due to limited by borehole diameter smaller, transverse vibration will not be huge [1-3], so the influence of drill string fatigue will be secondary. The influence of the torsional vibration of drill string fatigue is secondary too, for this purpose, this paper mainly studies the longitudinal vibration of drill string.

Many countries scholars have conducted in-depth study on the issue of the drill string. Some scholars has carried on the experiment and analysis, and some of them presented vibration equation of the drill string [4-7]. But previous studies are generally in lower drill as the research object [8-11], this causes the loss of the low order natural frequency and the inherent vibration mode of the drill string. The result can only be used for qualitative analysis, but cannot be used for quantitative description. In this paper, the whole drill string as the research object, Borehole is regarded as a three-dimensional space curve that can be arbitrarily curved, build the mechanical model of axial vibration for the deep wells. Meanwhile, analyze and calculate the dynamic response of the drill string by adopting the method of three-dimensional dynamic finite element theory improved the Ritz vector method, present the fatigue limit stress formula of the drill string, thus provide a theoretical basis for quantitatively assessing and predicting the fatigue strength of the drill string.
1 METHOD AND THEORETICAL EQUATION

1.1 MECHANICAL MODEL OF THE DRILL STRING LONGITUDINAL VIBRATION ANALYSIS

The mechanical model is based on the overall drill string in this article, this is, the whole drill string from the wellhead to the bottom as research subjects, it can be any combination of the drill collars, drill pipe, joints, stabilizer and shock absorbers and other various parts of the drill. Taking into account the influence of the surface structure on the drill string vibrations, The derrick, wire rope and the traveling block and the other structure on the ground are simplified to the equivalent structure which the axis free, but the transverse immobile, As shown in Figure 1.

The actual borehole of the deep wells is not an ideal straight line, but an arbitrary curved three-dimensional space curve. So it is reasonable to consider the drill string as the three-dimensional which is coincided with the borehole axis. At the same time, should consider the impact on the drill string fatigue strength of the drill string bending stress caused by the initial curvature.

In the process of drilling, drilling bit beat in the bottom hole. Impose a longitudinal vibration force on the bottom of the drill string. The vibration force can approximate as a simple harmonic vibration force.

\[ P = P_0 \sin(\omega t) \]  \hspace{1cm} (1)

In the formula, \( P_0 \) is the excitation force amplitude. \( \omega \) is Circular frequency of excitation force. \( t \) is time variable.

1.2 THE FINITE ELEMENT EQUATIONS OF THE DEEP WELL DRILL STRING LONGITUDINAL VIBRATION

Discrete deep well drilling string along the axis into several space straight beam element. Using dynamic finite element method to vibration analysis .Get the longitudinal vibration equation of whole drill string structure are as follows:

\[ M\ddot{U} + C\dot{U} + KU = P_0 \sin(\omega t) \]  \hspace{1cm} (2)

The \( M, C \) and \( K \) in the formula is he overall quality of the whole drill string matrix, damping matrix and stiffness matrix respectively. \( \dot{U} , \ddot{U} , U \) is node acceleration vector, the velocity vector and displacement vector respectively. \( P_0 \) is the excitation amplitude vector. \( \omega_0 \) is the longitudinal excitation frequency at bit. Using the method of modal superposition addition to solve the equation (2).The first thing is to determine inherent vibration mode and natural frequency of the drill string. We should solve the undamped free vibration equation at first:

\[ M\ddot{U} + KU = 0 \]  \hspace{1cm} (3)

Assuming the form of the solution is \[^3\]

\[ U = \Phi \sin(\omega t) \]  \hspace{1cm} (4)

According to this equation, the frequency equation is:

\[ K - \omega^2 M = 0 \]  \hspace{1cm} (5)

According to this equation, we can calculate the n solution of the Natural frequency of the drill string. Arrange this n
natural frequency.

\[ \omega_1 \langle \omega_2 \langle \omega_3 \langle \cdots \langle \omega_n \rangle \]  

Further, we can use the improved Ritz vector method to solve the n inherent vibration mode corresponding to the n natural frequency \( \phi_i (i=1, 2, \ldots, n) \).

1.3 THE MODAL SUPERPOSITION METHOD OF THE DEEP WELL DRILLING TOOL DYNAMICS ANALYSIS.

Express the corresponding displacement of drill string with the linear combination of the first order natural vibration mode:

\[ U_m = y_1 \phi_1 + y_2 \phi_2 + \cdots + y_m \phi_m = [\phi_1 \phi_2 \cdots \phi_m] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \Phi_m y \]  

\( y_i (i=1,2,\cdots,m) \) is the generalized coordinates related to the damping. Substituting this formula into formula (2), and we can solve the m independent second order nonhomogeneous linear differential equations.

\[ \ddot{y}_j + 2\omega_j \gamma_j \dot{y}_j + \omega_j^2 y_j = P^*_j \sin(\omega_{0j}t) \quad (j=1,2,\cdots,m) \]  

In this formula, \( P^* = \Phi^T P \). The Special solution of the last-written is:

\[ y_j = A_j \sin(\omega_{0j}t) + B_j \cos(\omega_{0j}t) \]  

In this formula:

\[ A_j = \frac{P^*_j}{\omega_j^2} \begin{bmatrix} 1 - \frac{\omega_0^2}{\omega_j^2} \\ 1 - \frac{\omega_0^2}{\omega_j^2} + \left( 2\gamma_j \frac{\omega_0}{\omega_j} \right)^2 \end{bmatrix} \quad (j=1,2,\cdots,m) \]  

\[ B_j = -\frac{P^*_j}{\omega_j^2} \begin{bmatrix} 2\gamma_j \frac{\omega_0}{\omega_j} \\ 2\gamma_j \frac{\omega_0}{\omega_j} + \left( 2\gamma_j \frac{\omega_0}{\omega_j} \right)^2 \end{bmatrix} \]  

Thus, in response to displacement of the drill string is:

\[ U_m = \Phi A \sin(\omega_{0j}t) + \Phi B \cos(\omega_{0j}t) \]  

Accordingly, the response can be further calculated forces and stress response, as drill string dynamic strength assessment basis.

1.4 DAMPING CALCULATION DRILL STRING

Drill string in contact with the wall friction was not much, but it is generally different damping mud. Rayleigh damping used here, assuming that the damping matrix:

\[ C = C_1 + C_2 \]  

Where \( C_1 \) is mud damping and structural damping; \( C_2 \) is generated by the contact friction damping.
\[ C_1 = \alpha M + \beta K \]  

(13)

Right side of equation coefficients \( \alpha, \beta \) is an arbitrary scale factor, the overall mass matrix, stiffness matrix structure of the drill string.

It can be seen from this formula is a linear damping mode, which is the ratio of the two mass and structural stiffness and structural damping associated linear combination. The former considers only the inside of the structural material damping, ignoring the viscous fluid damping structure surrounding the object; which only consider the viscous damping fluid, a damping structure is ignored within the material. In theory, the general model of single-proportional damping unreasonable, the result is not entirely realistic, reasonable combination of the two linear damping formula (2-2).

\[ C_2 \ddot{U} = R_t \]  

(14)

Where \( R_t \) represents the contact friction vector, each element of which represents the frictional resistance on the corresponding node, the drill string static analysis results, and then converted to an equivalent diagonal damping matrix based on the node speed. So then according to formula (3-25) to obtain the total damping matrix.

2. DRILL STRING FATIGUE STRENGTH ANALYSIS THEORY

2.1 DRILL STRING PRELOAD STRESS

For different types of drill collars and drill pipe, which the buckle tightening torque is different, there are norms to be investigated. But actually buckle tightening torque requirements and specifications buckle tightening torque on the site of operation is different, in fact deducted in the calculation applied tightening torque. In the role of the buckle tightening torque, tensile stress is generated in the vicinity preload stress slots and threaded root public deductible approximated as:

\[ \sigma_y = \frac{M_0}{2fRA} \]  

(15)

Where is the buckle tightening torque, it drill collars, drill pipe type and buckle type related; to face contact drill collars and drill pipe friction coefficient; was threaded root diameter; sectional area of threaded root. Here trapezoidal thread ignores the impact angle of inclination of the tooth face, because the tilt angle trapezoidal thread tooth face is very small, its impact is negligible.

2.2 WORK DRILL STRING STATIC STRESS

In the mean weight WOB and the role of the cross-section of the drill string will produce a positive tension and compression stress, which is calculated as:

\[ \sigma_p = \frac{N_p}{A} \]  

(16)

Where in the static axial force, the cross-sectional area of the drill string, if the threaded joint is effective when the cross-sectional area.

2.3 DRILL STRING NORMAL STRESS AMPLITUDE LONGITUDINAL VIBRATIONS

Vibration analysis by the drill column can be harmonic alternating vertical drill string vibration amplitude axial force \( D \), whereby the drill string can be calculated longitudinal vibration amplitude of normal stress:

\[ \sigma_1 = \frac{N_D}{A} \]  

(17)

This stress is a major factor in fatigue failure caused by the drill string.

2.4 INITIAL DRILL STRING BENDING NORMAL STRESS

Wellbore curvature caused by the initial drill string bending stresses as

\[ \sigma_0 = \frac{DE}{2\rho} \]  

(18)
Where is the radius of curvature of the wellbore, the outer diameter of the drill string, the drill string is elastic modulus. The initial drill string bending stresses are stresses in the turntable drilling is alternating, belonging to dynamic stress; directional drilling when it is a positive constant stress, are static stress. For straight-deep well, the initial bending stresses in the drill string is very small and negligible.

2.5 CALCULATE THE CROSS SECTION OF THE DRILL STRING STRESS

Cross-section of the drill string stress can be divided into two parts: First, the average stress, and second alternating stress amplitude. If using a power drill, power drill should also consider the impact of rotation.

Turntable does not turn, turn turbo drill
Besides turntable turbo drill turn

\[
\begin{align*}
\sigma_s & = \sigma_r + \sigma_b + \sigma_i, \\
\sigma_s & = \sigma_i + \lambda \sigma_b \quad (19)
\end{align*}
\]

Where the ratio of the rotation frequency and the frequency of the exciting force drill turntable.

2.6 DRILL STRING FATIGUE STRENGTH CONDITIONS

Drill string at work, stress at each cross-section of a typical asymmetric cycle state, in a cycle characteristics \( r \), promised a cross-section of the drill string with fatigue (lasting) limit is calculated using the following formula[12];

\[
[\sigma_r] = \frac{2[\sigma_{+1}] [\sigma_{-1}]}{(1-r)[\sigma_{+1}] + (1+r)[\sigma_{-1}]} \quad (20)
\]

\[
r = \frac{\sigma_m - \sigma_a}{\sigma_m + \sigma_a} \quad (21)
\]

The \( r \) in the formula is characteristic value of asymmetrical stress cycling. \([\sigma_{+1}]\) is the allowable stress of (static) constant load \( \sigma_s \) and static load safety factor \( n_s \). \([\sigma_{-1}]\) is the drill string in the symmetry of allowable stress cycle conditions endurance limit \( \sigma_s \), which is related to the stress concentration factor of the drill string \( \beta \), the size coefficient \( \varepsilon_d \) and the coefficient of surface processing \( \varepsilon_h \). Considering the above several kinds of factors, which affect the endurance limit, the drill string in the symmetry of allowable stress cycle conditions endurance limit is

\[
[\sigma_{-1}] = \frac{\varepsilon_d \varepsilon_h}{n_d \beta} \sigma_{-1} \quad (22)
\]

The \( n_d \) in the formula is the dynamic basic safety coefficient \( n_d \), \( \sigma_{-1} \) is the drill string material tensile compression endurance limit \( \sigma_s \), which is related to the string material strength limit \( \sigma_b \) and the yield limit \( \sigma_s \), it can be expressed by

\[
\sigma_{-1} = 0.27(\sigma_s + \sigma_b) \quad (23)
\]

The drill string material yield limit \( \sigma_s \) and ultimate strength \( \sigma_b \) is determined by the drill string material mechanical properties, which can check from the material mechanical properties of the drill string table, so the structure of the drill string allowable endurance limit \([\sigma_r]\) can be calculated by formula (20). Because the alternating load of the drill string which may be accompanied by the impact is not stable \( \sigma_s \), the allowable endurance limit shall also divided by the number of additional safety coefficient \( n_f \), which’s value is changed by asymmetry, stress changes of the more violent, this coefficient is higher. Always is 1.2. For this, we can get the allowance last limit of the drill string structure, the fatigue strength condition of the deep drill established by this is
\[ \sigma_{\text{max}} = \sigma_m + \sigma_s \leq \lceil \sigma_r \rceil \]  

(24)

The \( \omega_{\text{max}} \) is the biggest working allowance strength of the drill string. If only just one which don’t be content with \( \omega_{\text{max}} \) and \( \omega_n \), the allowable stress \( \sigma_s = 773 \text{MPa}, \ \sigma_b = 879 \text{MPa} \). The calculation results are shown in table 1.

### RESULTS AND DISCUSSION

#### 3.1 EXAMPLE

According to S - 7 Wells of real drilling, engineering design and application of the above theory and the corresponding software of the shaft of the drill string longitudinal vibration and fatigue strength is analyzed and forecast and evaluation. This well deepest depth is 3880m, the density of the mud is 1.14-1.270 g/cm³, the material of the drill pipe is \( \sigma_s = 773 \text{MPa}, \ \sigma_b = 879 \text{MPa} \). The calculation results are shown in table 1.

Can be seen from Table 1 with the gradual deepening of the drilling and the drilling string lengthen, the natural frequencies of the drill string in decreases, and when the excitation frequency of drill string and a natural frequency phase recently, the drill string will be violent vibration. Excitation frequency resonance region will be from the fifth (the fourth natural frequency and natural frequency between 5) into the sixth new non-resonant area (natural frequency in the fifth and sixth) between natural frequency. Along with the further deepening of drilling footage, excitation frequency will also from the sixth non-resonant area into the seventh non-resonant area, meanwhile to skip the sixth resonance region. Instances of penetration gradually close to 1585 m, the vibration of the drill string has a tendency to increase. In the depth of 1585 m, the rotary speed of 120 r/min, the excitation frequency of drill string for omega p = 40.09 rad/s, obviously the excitation frequency and the fifth order natural frequency (40.164) is very close, so the intense vibration or resonance happens. The maximum axial force is very big, is 177.92kN, occurred in the bit of 130.8 m. Drill string fatigue dangerous section develops at the site of about 117.3 m from the bit, the cross section of the maximum dynamic stress \( \omega_{\text{max}} = 358.7 \text{MPa} \), the name of the safety coefficient is 0.83, less than 1, is likely to have the occurrence of fatigue fracture failure. To avoid the resonance, need to change the rotary speed to adjust the bit the excitation frequency, excitation frequency and the fifth order natural frequency stagger, won't appear resonance phenomenon. For this table and another is given rotary speed 105 r/min, the scheme, the scheme can avoid the drill string in Ci-Jing deep resonance.

<table>
<thead>
<tr>
<th>The serial number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well depth (m)</td>
<td>1360</td>
<td>1585</td>
<td>1585</td>
<td>1960</td>
<td>2160</td>
<td>3600</td>
</tr>
<tr>
<td>Bit type</td>
<td>Φ311PDC</td>
<td>Φ311PDC</td>
<td>Φ311PDC</td>
<td>Φ311PDC</td>
<td>Φ216</td>
<td>Φ216</td>
</tr>
<tr>
<td>Vibration state</td>
<td>The micro vibration resonance</td>
<td>The micro vibration</td>
<td>The micro vibration</td>
<td>The micro vibration</td>
<td>The micro vibration</td>
<td>The micro vibration</td>
</tr>
<tr>
<td>wob(kN)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>Rotary speed (r/min)</td>
<td>120</td>
<td>120</td>
<td>105</td>
<td>120</td>
<td>120</td>
<td>65</td>
</tr>
<tr>
<td>Axial force ( N_{\text{max}} ) (kN)</td>
<td>12.57</td>
<td>177.92</td>
<td>13.68</td>
<td>6.53</td>
<td>5.44</td>
<td>8.45</td>
</tr>
<tr>
<td>( L_{\text{max}} ) (m)</td>
<td>63.3</td>
<td>130.92</td>
<td>69.3</td>
<td>225.8</td>
<td>1339.9</td>
<td>114.63</td>
</tr>
<tr>
<td>( \sigma_{\text{max}} ) (MPa)</td>
<td>338.8</td>
<td>358.7</td>
<td>335.8</td>
<td>316.5</td>
<td>30.567</td>
<td>31.115</td>
</tr>
<tr>
<td>The allowable stress ( \sigma_s ) (MPa)</td>
<td>506.3</td>
<td>296.2</td>
<td>494.0</td>
<td>476.6</td>
<td>481.0</td>
<td>515.6</td>
</tr>
<tr>
<td>Nominal safety coefficient ( \rho_{\text{min}} )</td>
<td>1.49</td>
<td>0.83</td>
<td>1.47</td>
<td>1.51</td>
<td>1.46</td>
<td>1.27</td>
</tr>
<tr>
<td>( L_{\text{max}} ) (m)</td>
<td>117.3</td>
<td>117.3</td>
<td>99.3</td>
<td>1670</td>
<td>1870</td>
<td>164</td>
</tr>
<tr>
<td>Disturbance frequency (rad/s)</td>
<td>44.29</td>
<td>40.09</td>
<td>38.58</td>
<td>34.92</td>
<td>32.79</td>
<td>20.42</td>
</tr>
<tr>
<td>the ( \omega_{\text{max}} ) (rad/s)</td>
<td>4.312</td>
<td>3.840</td>
<td>3.840</td>
<td>3.251</td>
<td>3.005</td>
<td>2.980</td>
</tr>
<tr>
<td>drill ( \omega_{\text{max}} ) (rad/s)</td>
<td>13.816</td>
<td>12.115</td>
<td>12.115</td>
<td>10.085</td>
<td>9.296</td>
<td>6.003</td>
</tr>
<tr>
<td>string ( \omega_{\text{max}} ) (rad/s)</td>
<td>24.332</td>
<td>21.157</td>
<td>21.157</td>
<td>17.421</td>
<td>15.938</td>
<td>10.155</td>
</tr>
<tr>
<td>inherent ( \omega_{\text{max}} ) (rad/s)</td>
<td>35.285</td>
<td>30.567</td>
<td>30.567</td>
<td>25.043</td>
<td>22.858</td>
<td>14.419</td>
</tr>
<tr>
<td>frequency ( \omega_{\text{max}} ) (rad/s)</td>
<td>46.488</td>
<td>40.164</td>
<td>40.164</td>
<td>32.808</td>
<td>29.908</td>
<td>18.756</td>
</tr>
<tr>
<td>( \omega_{\text{max}} ) (rad/s)</td>
<td>57.921</td>
<td>49.911</td>
<td>49.911</td>
<td>40.665</td>
<td>37.036</td>
<td>27.546</td>
</tr>
</tbody>
</table>
Notion: LNman(m) and Lnmin (m) in the table are respectively presented the distance which is from the drill string maximum axial force Nmax (kN) to the bit of distance and fatigue in the dangerous section to the distance of the drill bit; σmax(MPa) and [σpl](MPa) are respectively presented the fatigue dangerous section in the maximum working stress and allowable stress fatigue

3.2 ANALYSIS OF VIBRATION REDUCTION MEASURES

(1) Change the speed of the drill string. As before, when the violent vibration of the drill string, the changing speed of vibration reduction is one of the most simple and effective measures. Speed change will change the bit the excitation frequency, will not change the natural frequency of the drill string. The effect which is very close to the excitation frequency and inherent frequency staggered, to achieve the purpose of reducing vibration. Change can be reduce the speed, rotational speed can be increased speed, reduction should not be too big, so as to avoid vibration resonance region from the current to the adjacent another resonance zone, the ideal of the frequency of the exciting force should be a fall in between the two resonance region near the resonance zone of intermediate value, damping effect is best.

(2) Increase the weight on bit. When the violent vibration of the drill string, if the weight is not too big, the appropriate increase the weight on bit to vibration also to have certain advantages, but time is too long. Appropriate increase the weight on bit can reduce bit bounce, can achieve the purpose of reducing vibration.

(3) Change the drill bit. The violent vibration of the drill string, sometimes is a bit of broken teeth. Especially cone bit off after the tooth, the rotation is not very smooth, beating a lot and penetration is very small, long drill grinding beating, will cause the drill string fatigue fracture. Appear this kind of circumstance must be timely replacement of bit, the driller with rich experience in drilling is required to timely detection and timely replacement.

(4) Change the tool. The violent vibration of the drill string, the first to think of drilling tool structure is reasonable, if too little drill collar should be appropriately increased the number of drill collar, upper drill collar to put heavier drill pipe at the same time.

CONCLUSION

In the longitudinal vibration of the drill string fatigue strength assessment and prediction of available the following understanding:

(1) Establish the deep well drilling string mechanical model of longitudinal vibration analysis and the theory of finite element analysis method is correct and effective.
(2) Of the drill string fatigue limit stress calculation formula, right of drill string fatigue strength provides theory basis for quantitative evaluation and prediction.
(3) Change in drilling construction speed of vibration reduction is one of the most simple and effective measures, but the speed increase or decrease amplitude should not be too big, in order to avoid the resonance of vibration from the current area to another, the resonance zone adjacent to create new resonance.
(4) The drill string vibration in the borehole mud, its damping is bigger. How to accurately select damping coefficient, on the drill string vibration is also very important, need to be further in-depth study.

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