



Research Article

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## Field-goal percentage influence factors correlation analysis and counter measures based on optimization model

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### ABSTRACT

*Basketball is a kind of strong antagonistic sports event that centered on shoots, layup and slam dunk. Two teams participate competition, both teams have 5 players entering into the court, the final purpose is to shoot basketball into opponent hoop and get scores, and need to interfere opponent get ball and reduce opponent scores. Shooters' throwing height, projection angle, projection speed are the main influence factors that affect field-goal percentage. Through establishing relative mathematical model and getting relative data, analyze player throwing speed, angle and height relations, apply the scientific data as reference basis in practical operation, carry out targeted training so that achieve fast improving field-goal percentage.*

**Key words:** basketball shoot, mathematical model, projection parameter

### INTRODUCTION

Shoot is a kind of scoring method that player applies every special and reasonable motion in entering ball into opponent hoop in basketball match. Shoot is a key technique in basketball and also the unique way to get scores. Attack team player's purpose of utilizing every kind of techniques and tactics is to create more shoot opportunity so that get higher scores in multiple shooting opportunities [1-3]; Defense team positive defending is to interfere opponent shooting so that overwhelm opponent scores [4]. With basketball development, player height, physical quality and technical levels improvement propels to shoot techniques continuously development; Throwing parts change from low to high, throwing speed changes from slow to fast, shooting way is also becoming more and more, field-goal percentage is continuously increasing accordingly [5].

Shooting techniques refer to body each part comprehensive coordination and exertion process when player shoots. Force aggregation starts from shooting preliminary posture, lower limbs grounds and exertion, then stretch body along hoop throwing directions, especially with the help of spine extension inertia to propel to lower limbs, trunk and upper limbs coherent and synergic cooperation, put body each part muscle strength finally accumulated on arms, wrist and fingers, tip the ball out by extending arms, turning over wrists as well as fingers plucking motions [6].

Any team organization cannot ignore one problem from the start is improving medium and long distance basketball field-goal percentage. In teaching and training, shooting is also a kind of basic technique that teachers or coaches cannot use exact quantitative expression to pass on to students or players. From the perspective of physiology, shoot training is a gradually forming, dynamic stereotype process [2]. It includes generalization, differentiation, consolidation and automation these phases. Most of people all hope to achieve dynamic stereotype in shorter time, so that get stable and correct field-goal percentage. However, constrained by each aspect condition, experiencing several years hard training, some players feel difficult to achieve ideal expectation results, so that only rely on player intuitional observation, abstract understanding and groping in training not only influences learning process but also cannot surely arrive at expected expectations. [4] Therefore, if it has already calculated correlation data and give learners quantizable data as guiding so that it is more easier for them to fast seizing shooting technology and

improve field-goal percentage. This research implements analysis on projection speed, projection height and projection angle these three main factors which effect when shooting can hit or not.

## SHOOTING MATHEMATICAL MODEL ESTABLISHMENT AND ANALYSIS

### Hit rate improvement principle analysis

Previously, American one professor analyzes data that one university students carry out on site shot and on site jump shot, he finds that make successful baskets, their projection angles are very close to so-called “minimum speed angle”. “Minimum speed angle” refers that guide player to shoot ball into hoop with a special angle and with minimum projection speed, minimum projection angle that is optimal projection angle. In the different shooting distances and projection heights, shooting should respectively adopt a kind of suitable optimal projection angle, though their sizes are not the same, they have a common feature that they all very close to minimum speed angle.

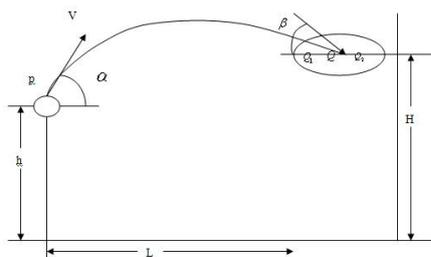


Figure 1: Shoot from shooting point schematic figure

Figure 1 is shoot from shooting point schematic figure, shooting point is  $P$ , hoop center is  $O$ , according to set size, point  $P$  and  $O$  horizontal direction distance is  $L$ :

$$L = 4.225m[5.80m - 1.20m - 0.225m(\text{Radius of the rim}) - 0.15m(\text{length of the bracket})]$$

Now get measurement data, basketball diameter  $d = 0.246m$ , hoop distance  $D = 0.45m$ , point  $O$  height  $H = 3.05m$ , ignoring errors, assume that shooter projection point height is between  $1.60m$  and  $2.10m$ , player projection speed is  $v = 8.0 \sim 9.0m/s$ , then establish mathematical model, research cases as following :

- Basketball and hoop sizes that effect on shooting, ball center hits hoop center conditions, solve corresponding eligible projection angle  $\alpha$ , incident angle  $\beta$ , then solve by calculation the peak value that basketball flight arc achieves.
- Under circumstance that guarantee ball hits hoop, the condition not ensure ball center hits hoop center, when throwing ball generates too forward or too backward, projection angle and projection speed permissible maximum deviation that generates.
- Air resistance influences, air resistance would generate inhibition on throwing ball, so it needs to work out real projection speed and projection angle.
- Shooting distances influences, to solve corresponding projection speed and projection angle.

### Research method

According to parabolic body physics motion law, establish mathematical model.

Use mathematics non-linear equations, quadratic function, ordinary differential equation, *Matlab* optimization toolkit *fsolve* to solve problems. After throwing ball, tentatively ignore statuses that the ball its own rotation as well as touch the rebound and enter into hoop.

### Problem analysis

Hoop and basketball sizes, as Figure 2, hoop diameter is  $D$ , basketball diameter is  $d$ . It is obvious even basketball center hits hoop, angle would be small. Following by basketball touching hoop proximal  $A$  with ball impulse, it affects the ball so that it cannot smoothly enter into the hoop.

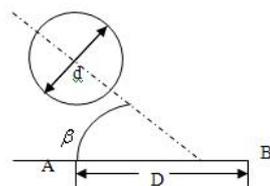


Figure 2: Basketball enters into the hoop

In case that ball center is allowed to deviate hoop center and entering into hoop, forward(Figure 1 的 point Q) maximum distance is Figure 3  $\Delta x$ ,  $\Delta x$  can calculate from incident angle  $\beta$ . According to  $\Delta x$  and ball center trajectory relations between  $x$  and  $\alpha$ , it can get projection angle  $\alpha$  permissible maximum deviation  $\Delta\alpha$ . Projection speed  $v$  permissible maximum deviation  $\Delta v$ , it can be similarly handled.

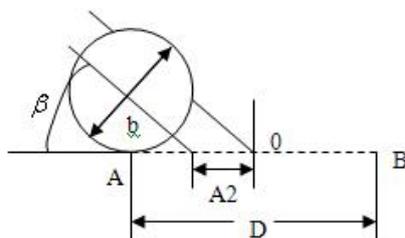


Figure 3: Ball center forward

Considering ball affected by air resistance, use differential equation to solve ball center movement trajectory. Due to resistance is smaller, it can be properly simplified. Then make every kind of calculations similarly as front.

## RESULTS AND DISCUSSION

### Case one, ball center hits hoop center

In view of basketball and hoop sizes, discuss ball center hits hoop center conditions so that solve eligible projection angle  $\alpha$  and incident angle  $\beta$  as well as ball flight arc top point.

1) At first ignoring basketball and hoop sizes as well as air resistance factor, solve corresponding eligible projection angle  $\alpha$  and incident angle  $\beta$ . With projection ball center  $P$  as origin of coordinates,  $X$  axis is in the horizontal direction,  $Y$  axis is in the vertical direction, basketball is throwing out at the time that  $t = 0$  with projection speed  $v$  and projection angle  $\alpha$ , it can be regarded that particle( ball center) makes physics oblique projectile motion, establish oblique projectile equation and get projection angle  $\alpha_1$  and  $\alpha_2$ . If assume  $\alpha_1 > \alpha_2$ , then get  $\alpha_1$  is increasing function of  $h$  and  $v$ . First workout different projection angles minimum projection speed  $v_{\min}$  and corresponding projection angle  $\alpha_0$ , value projection height  $h = 1.6 \sim 2.1m$ .

$$v^2 \geq g \left[ H - h + \sqrt{L_2 + (H - h)^2} \right] \quad (1)$$

$$\tan \alpha = \frac{v^2}{gL} \left[ 1 \pm \sqrt{1 - \frac{2g}{v^2} (H - h + \frac{gL^2}{2v^2})} \right] \quad (2)$$

Utilize formula (1), it get  $v_{\min}$ , Utilize formula (2), it get  $\alpha$ , results can refer to Table 1.

Table 1: Different throwing heights minimum throwing speeds and corresponding throwing angles

h/m	v/m s-1	$\alpha(^{\circ})$
1.6	7.614824	54.47099
1.7	7.529762	53.86003
1.8	7.445074	53.24064
1.9	7.360801	52.61321
2.0	7.276983	51.97822
2.1	7.193663	51.33616

As Table 1 shows, there is probability to make clean shot that only projection height is larger than corresponding projection angle. Because model assumption projection speed is equal to  $8m/s$ , meet minimum projection speed and minimum speed gets close to hypothesis conditions(hypothesis conditions are at work), so projection speed generally should not less than  $8m/s$ .

2)  $\beta$  is hoop incident angle. Angle  $\alpha_1$ 、 $\alpha_2$  correspond to different projection angles, then they have different incident angle  $\beta_1$ 、 $\beta_2$ , assume that  $\beta_1 > \beta_2$ . Considering standard hoop and basketball sizes, as Figure 2. If incident angle is relative small, and then basketball cannot be shot into hoop. According to  $d$  and  $D$ , it works out  $\beta > 33.1^{\circ}$ . For projection speed  $v = 8.0 \sim 9.0m/s$  and projection height  $h = 1.6 \sim 2.1m$ , calculate projection angle  $\alpha_1$  and its corresponding incident angle  $\beta_1$ 、 $\alpha_2$  and its corresponding incident angle  $\beta_2$ , list them into following table. From Table, it is clear that due to  $\beta_2$  not meet the condition that above  $33.1^{\circ}$ , the corresponding  $\alpha_2$  would be false, and then projection angle can only be  $\alpha_1$ .

$$\tan \beta = \tan \alpha - \frac{2(H-h)}{L} \quad (3)$$

In formula (2), it gets  $\alpha_1$  by taking plus, and it gets  $\alpha_2$  by taking minus. Use formula (3), it works out corresponding  $\beta_1$  and  $\beta_2$ .  $M_H$  represents ball flight top point value, result refers to Table 2.

Table 2: Different throwing speeds and throwing heights corresponding throwing angles and incident angles

v/m s-1	h/m	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$	$\beta 1(^{\circ})$	$\beta 2(^{\circ})$	$M_H/m$
8.0	1.6	64.73264	44.20934	55.07715	15.98087	4.270385
8.0	1.7	65.35122	42.36885	57.00624	15.27370	4.397356
8.0	1.8	65.89125	40.59002	58.67192	14.84681	4.520498
8.0	1.9	66.37155	38.85488	60.13390	14.63967	4.640755
8.0	2.0	66.80465	37.15179	61.43314	14.61042	4.758754
8.0	2.1	67.19938	35.47294	62.59920	14.72849	4.874935
8.5	1.6	69.43479	39.50719	63.19203	7.865998	4.831371
8.5	1.7	69.74025	37.97982	64.21676	8.063171	4.944218
8.5	1.8	70.02438	36.45689	65.15158	8.367153	5.056026
8.5	1.9	70.29004	34.93639	66.00897	8.764600	5.166940
8.5	2.0	70.53951	33.41693	66.79909	9.244464	5.277078
8.5	2.1	70.77469	31.89763	67.53026	9.797422	5.386535
9.0	1.6	72.51553	36.42645	68.10499	2.953034	5.359604
9.0	1.7	72.70707	35.01300	68.76111	3.518830	5.467484
9.0	1.8	72.88920	33.59207	69.37301	4.145721	5.574907
9.0	1.9	73.06281	32.16361	69.94535	4.828220	5.681918
9.0	2.0	73.22867	30.72778	70.48214	5.561421	5.788557
9.0	2.1	73.38741	29.28490	70.98680	6.340881	5.894858

From Table 2, it is clear that when projection speed is  $8.0m/s$ , projection height increases from  $1.6m$  to  $2.1m$ , projection angle is also corresponding increasing from  $64.73^{\circ}$  to  $67.20^{\circ}$ , variation is  $2.47^{\circ}$ , and its incident angle is also corresponding increasing  $7.52^{\circ}$ ; when speed is  $9.0m/s$ , throwing height increases from  $1.6m$  to  $2.1m$ , projection angle also increases from  $72.52^{\circ}$  to  $73.39^{\circ}$ , variation is  $0.87^{\circ}$ , and its incident angle also corresponding increases  $2.88^{\circ}$ .

It can be concluded that if projection speed and height increase, projection angle changes are in decreasing tendency; Incident angle increasing may be caused by ball flight arc increasing. Projection angles changes lead to incident angles corresponding changes indicate that projection angle has an influence on ball arc change.

In order to improve shooting stability, projection speed should be change in small range, and through comparison, get that projection speed should not less than  $8.0m/s$ . With projection speed, height increasing, projection angle changes are in decreasing tendency; when projection speed is fixed, larger projection angle shoots ball arc would be higher.

### Case two, projection angle and projection speed maximum deviation

$$\Delta\alpha = \frac{gL - v_2 \sin\alpha \cos\alpha}{L(v^2 - gL \tan\alpha)} \quad (4)$$

$$\Delta v = \frac{gL - v^2 \sin\alpha \cos\alpha}{gL^2} \quad (5)$$

$$\left| \frac{\Delta v}{v} \right| = \left| \Delta\alpha \left( \frac{v^2}{gL} - \tan\alpha \right) \right| \quad (6)$$

According to formula (4) and above calculated  $\alpha_1$ , work out maximum projection angle deviation  $\Delta\alpha$ , use formula (5) and formula (6) calculate projection speed maximum deviation  $\Delta v$  and  $\left| \frac{\Delta v}{v} \right|$ ,  $\Delta x = \frac{D}{2} - \frac{d}{2\sin\beta}$  only list  $h=1.8, 2.0m$  result into Table 3.

Table 3: Throwing angle and speed maximum deviation

h/m	$\alpha(^{\circ})$	v/m-s-1	$\Delta\alpha(^{\circ})$	$\Delta v$	$\left  \frac{\Delta\alpha}{\alpha} \right  \times 100\%$	$\left  \frac{\Delta v}{v} \right  \times 100\%$
1.8	65.89125	8.0	-0.67564	0.064989	1.025380	0.812362
1.8	70.02438	8.5	-0.53018	0.079138	0.757132	0.931035
1.8	72.88920	9.0	-0.44185	0.089678	0.606197	0.996427
2.0	66.80465	8.0	-0.64386	0.070841	0.963802	0.885510
2.0	70.53951	8.5	-0.51488	0.082888	0.729912	0.975152
2.0	73.22867	9.0	-0.43241	0.092503	0.590499	1.027814

As Table 3 shows, data indicates that when projection height arrives at  $1.8m$ , projection speed increases from  $8.0m/s$  to  $9.0m/s$ , projection angle permissible deviation ratio change is relative remarkable that is 0.33%, speed variation ratio change is 0.18%. Analyze projection speed and projection angle, it gets that large projection angle change is affected by projection speed small scale variation.

It is concluded that projection speed variations is remarkable than projection angle, both projection speed and projection angle permissible deviations are quite small.

### Case three, considering air resistance

According to basketball features, it can only consider horizontal direction resistance, and resistance is in direct proportion to speed, horizontal direction movement at that time can solve by differential equation, list differential equation as following:

$$\begin{aligned} x(t) &= v \cos \alpha t - \frac{kv \cos \alpha t^2}{2} \\ y(t) &= v \sin \alpha t - \frac{gt^2}{2} \end{aligned} \quad (7)$$

Solve formula (7); it gets non-linear equations as following:

$$v \cos \alpha t - \frac{kv \cos \alpha t^2}{2} - L = 0$$

$$v \sin \alpha t - \frac{gt^2}{2} - (H - h) = 0 \quad (8)$$

Let air resistance coefficient  $k = 0.05(1/s)$ , utilize non-linear equations and *Matlab* optimization toolkit *fsolve* making solution, its calculation results refer to Table 4 (rewrite results ignoring resistance on the last two rows for comparison). *Matlab* Program process is omitted here.

**Table 4: Throwing angle when considering air resistance**

v/m s-1	h/m	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$	$\alpha 1(^{\circ})$	$\alpha 2(^{\circ})$
8.0	1.6	63.40379	44.67552	64.73264	44.20934
8.0	1.7	64.09375	42.77722	65.35122	42.36885
8.0	1.8	64.69099	40.96589	65.89125	40.59002
8.0	1.9	65.20960	39.20403	66.37155	38.85488
8.0	2.0	65.67576	37.48620	66.80465	37.15179
8.0	2.1	66.09634	35.79646	67.19938	35.47294
8.5	1.6	68.45080	39.66407	69.43479	39.50719
8.5	1.7	68.77078	38.14038	69.74025	37.97982
8.5	1.8	69.07166	36.61129	70.02438	36.45689
8.5	1.9	69.34745	35.09499	70.29004	34.93639
8.5	2.0	69.60441	33.58149	70.53951	33.41693
8.5	2.1	69.84872	32.07001	70.77469	31.89763
9.0	1.6	71.66937	36.47657	72.51553	36.42645
9.0	1.7	71.86844	35.07505	72.70707	35.01300
9.0	1.8	72.05524	33.66427	72.88920	33.59207
9.0	1.9	72.23582	32.24225	73.06281	32.16361
9.0	2.0	72.40420	30.81789	73.22867	30.72778
9.0	2.1	72.56642	29.37826	73.38741	29.28490

As Table 4 shows, on the condition that throwing speed at 8.0m/s, throwing height at 1.6 m, when considering air resistance, its corresponding throwing angle by comparing with ignoring resistance is 1.33° smaller; Similarly, on the condition that throwing speed at 9.0m/s, throwing height at 2.0m, when considering air resistance, its corresponding throwing angle by comparing with ignoring resistance is 0.82° smaller.

Synthesize indications, when considering air resistance influences with same throwing speed and angle, throwing angle reduction scale are basically changing between 1-2°.

#### Case four, considering different distances

Here take different values of L, use oblique projectile equation in calculating, it gets different projection height s minimum projection speed as well as corresponding projection angle, take L value 5.0 and 6.25 as examples, calculation results as Table 5.

As Table 5 shows, when throwing height values 1.7m, shooting distance changes from 5.0m (Middle distance shot) to 6.25m (three-point), throwing minimum speed is increasing from 8.08m/s to 8.78m/s, and its corresponding throwing angle is decreasing from 53.09° to 51.53°.

It is clear that when projection height is fixed, if throwing distance increases, projection speed would correspondingly increase, projection angle will accordingly decrease; ball flight arc would accordingly enlarge.

**Table 5: Different distances different throwing heights corresponding minimum throwing speed and throwing angle**

L/m	h/m	v/m s-1	$\alpha(^{\circ})$	MH(m)
5.0	1.7	7.999040	52.55479	3.757736
5.0	1.8	7.921997	52.01812	3.789263
5.0	1.9	7.845339	51.47638	3.822079
5.0	2.0	7.769092	50.92989	3.856213
5.0	2.1	7.693283	50.37898	3.891694
6.25	1.7	8.711633	51.09432	4.044791
6.25	1.8	8.643668	50.65497	4.079730
6.25	1.9	8.576025	50.21291	4.115756
6.25	2.0	8.508722	49.76832	4.152887
6.25	2.1	8.441772	49.32140	4.191137

### CONCLUSION

- 1) When player shoots, minimum projection angle should be correspond to minimum projection speed, both would slight decrease with projection height increasing, projection speed might as well above or equal to  $8m/s$  (Table 1).
- 2) When assuming projection as a constant, if projection speed is relative large, projection angle can be large so that ball flight arc would become larger, permissible deviation angle is relative small. In this way due to speed influences on projection angle that causes projection angle would not so large, it will maintain between  $7-9^\circ$ ; When projection speed is fixed, the larger projection height is, the larger projection angle would be, and the larger ball flight arc would be. But with projection speed increasing, projection height influences on projection angle would decrease; the influence is approximately maintaining around  $1^\circ$  (Table 2, Table 3).
- 3) For same projection speed and projection height, if consider resistance influences, projection angle would be  $1-2^\circ$  smaller, ball flight arc would accordingly decrease; Then in training and competition, maintain projection speed stability (or change in small scale) is crucial to guarantee field-goal percentage (Table 4).
- 4) When shooting speed is in a certain range, projection height increases and it become larger, then projection angle would have smaller change; angle affected by distance is between  $1-2^\circ$ . Therefore, when shooting distance increasing, shooting speed increasing in reasonable range, corresponding projection angle should decrease  $1-2^\circ$  so that ensure throwing ball optimal flight arc (Table 3, Table 5).
- 5) Correct and proper backspin is of great importance to ball that throws. Backspin ball underneath pressure is larger than above pressure, so that it will give ball a resultant action from bottom to top which holds up ball upwards, so that ball flight arc would also accordingly increase and entering hoop angle increases; meanwhile, circle round ball can decrease, ball forward impulse force and air resistance to ball make ball fly uniformly as much as possible along with expected operation direction.
- 6) Backspin can make up ball fast speed shortcoming when shooting, because ball touches hoop sides, ball speed would decrease. Therefore, backspin spinning result that reduce possibility of popping ball while accordingly increase possibility of making the hoop; after that, backspin ball hits the backboard and gets rebound, ball strength on backboard rebounding will alleviate.

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