



Research Article

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Distribution network design in battlefield environment with loss consideration

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ABSTRACT

We study the distribution network design problem in a battlefield environment. The problem is to decide where to locate the distribution nodes and which distribution nodes or supply nodes the combat units are allocated to minimize the total setup cost and transportation cost. Some commodities would be destroyed by the enemy during the distribution process, and the loss quantity is assumed to follow a function of the exposure time in the distribution process. An Integer Programming model is proposed to formulate the problem, which is solved by Lagrangian heuristic algorithm. One hundred numerical examples including up to 20 supply nodes, 80 distribution nodes and 400 combat nodes are randomly generated based on data investigated from military wars and games, and all of them are used to test the solution approach. Computational results show that the algorithm can present very good near-optimal solution within short computational time.

Key words: Battlefield supply, network design, integer programming, Lagrangian relaxation

INTRODUCTION

The successful completion of combat missions depends heavily on the reliability and availability of the military logistic support system, as combat units can only exert all their power when they receive the right support commodities in the right place, at the right time, and in the right quantity [1-3]. In the military history, the backside logistic support system is always one of the most important strategic targets that are attacked by enemies. In recent decades, as the application and generalization of information technologies, more and more reconnaissance equipments with high precision and long-distance attacking weapons are developed and equipped in army [4-6]. Therefore, in current wars, the military logistic systems in battlefield suffer more frequent and severe attack from the enemy, which cause much loss when the commodities are transported from backside base to the front combat units.

Recent wars, such as the War in the Persian Gulf, and the Iraq War, have shown that military logistics face many new challenges in modern battlefield, which include frequent and small delivery batches, higher agility, and much shorter response time and so on. Many new logistic theories are developed to cover these new requirements, like Focused Logistics [7-10], Sense and Response Logistics [11-13]. To support the implement of these new theories, effective and efficient distribution networks are required.

We investigate the design of distribution network in the battlefield environment, which is shown in Figure 1. The distribution network consists of supply nodes (SNs), distribution nodes (DNs) and combat nodes (CNs). The supply node represents the military bases that hold large quantity of commodities for supporting front combat units, the distribution node represents the distribution center for transporting commodities from backside supply nodes to the front combat units, and the combat node represents combat unit that has requirement for commodities. The commodities can be sent to CNs directly or through the DNs. We assume that each CN's demand for one SN is served either by a single DN or directly by the SN, but not by both.

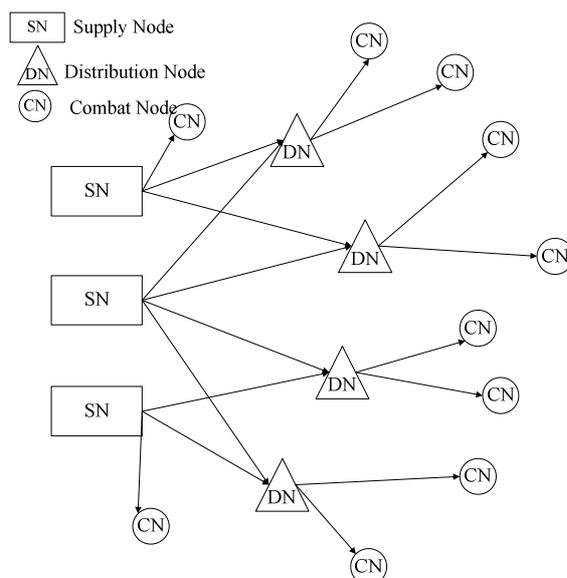


Fig. 1 The distribution network in the battlefield

In the proposed distribution network in Fig. 1, it is assumed that commodities in SNs are safe and can not be detected by the enemy, as the backside military bases are usually underground. When commodities are sent out and transported to CNs, they may be detected and attacked by the enemy during the distribution time, which would cause some losses. The more time of the commodities consume in the distribution process, the more quantity would be destroyed by the enemy.

The proposed problem can be viewed as a two-layer facility location problem. As an important area, many achievements have been obtained on distribution network design and facility location problem [14-15]. But fewer models are developed specially for distribution systems in the battlefield environment. Gue investigates a sea-based logistic system, and develops a multi-period, facility location and material flow model for the system. The proposed model can be used to construct the land-based distribution system over time to support a given battle plan with minimum inventory. Toyoglu et al. study an ammunition distribution problem in the battlefield and develop a novel three-layer commodity-flow location routing model that helps distribute multiple products, respects hard time windows, to the combat units. As far as we know, there is no work considering the loss caused by attacks from the enemy in the design of distribution systems [16-20].

The organization of this paper is as follows. In section 2, the distribution network design problem is formulated into Integer Programming model. The solution approach for the problem is developed in section 3 and the computational results are discussed in section 4. We conclude the paper in section 5.

MODEL FORMULATION

In the battlefield distribution network, each supply node supplies a different commodity and can satisfy all the demand from CNs. In practical war, military bases usually supply multiple kinds of commodities. Thus, some of the supply nodes here are virtual nodes. The commodities are first transported to the DNs from the SNs and then delivered to the CNs, and they can be also sent to the CN directly from the SN if the distance between them is close enough. The commodities form the same SN can either be delivered form SN or through one DN, but not from both or through more than one DN.

The following notations are used in the MIP model:

- $i=1,2,\dots,I$: index of SNs
- $j=1,2,\dots,J$: index of potential DNs
- $k=1,2,\dots,K$: index of CNs

Parameters:

- c_i = the unit cost of commodity i
- d_{ik} = demand for commodity i at combat node k
- td_j = the dwell time at DN j
- tpd_{ij} = the transportation time from SN i to DN j
- tdr_{jk} = the transportation time from DN j to CN k

tpr_{ik} = the transportation time from SN i to CN k
 cpd_{ij} = the cost of transporting per commodity from SN i to DN j
 cdr_{jk} = the cost of transporting per commodity from DN j to CN k
 cpr_{ik} = the cost of transporting per commodity directly from SN i to CN k
 f_j = the setup cost of DN j

The dwell time at DN j , td_j , is the overall time spent on the DN j from receiving the commodities until sending out them, which includes the unloading time, the sorting time, the upload time and so on.

The decision variables are defined as follows:

X_{ijk} 1 if the demand of CN k are transported from SN i through DN j , otherwise 0;
 Y_{ik} 1 if the demand of CN k are transported from SN i directly, otherwise 0
 Z_j 1 if DN j is setup, otherwise 0.

Here we make the assumption that the loss rate of commodities transported from SNs to CNs depends on the delivery time from SNs to CNs, and the longer of the exposure time, the more commodities would be destroyed by the enemy. Therefore the expected quantity that can safely arrived at the combat is described as follows,

$$DE=DIg(t) \tag{1}$$

where DE is the expected quantity that can safely arrived at the combat node, DI is the initial quantity sent out from the supply node, t is the total exposure time, and $g(t)$ is a nonincreasing function of the exposure time, which satisfies $0 < g(t) \leq 1$. Let $t_{ijk}=tpd_{ij}+td_j+tdr_{jk}$ = the exposure time from SN i to CN k through DN j . $t_{ik}=tpr_{ik}$ = the exposure time from SN i to CN k directly. Then, the required initial quantities sent out from supply nodes in different ways are $d_{ik}/g(t_{ijk})$ or $d_{ik}/g(t_{ik})$.

In fact the presented distribution network design problem can be viewed as a special case of two-level facility location problem. The mathematical optimization model for the problem is formulated as follows.

Min

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K (c_i + cpd_{ij} + cdr_{jk}) \frac{d_{ik} X_{ijk}}{g(t_{ijk})} + \sum_{i=1}^I \sum_{k=1}^K (c_i + cpr_{ik}) \frac{d_{ik} Y_{ik}}{g(t_{ik})} + \sum_{j=1}^J f_j Z_j \tag{2}$$

s.t.

$$\sum_{j=1}^J X_{ijk} + Y_{ik} = 1 \quad \forall i, k \tag{3}$$

$$X_{ijk} \leq Z_j \quad \forall i, j, k \tag{4}$$

$$X_{ijk}, Y_{ik}, Z_j \in \{0, 1\} \quad \forall i, j, k \tag{5}$$

The objective function is to minimize the overall cost. Constraint (3) ensures that the demand of CN k from SN i is satisfied by shipment through one DN or by shipment directly from the SN, but not by both. Constraint (4) ensures that the commodities can only be transported through the DN which has been opened. Constraint (5) is the integer restriction.

SOLUTION ALGORITHM

To introduce conveniently, we note that $a_{ijk} = (c_i + cpd_{ij} + cdr_{jk})d_{ik} / g(t_{ijk})$ and $b_{ik} = (c_i + cpr_{ik})d_{ik} / g(t_{ik})$.

The relaxed problem can be obtained by relaxing the constraint set (3).

$$\text{Min} \quad \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K a_{ijk} X_{ijk} + \sum_{j=1}^J f_j Z_j + \sum_{i=1}^I \sum_{k=1}^K b_{ik} Y_{ik} + \sum_{i=1}^I \sum_{k=1}^K \lambda_{ik} (1 - \sum_{j=1}^J X_{ijk} - Y_{ik})$$

Subject to (4) and (5). Then the relaxed problem is decomposed into independent subproblems.

1) Subproblem for SN i

$$\text{Min} \quad \sum_{k=1}^K (b_{ik} - \lambda_{ik}) Y_{ik}$$

Subject to $Y_{ik} \in \{0, 1\}, \forall i, k$. The optimal solution for this subproblem can be obtained by observing: if $(b_{ik} - \lambda_{ik}) < 0$,

$Y_{ik}=1$; Otherwise, set Y_{ik} to be zero.

2) Subproblem for DN j

$$\text{Min } f_j Z_j - \sum_{i=1}^I \sum_{k=1}^K (\lambda_{ik} - a_{ijk}) X_{ijk} + \sum_{i=1}^I \sum_{k=1}^K \lambda_{ik},$$

Subject to (4) and $X_{ijk}, Z_j \in \{0,1\}, \forall i,j,k$. The solution procedure of the second subproblem is as follows:

Step 0 (Initialization):

$Z_j=0; X_{ijk}=0$

Step 1 Iteration

For $i=1$ to I

For $k=1$ to K

If $\lambda_{ik} - a_{ijk} \geq 0$

Then $X_{ijk}=1$

end for

end for.

Step 2 Judge

If $\sum_{i=1}^I \sum_{k=1}^K (\lambda_{ik} - a_{ijk}) X_{ijk} \geq f_j Z_j$,

then $Z_j=1$

else set all $X_{ijk}=0$.

Here we use subgradient algorithm to obtain an upper bound for the problem. The main steps of the algorithm are as follows:

Step 0 Initialization

Let LR be the objective value of the Lagrangian relaxation problem, and set $LR=0$.

Let UB be the upper bound of the problem, and set $UB=+\infty$.

Let LB be the lower bound of the problem, and set $LB=-\infty$.

Set $\lambda_{ik}=b_{ik}+1$, and $\alpha=2$.

Step 2: Update lower bound

If $LR > LB$, then $LB=LR$.

Step 3: Update upper bound

If the relaxed solution satisfies constraint and its corresponding objective function value is $F' < UB$, then let $UB = F'$.

Step 4: Calculate new step size

Let $norm = \sum_i \sum_k (1 - \sum_j X_{ijk} - Y_{ik})^2$

If $norm > 0$

$stepsize = \alpha(UB - LR) / norm$

Otherwise, $stepsize = stepsize / 2$.

Step 1 Solve subproblems

Given λ_{ik} , solve subproblems for SN i and DN j , then obtain a new bound LR .

Step 2 Update lower bound

If $LR > LB$, let $LB=LR$.

Step 3 Update upper bound

If the relaxed solution is feasible, and the corresponding objective value, noted as F , is smaller than UB , let $UB=F$.

Step 4 Calculate new step size

$$\text{Let } norm = \sum_i \sum_k (1 - \sum_j X_{ijk} - Y_{ik})^2$$

If $norm > 0$

$$stepsize = \alpha(UB - LR) / norm$$

else $stepsize = stepsize / 2$.

Step 5 Update multipliers

$$\lambda_{ik} = \lambda_{ik} - stepsize * (1 - \sum_j X_{ijk} - Y_{ik}).$$

Step 6 Stopping criteria

If the number of iterations $> Ni$ or the max gap of λ_{ik} between two coterminous iteration is less than ε ,

then STOP, else GOTO step 1. Ni and ε is defined by user according requirement, in our problem

$Ni = 200$ and $\varepsilon = 0.01$.

Here we use a heuristic algorithm to obtain a feasible solution for the problem from the bound given by the subgradient algorithm. The main steps of the heuristic algorithm are as follows:

Step 0 Find an infeasible constraint and initialize

Find an infeasible pair (i, k) which $\sum_{j=1}^J X_{ijk} + Y_{ik} \neq 1$,

and let $X_{ijk} = 0 (\forall j)$, $Y_{ik} = 0$

Step 1 Find a node to allocate

Find a best DN j among current open DNs which $B_j = \min \{a_{ijk} \mid Z_j = 1\}$.

If $B_j < b_{ik}$, allocate CN k to DN j and $X_{ijk} = 1$,
else allocate CN k to SN i and $Y_{ik} = 1$

Step 2 Judge

If there are no infeasible pairs, STOP,
else GOTO step 0.

RESULTS AND DISCUSSION

The presented model and algorithm are tested on randomly generated numerical examples. And the computational experience for all examples is conducted on the IBM T420 laptop with Windows XP (Intel® Core™2 Duo CPU, 2GB of RAM).

In order to test the robust of the solution algorithm, fifty examples with the size of 5 SNs, 20 DNs and 50 CNs are generated, and fifty examples with the size of 20 SNs, 80 DNs and 400 CNs are generated. All these examples are randomly generated and solved by our solution approach. The computational performance is summarized in Table 1. The gap represents the percentage error between the feasible lower bound obtained by the heuristic algorithm and the upper bound obtained by the subgradient algorithm. From Table 1 we can see the Lagrangian heuristic approach works very well for all the examples and can present very good solution in short CPU time. Here we should also note that the results in table 1 are obtained in 200 iterations and better results can be obtained with more iteration.

Table 1 computational performance of the algorithm

	Gap			CPU time (second)		
	Max	Min	Average	Max	Min	Average
Small example	0.075%	0.002%	0.018%	0.852	0.645	0.702
Large example	0.264%	0.053%	0.098%	190.15	142.36	165.71

CONCLUSION

We studied the distribution network design problem in the battlefield environment, which considers the potential

destruction of commodities by the attacks from the enemy. The problem is formulated as an Integer Programming model. An efficient Lagrangian heuristic algorithm is developed to solve the problem. Fifty numerical examples for networks including 5 SNs, 20 DNs and 50 CNs, and fifty examples including 20 SNs, 80 DNs and 400 CNs are used to test the model and the solution approach. Computational performance analysis shows that the algorithm can present near optimal solutions for all examples in short CPU. In this paper, the loss function of commodities is assumed to be constant and known, while in practical wars it is usually uncertain. Therefore, the stochastic situation of the distribution network design problem in battlefield environment would be a valuable extension to this study.

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