Development of self-adaptively loading for planetary roller traction-drive transmission

Desheng Chen and Xuewen Du

College of Vocational and Technical Education, Zhejiang University of Technology, Hangzhou, China

ABSTRACT

Traction drives offer certain advantages over gear drives. In particular, they generally run more quietly than geared units and provide an opportunity for higher speed and higher efficiency by eliminating sliding motion between contacting elements. Generating adequate normal load at the frictional contact is essential for traction drives. The best way to generate the normal force is the self-adaptively loading to prevent the rollers under a continuous and high-stress load. In this paper, a novel design of planetary traction-drive device was developed. It is a combination of a planetary cylindrical roller configuration and a logarithmic spiral profile connection. In this device, the sun roller is hollow, and connected to the input shaft by a logarithmic spiral profile. So as the input shaft rotates relative to the sun roller according a certain direction, a wedging action takes place between the input shaft and the sun roller. This makes the sun roller generate a certain amount of elastic deformation, so an appreciable amount of contact load is generated between the contacting elements. The greater the input torque is, the stronger the strength of the wedging action, and the greater the contact load, and vice versa. So this device is self-actuating and self-adaptive. This design offers an alternative way to solve the problem of self-adaptively loading of the planetary cylindrical roller traction drive.

Keywords: Traction-drive, planetary roller mechanism, self-adaptively loading, profile connection, logarithmic spiral

INTRODUCTION

Traction drive possesses desired performance attributes that pertain to its unique operating principles. These attributes include high mechanical efficiency, little or no backlash, low noise and vibration and high-speed capability. The power density of a traction drive can be very high when operated at elevated speeds. These performance features, in conjunction with its inherent manufacturing simplicity, make traction drives suitable candidates for a host of applications.

Traction drives can be broadly classified into variable-speed ratio drives and fixed-speed ratio drives. Generating adequate normal load at the frictional contact is essential for traction drives. Planetary configuration has been the preferred arrangement for fixed-speed ratio drives because the internal forces are well balanced. A common practice is to use tapered raceways along the axial direction. By moving these raceways axially, a radial displacement and thus normal force are generated. Examples of such designs were disclosed in [1, 2].

Since the envelopes of the tapered raceways in most design configurations do not converge to a common point, this results in a so-called spin motion between the contacting surfaces. The spin motion not only offsets the high-efficiency otherwise offered by friction drives, but also causes component wear and high break away torque. Recently, a zero-spin design of planetary traction drive was proposed by Ai and Rybko [3]. This design employed the on-apex concept similar to that of a tapered roller bearing and offered spin free rolling motion at the frictional contacts. Two rows of planetary rollers were used in a back-to-back arrangement to balance the internal axial force.
on the planetary rollers. This design featured a torque-actuated roller-ramp loading mechanism, offering improved torque capability. It looks as if the problem of spin motion has been solved, but when the shape of the tapered planetary rollers were revised in shape in order to improve the contact stress distribution, the spin motion would reoccur.

Cylindrical annular rollers could eliminate the spin motion. However, the torque-actuated roller ramp mechanism is not applicable to this case. Instead, the normal load was generated by using oversized rollers before. The magnitude of the load was controlled by precisely pre-set internal interference. This imposes severe constraints on manufacturing and assembling. Moreover, when a traction drive of such design is operated at partial torque, contact components are unnecessarily overloaded. This places the rolling components under a continuous, high-stress load and reduces the durability of the traction drive.

So in a word, if tapered rollers were used in the planetary roller mechanism, the spin motion would occur likely; while if cylindrical rollers were used, the spin motion would not happen, but it would be required to solve the problem of self-adaptive loading for the planetary roller mechanism. This problem has been studied by Ai et al [4] who developed an eccentric cylindrical planetary design and by Flugrad and Qamhiyah [5] who used a different principle involving pairs of intermediate rollers to produce the normal force on torque demand. Also, this problem is the subject of this paper, in which a new way to solve this problem will be presented.

OPERATING PRINCIPLES

With the development of numerical control machining technology, logarithmic spirals have been used for profile keyless connections [6], as seen in Figure 1. Inspired by this, a new type of planetary cylindrical roller traction-drive device was developed, as shown in Figure 2. It is composed of a sun roller, three planetary rollers, an outer ring, an input shaft, and a planet carrier (output shaft). The sun roller is hollow, and is attached to the input shaft through a logarithmic spiral profile. When the input shaft rotates relative to the sun roller in a certain direction, a wedging action takes place between the input shaft and the hollow sun roller. This makes the sun roller produce a certain amount of radial displacement. Thus, an appreciable quantity of normal load is generated between the sun roller and the planetary rollers as well as between the planetary rollers and the outer ring. The greater the input torque, the stronger the strength of the wedging action, and then the greater the contact load, and vice versa. So this
configuration is self-actuating. Here, as cylindrical rollers are used in the planetary traction-drive device, such a spin motion as would present in the case where tapered rollers were used will not occur.

GEOMETRIC DESIGN FOR PROFILE CONNECTION

In mathematics, a logarithmic spiral refers to the track of a moving point its moving direction keeps a fixed angle $\lambda$ with its polar radius, as seen in Figure 3. The polar coordinate equation of logarithmic spirals can be expressed as $\rho = ae^{m\theta}$, where $a$ and $m$ are constants, $\theta$ is referred to as the polar angle, and $\rho$ is referred to as the polar radius.

In a logarithmic spiral, $a$ decides the size of the logarithmic spiral, while $m$ decides the shape. When a logarithmic spiral is used in a profile connection, the value of $m$ decides the magnitude of the wedge angle $\alpha$, which in turn decides the frictional self-locking status of the curved wedge. Here, the profile connection is used for self-adaptive loading, so it is not permissible for it to be self-locking. That is to say, the following relationship must be satisfied:

$$\tan \alpha = \tan g(90^\circ - \lambda) = 1/ \tan \lambda = m > f$$

where $f$ is the coefficient of friction between the shaft and the hub.

If a logarithmic spiral profile connection is used to transmit torque, its cross-section contour only requires being capable of centring, and normally consists of three segments of logarithmic spiral. Here, the logarithmic spiral profile connection is to be used for the planetary cylindrical roller traction drive. The aim of doing so is to obtain the radial elastic deformation of the sun roller and then to obtain the normal force needed for traction drive through the profile connection between the input shaft and the hollow sun roller. Under the condition that the input torque is constant, the greater the amount of radial elastic deformation of the hollow sun roller, the greater the normal forces between the rollers of the traction-drive device, and then the higher the capacity of the transmission. In order to obtain a greater elastic deformation under the condition that the minimum wall thickness of the hollow sun roller satisfies the tensile strength, a thin average wall thickness is desired. If only three segments of logarithmic spiral were employed in the cross-section outline of the profile connection, the length of each segment of logarithmic spiral would be longer, and the average wall thickness of each segment of wall would be thicker, and then the elastic deformation of the hollow sun roller would be smaller. Increasing the number of segments of logarithmic spiral would effectively reduce the average wall thickness. Considering that the wedging stroke is not long, six segments of logarithmic spiral could be selected. Figure 4 shows the cross-section contour of the profile connection. It consists of six segments of logarithmic spiral, and in each segment of logarithmic spiral, the value of $\theta$ is defined as from $\theta_1 = 0$ to $\theta_2 = 11\pi/36$, and a straight line or arc is adopted to connect the two adjacent segments in order to reduce the stress concentration.
In order for the planetary traction-drive device to work properly, relative sliding is not permissible between the rolling elements. This can be referred to as the frictional self-locking condition for this device, and can be further explained as follow: When a torque is applied to the input shaft, a wedging action takes place between the input shaft and the hollow sun roller. The wedging action makes the hollow sun roller produce a certain amount of radial elastic deformation. This makes the hollow sun roller apply a contact load to the planet rollers and outer ring. This contact load is needed for the planetary traction drive. To prevent sliding between the rolling elements, the frictional forces generated by the contact load should overcome the working torque applied to the planetary traction-drive device.

**Contact Stress of Profile Connection** $\sigma_p$

Provided there is an ideal condition to the fit between the input shaft and the hollow sun roller, and a torque is applied to the input shaft, as shown in Figure 5, then an appreciable amount of contact stress is thus generated at the six sections of working surface of the profile connection. Taking any point $P$ from the $AB$ section, so in the infinitesimal $ds$ that contains the point $P$, the summing of the moments about point $o$ will produce the following expression:

$$dT = \rho (\sigma_p \sin \gamma ds + \sigma_p f \cos \gamma ds) = (\sin \gamma + f \cos \gamma)\sigma_p \rho \cos \theta ds$$

where $\sigma_p$ is contact stress between two fitted surfaces, $\rho$ is polar radius of the point $P$, $l$ is fit length between two fitted surfaces, $\gamma$ is included angle between $\sigma_p$ and $\rho$, and $f$ is coefficient of friction between two fitted surfaces.

From the characteristics of the logarithmic spiral, we have

$$\gamma = \alpha = \pi/2 - \lambda, \rho = ae^{m\theta} \text{ and } ds = \rho d\theta$$

Substituting these relationships into Equation (1) leads to

$$dT = (\sin \gamma + f \cos \gamma)\rho^2 \sigma_p \cos \theta l d\theta$$

$$= (\sin \alpha + f \cos \alpha)\rho^2 \sigma_p l d\theta$$
So, the total moment exerted by the three pieces of working surface of the profile connection is as follows:

\[
T = 6 \int_0^{\frac{1}{36}} dT = 6 \int_0^{\frac{1}{36}} a \sigma_p l (\sin \alpha + \cos \alpha) e^{2m\theta} d\theta = \frac{3a \sigma_p l (\sin \alpha + f \cos \alpha)(e^{\frac{1}{18}} - 1)}{m}
\]

Thus, the contact stress is

\[
\sigma_p = \frac{mT}{3a l (\sin \alpha + f \cos \alpha)(e^{\frac{1}{18}} - 1)}
\]  

(2)

Contact Load between Rollers \(P\)

In this design of a planetary roller traction drive, as the contact load for the traction drive is produced through the elastic deformation of the hollow sun roller, so its value can be calculated by analysing the elastic deformation of the hollow sun roller. In order to simplify the theoretical calculation, the hollow sun roller its wall thickness is actually not uniform, can be seen a uniform thickness sleeve, as seen in Figure 6. In this simplified model, the inner wall of the sleeve is subjected to a uniformly distributed load \(p\) and constrained by the input shaft; the outer wall is constrained by three equally distributed planetary rollers, so it is subjected to three equally distributed contact loads (reactions) \(P\). Obviously, this sleeve is an internal-force statically indeterminate system. According to the principle of deformation superposition, two deformation quantities, \(u\) and \(\delta\), can be calculated respectively. Here, \(u\) is the radial deformation quantity of the sleeve produced by the internal pressure \(p\), and \(\delta\) is the contact deformation quantity between the rollers caused by the three contact loads \(P\). Then, according to the deformation compatibility condition of the sleeve at point \(C\), that is \(\delta = u\), the contact load \(P\) could be calculated.

**Fig. 6 Free-body diagram of sleeve**

**Calculation of Radial Deformation Quantity \(u\)**: The calculating model of the radial deformation quantity of this thin-walled sleeve is shown in Figure 7. The calculating formula of the radial deformation of a thin-walled sleeve is [7]

\[
u = \frac{p r_1^2}{E t}
\]  

(3)

where \(r_1\) is the radius of the sleeve, \(t\) is the wall thickness of the sleeve.
Calculation of Contact Deformation Quantity $\delta$: The calculation model of the contact deformation quantity of this planetary roller traction-drive device is shown in Figure 8. There are two places at which the contact deformation occurs: one is between the sun roller and the planetary rollers; the other is between the planetary rollers and the outer ring. The quantities of contact deformation at the two places are named $\delta_i$ and $\delta_e$ respectively. In engineering, Palmgren’s formula is usually used to calculate the line-contact elastic deformation [8], so we get

$$\delta_i + \delta_e = 2 \times 3.847 \times 10^{-5} \times \frac{P^{0.9}}{L^{1.8}}$$

where $L$ is the contact length of two contacting bodies.

Deformation compatibility equation: Provided that only the contact elastic deformation is considered while the whole elastic deformation is neglected for the planetary roller and the outer ring, then the deformation compatibility equation can be expressed as

$$\delta_i + \delta_e = u$$

Self-locking Condition for Traction-drive Device

If $\mu$ is referred to as the coefficient of friction between the rollers of the traction-drive device, the condition to prevent sliding between the rollers is as below:

$$3\mu P r_i \geq T$$

Trying to solve the equations (2), (3), (4), (5) and (6) simultaneously, and noting that $\sigma_p = p$, theoretically the frictional self-locking condition for the planetary roller traction-drive device can be obtained, but it is difficult to
write out the analytic solution.

STRENGTH DESIGN
In order to simplify calculations, the following strength conditions are based on an assumption of simple stress.

Compressive Strength Condition
The compressive strength condition for the profile connection pair is
\[ \sigma_p \leq [\sigma_p] \]  
(7)
where \( \sigma_p \) is the compressive stress of the profile connection pair, \([\sigma_p]\) is the allowable compressive stress.

Tensile Strength Condition
The hollow sun roller is taken as a thin-walled sleeve, and squeezed by the profile shaft, so it is subjected to a circumferential tensile stress. The circumferential tensile stress may make the hollow sun roller produce the plastic deformation and even rupture. Therefore, the tensile strength condition for the hollow sun roller is
\[ \sigma_\theta = \frac{r_i}{t} \sigma \leq [\sigma] \]  
(8)
where \( \sigma_\theta \) is the circumferential tensile stress of the hollow sun roller, \([\sigma]\) is the allowable tensile stress.

Contact strength Condition
According to the Hertz theory, the contact strength condition for two cylindrical bodies is
\[ \sigma_H = \sqrt{ \frac{\rho_1}{L} \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) } \leq [\sigma_H] \]  
(9)
in which, \( \sigma_H \) is maximum contact stresses for two bodies, \([\sigma_H]\) is allowable contact stresses for two bodies, \( \rho_1 \) and \( \rho_2 \) are radius of curvature for two bodies respectively, \( \mu_1 \) and \( \mu_2 \) are Poisson’s ratios for two bodies respectively, \( E_1 \) and \( E_2 \) are modulus of elasticity for two bodies respectively.

DESIGN EXAMPLE
If the device as shown in Figure 2 is to be used for a speed reducer, and taking the sun roller as the input member, the planet carrier as the output member and the outer ring as the stationary member, then the speed ratio of this device would be
\[ \frac{r_1}{r_1 + r_2} = \frac{1}{i} \]  
where \( r_1 \) and \( r_2 \) are the radius of the sun roller and the outer ring. Supposing \( i = 4 \) is required for a certain application, then a radius of 21mm for the sun roller and 63mm for the outer ring will satisfy the speed requirement. Once the radius of the sun roller is known, the thickness of the sleeve at the thinnest wall \( t \) and the constant of logarithmic spiral \( a \) could be determined. The thinnest wall \( t \) can be calculated through the use of Eq. (8), and the constant of the logarithmic spiral \( a \) can be calculated through the following formulas:
\[ a = r_i - (t + \Delta \rho) \]  
in which, \( \Delta \rho = a (e^{\alpha_1} - e^{\alpha_2}) = a (e^{1/36} - 1) \).

After the geometric parameters of the planetary roller traction-drive device are determined, solving equations (2), (3), (4), (5) and (6) simultaneously, and noting that \( \sigma_p = \sigma \), the required minimum friction coefficient to ensure self-locking for the planetary roller traction drive can be obtained.

The input shaft is made of oil-quenched and tempered (OQ&T) 5140 steel (American Iron and Steel Institute (AISI)), and the hollow sun roller is made of OQ&T 52100 steel. From the American Society of Metals (ASM) Metals Reference Book [9], the yield strength of OQ&T 5140 steel is \( \sigma_y = 550 \text{MPa} \), and the yield strength of OQ&T
52100 steel is $\sigma_S = 1700 \text{MPa}$. If the safety factor of the profile connection is selected as $S = 1.25$, then the allowable compressive stress between the input shaft and the hollow sun roller is $[\sigma_p] = \sigma_S / S = 1700 / 1.25 = 1360 \text{MPa}$, and the allowable tensile stress of the hollow sun roller is $[\sigma_t] = \sigma_S / S = 1700 / 1.25 = 1360 \text{MPa}$. As stated in [5], the allowable contact stress of the rollers and raceways is $[\sigma_H] = 4000 \sim 4500 \text{MPa}$.

Having determined the geometric parameters of the traction-drive device, substituting the input torque $T$ into Equation (2), $\sigma_p$ is obtained; substituting $p = \sigma_p$ into Equation (3), $u$ is obtained; substituting $\delta_e = u$ into Equation (4), $P$ is obtained; substituting $P$ and $T$ into Inequality (6), the frictional locking condition, that is, the minimum value of $\mu$ could be obtained; substituting $p = \sigma_p$ into the left term of Inequality (8), $\sigma_h$ is obtained; substituting $P$ into the left term of Inequality (9), $\sigma_H$ is obtained. The calculations have been summarized in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>$T$ (N·m)</th>
<th>$\alpha$ (mm)</th>
<th>$\mu$</th>
<th>$\sigma_p$ (MPa)</th>
<th>$\sigma_h$ (MPa)</th>
<th>$\sigma_H$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td>0.24</td>
<td>16.67</td>
<td>674</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>0.22</td>
<td>33.33</td>
<td>269</td>
<td>992</td>
</tr>
<tr>
<td>400</td>
<td>17.0</td>
<td>0.205</td>
<td>66.66</td>
<td>539</td>
<td>1457</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td>0.19</td>
<td>133.33</td>
<td>1078</td>
<td>2142</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T$ (N·m)</th>
<th>$\alpha$ (mm)</th>
<th>$\mu$</th>
<th>$\sigma_p$ (MPa)</th>
<th>$\sigma_h$ (MPa)</th>
<th>$\sigma_H$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td>0.18</td>
<td>15.67</td>
<td>783</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>0.16</td>
<td>31.34</td>
<td>342</td>
<td>1151</td>
</tr>
<tr>
<td>400</td>
<td>17.5</td>
<td>0.15</td>
<td>62.69</td>
<td>685</td>
<td>1692</td>
</tr>
<tr>
<td>800</td>
<td></td>
<td>0.14</td>
<td>125.37</td>
<td>1370</td>
<td>2487</td>
</tr>
</tbody>
</table>

The results of calculation for $\mu$ in Table 1 and Table 2 show:

(1) Under a certain wall thickness condition, as the input torque increases, the value of the minimum friction coefficient needed to satisfy the self-locking condition for the traction drive will decline slightly. This explains that the self-locking condition for the traction drive is mainly relevant to the geometric parameters and the elastic modulus of the material, and has little relevance to the input torque.

(2) Under a certain input torque condition, as the wall thickness reduces, the value of the minimum friction coefficient needed to satisfy the self-locking condition for the traction drive will decline obviously. This is because, as the wall thickness reduces, the radial elastic deformation of the sun roller increases. Thus, the contact loads between the rollers increases, so the value of the minimum friction coefficient needed declines. But the reduction in the wall thickness is limited by the tensile strength of the hollow sun roller.

Based on the preceding analysis, it is shown that the calculation results are logical. This shows that the foregoing theoretical calculation has certain reliability.

The results of calculation for $\sigma_p$, $\sigma_h$ and $\sigma_H$ have also been listed in Tables 1 and 2, so as to compare them with their allowable stress, and then to judge the allowable maximum value of the input torque.

CONCLUSION

Based on the logarithmic spiral profile connection, a novel cylindrical roller traction drive device was developed. As cylindrical rollers are used in this design, such a spin motion as would present in the planetary tapered roller traction drives will not occur, and as a logarithmic spiral profile connection is utilized in this device, the device is self-actuating and self-adaptive, so a new way of self-adaptive loading for planetary cylindrical roller traction drives has been obtained. In this way, the device has higher capacity to transmit power compared to the eccentric cylindrical planetary device [4] due to its multi-roller drive and has fewer transmission elements compared to the
device proposed by Flugrad and Qamhiyah [5].

This research not only expands the method of self-adaptive loading for cylindrical roller planetary traction drives, but also extends the utilization of logarithmic spiral profile connections.

Future research is planned to experimentally measure the efficiencies for the proposed design. The stress distribution between the rolling elements will be determined by finite element analysis. The stresses will then be used to predict the expected fatigue life of the system, and experimental tests will be conducted to verify the life predication.

Acknowledgment
This research was supported by Zhejiang Provincial Natural Science Foundation of China (ZJNSF No. Y1110418).

REFERENCES