ABSTRACT
Basketball field goal percentage has already become the key point in many scholars’ research; this paper carries out deeply analysis of basketball shooting through release angle and release speed two main factors, establishes kinematics model, and analyzes basketball process trajectory. Apply kinematics equation and combine with mathematical knowledge, it deduces general shooting angle range and release speed range. By establishing relative mathematical model and get correlation data, analyzing athlete release speed, angle and height relationships, take these scientific data as references in practical operation, implement targeted training so as to achieve fast improving field goal percentage.

Key words: Basketball shooting, release angle, release speed, movement trajectory, Matlab simulation

INTRODUCTION
By far, many foreign scientific research institutions have made investigation on basketball players from combination, technique, nutrition, tactics and other aspects, all-around and multiple angle considering shooting effects influence factors, established proper mathematical model to do research analysis, and built international well-known game similar to NBA. Though foreign research considered factors are comprehensive, their analyses are thorough and let players better playing, due to China players’ physical quality has great difference by comparing with foreign players, such kind model cannot copy. Besides, domestic researches major in shooting problems are less. Though Guo Ding-Wen in document made better statements on how to shoot basketball can improve hitting rate, it didn’t provide practical effective model targeted this problem so that helpful for analyzing problems [1, 2]. Document [3, 4] applied statistics establishing models respectively with players’ attack and defense ability, scoring ability, several technical indicators and players’ competition ability these aspects as starting point, and mainly targeted CBA and other professional game players’ height, physical ability and other aspects factors to make analysis, though it possesses certain practicality, it is short of general employing to China players, which still needs to be further researched.

On the background of above research, this paper discusses basketball shooting problems, applies sports dynamics and mathematical knowledge combining, establishes scientific reasonable basketball shooting model, starts from basketball shooting uppermost influence factors as shooting release angle, release speed, release height as well as basketball sphere center and rim center horizontal distance, basketball incidence angles relationships, it analyzes field goal percentage multiple influence factors an makes reasonable hypothesis; in case that reasonable estimating releasing point and rim center distance and maintaining stable release speed, it defines shooting best release angle and best release speed, and gets one conclusion that can let field goal percentage improve with minimum physical ability consumption when shooting.
SHOOTING TECHNICAL PRINCIPAL AND MODEL HYPOTHESIS

Upward holding and taking-off motion coordination is the key to shooting, which ensures basketball to be fast and stable shot on the top point in the air [2]. Shooting process is a parabolic process, it can regard ball flight arc as a parabola; experiment shows if shooting parabola is excessive high, ball flight time will get longer accordingly and distance will get larger, enlarge air resistance and wind force, ball flight direction is not easier to control, furthermore it will affect field goal percentage [3]. If basketball flight parabola is too low, ball incident angle gets small, it is not easy to make the basket.

Reasonable consider release angle and release speed is the key to solve problems [4], at this time, basketball in flight, it suffered air resistance, wind force influence and other lots of secondary factors which can be ignored. Correlation hypothesis is as following:

(1) Assume that basketball and backboard collision when shooting is perfect elastic collision [5], no energy lost;
(2) Athletes master proficient shooting techniques, they can correct judge release point and rim center horizontal distance and can control ball release angle and corresponding release speed according to practical requests;
(3) Athlete has better psychological quality [6]; defender’s defending not affect field goal percentage;
(4) Shooting movement curve and rim center are in the same plane;
(5) Ignore air resistance, basketball rotation in the air not affecting shooting effects;
(6) Regard basketball as a particle, and the particle position lies in the ball gravity center.

MOLD ESTABLISHMENT AND SOLUTION

Air ball shooting status analysis

Establish rectangular plane coordinate system with basketball sphere center as origin when releasing basketball, as following Figure 1:

![Diagram of basketball movement trajectory](image)

Figure 1: Basketball movement trajectory rectangular plane coordinate system

From dynamics, it is known that arc $OP_2$ equation is general movement orbit equation; it can use parameter equation (1) describing $t$ time ball location, that is:

\[
\begin{align*}
\dot{x} &= vt \cos \theta, \\
\dot{y} &= vt \sin \theta - \frac{1}{2} gt^2,
\end{align*}
\]

Eliminate parameter $t$ it gets:

\[
y = x \tan \theta - \frac{g}{2v^2 \cos^2 \theta} x^2,
\]

If basketball sphere center just passes through rim center, input $(x_0, y_0)$ into (2), it gets by sorting that:
\[ v^2 = \frac{g\alpha^2}{2\cos^2 \theta(x_0 \tan \theta - y_0)} \] \hspace{1cm} (3)

Use Matlab software drawing out formula (3) figure as following Figure 2 shows:

![Matlab Simulation Figure](image)

By analyzing parabola, it is clear that incident angle \( a \) increases (decreasing) with release angle increasing (decreasing). When \( q \) reduces to below 30°, basketball will collide with rim, it is difficult to drop into rim[7, 8], therefore it should consider make the basket case, it only need to consider the case when \( q \) is above 30°.

From Figure 2 analysis, it gets that when \( q \) is above 30°, \( v \) increases with \( q \) increasing, it indicates that in order to let basketball pass through rim center, ball release speed should increase with basketball release angle increasing. By kinematics knowledge, combining with mathematical knowledge to analyze, it finds when ball vertical incidents into the rim, basketball available passing range would be the whole rim. When \( a \) is less than 90° [9-10], basketball can change into oval by rim range. It is clear that, \( v \) increases, \( q \) increases, \( a \) is also increasing, basketball available passing range is also increasing, so that improve field goal percentage; on the contrary, it will reduce basketball available passing range. But whether is the larger \( v \)、\( q \) the better, in the following it will further discuss [11].

According to formula (3), it can solve when basketball release angle is 90°, basketball shot into rim release speed should arrive at 20m/s, the speed goes beyond any athletes any shooting ways achievable speeds. Therefore, it can be known, \( v \)、\( q \) cannot unlimited increase. To let shooting effects be the best, how large \( v \)、\( q \) is can conform to actual, according to formula (2) given \( op_1 \) equation as:

\[ y = x \tan q - \frac{g}{2v_{o1}^2 \cos^2 q} x^2 \] \hspace{1cm} (4)

From curve \( op_1 \) passing through point \( P_1 \), it has

\[ \frac{g}{2v_{o1}^2 \cos^2 q} = \frac{(s_0 - R) \tan q - (H_0 - h_0)}{(s_0 - R)^2} \] \hspace{1cm} (5)

So \( op_1 \) equation is:

\[ y = x \tan q - \frac{(s_0 - R) \tan q - (H_0 - h_0)}{(s_0 - R)^2} x^2 \] \hspace{1cm} (6)

Similarly, \( op_2 \) passes through point \( P_2(s_0 + R, H_0 - h_0) \), and \( op_2 \) equation is:
\[ y = x \tan q - \frac{(s_0 + R) \tan q - (H_0 - h_0)}{(s_0 + R)^2} x^2, \]  

(7)

Write down \( \text{op}_1 \), \( \text{op}_2 \) equation, straight line \( \text{op}_1 \) equation is:

\[ y = \frac{H_0 - h_0}{s_0 - R} x, \]  

(8)

Straight line \( \text{op}_2 \) equation is:

\[ y = \frac{H_0 - h_0}{s_0 + R} x, \]  

(9)

Solve entry part area, \( A(q) \) has:

\[ A_{\Delta \text{op}_1} = A_1 = \int_{0}^{s_0 - R} \left[ x \tan \theta - \frac{(s_0 - R) \tan \theta - (H_0 - h_0)}{(s_0 - R)^2} x^2 - \frac{H_0 - h_0}{s_0 - R} x \right] dx \]  

(10)

\[ A_{\Delta \text{op}_2} = A_2 = \int_{0}^{s_0 + R} \left[ x \tan \theta - \frac{(s_0 + R) \tan \theta - (H_0 - h_0)}{(s_0 + R)^2} x^2 - \frac{H_0 - h_0}{s_0 + R} x \right] dx \]  

(11)

\[ A_{\Delta \text{op}, p_2} = A_3 = \frac{1}{2} [2R(H_0 - h_0)] = R(H_0 - h_0), \]  

(12)

So:

\[ A(q) = A_1 * A_2 * A_3 = \frac{2}{3}s_0 R \tan q - \frac{4}{3} R(H_0 - h_0). \]  

(13)

From formula(13), it gets the bigger \( \tan q \) is, the bigger \( A(q) \) would be, while actually, shooting initial speed has limits, according to formula (3), it is known that release angle has also limits, therefore \( \tan q \) only may change in some range. Through converting \( A(q) \) into initial \( v \) function to solve maximum value, so as to solve value inside the regulation range that \( \tan q \) let \( A(q) \) arrives the maximum. Kinematic equation is formula(14):

\[ y = x \tan q - \frac{g}{2v^2 \cos^2 q} x^2, \]  

(14)

Given it passes through point \((s, H_0 - h_0), (s_0 - R, s_0 + R)\), input formula (14) it gets:

\[ H_0 - h_0 = s \tan q - \frac{g}{2v^2 \cos^2 q} s^2, \]  

(15)

So that:

\[ \frac{g}{2v^2}(1 + \tan^2 q) = \frac{s \tan q - (H_0 - h_0)}{s^2}, \]  

(16)

It is mono basic quadratic equation about \( \tan q \), take its smaller root: \( q_+ \)
\[ \tan q = \frac{1}{g_s} \left( v^2 - \sqrt{v^4 - 2v^2 (H_0 - h_0)g - g^2 s^2} \right), \]  

(17)

From which, \( v^2 \) should meet:

\[ v^4 - 2v^2 (H_0 - h_0)g - g^2 s^2 \geq 0 \]  

(18)

Solve in equation, it gets:

\[ v^2 \geq g(H_0 - h_0 + \sqrt{(H_0 - h_0)^2 + s^2}). \]  

(19)

Because:

\[ \frac{d \tan q}{d(v^2)} = \frac{\sqrt{v^4 - 2v^2 (H_0 - h_0)g - g^2 s^2} - v^2 + (H_0 - h_0)g}{g_s \sqrt{v^4 - 2v^2 (H_0 - h_0)g - g^2 s^2}} < 0, \]  

(20)

It gets, \( \tan \theta \) is monotone decreasing function, when \( v^2 \) is minimum, \( \tan \theta \) has the maximum value, due to:

\[ v^2 \geq g(H_0 - h_0 + \sqrt{(H_0 - h_0)^2 + s^2}) = v_m^2(s), \]  

(21)

So it has:

\[ \max \tan q = \frac{1}{g_s} v_m^2(s) = \frac{H_0 - h_0}{s} + \sqrt{\left(\frac{H_0 - h_0}{s}\right)^2 + 1} = \tan q_0(s), \]  

(22)

\[ \frac{d \tan q_0(s)}{ds} = -\frac{1}{s^2} [H_0 - h_0 - \frac{1}{2} (H_0 - h_0)^2] < 0 \]  

(23)

From formula (23), it gets \( \theta_0(s) \) is monotone decreasing function about \( s \), so:

\[ q_0(s) \leq \arctan\left[ \frac{H_0 - h_0}{s_0 - R} + \sqrt{\left(\frac{H_0 - h_0}{s_0 - R}\right)^2 + 1} \right] \]  

(24)

On the other hand:

\[ q_0(s) \geq \arctan\left[ \frac{H_0 - h_0}{s_0 + R} + \sqrt{\left(\frac{H_0 - h_0}{s_0 + R}\right)^2 + 1} \right] \]  

(25)

To sum up, projection angle normally is controlled in the range of following formula (26):

\[ \arctan\left[ \frac{H_0 - h_0}{s_0 + R} + \sqrt{\left(\frac{H_0 - h_0}{s_0 + R}\right)^2 + 1} \right] \leq q \leq \arctan\left[ \frac{H_0 - h_0}{s_0 - R} + \sqrt{\left(\frac{H_0 - h_0}{s_0 - R}\right)^2 + 1} \right] \]  

(26)

Correspondingly, according to formula(3), release speed \( v \) should be controlled in the range of formula (27):

\[ \sqrt{2g[H_0 - h_0 + \sqrt{(H_0 - h_0)^2 + (s_0 - R)^2}] \leq v \leq \sqrt{2g[H_0 - h_0 + \sqrt{(H_0 - h_0)^2 + (s_0 + R)^2}] \]  

(27)
Analyze status when shooting a bank shot

Assume that basketball board back has a same rim, according to hypothesis (1), fill out backboard back parts, at this time, basketball curve similarly forms into a parabola, and the parabola passes through backboard rear rim. Correspondingly, player release point and virtual rim center horizontal distance will change; it should consider air ball shooting status, original $s_0$ changes into $s_0+0.575$.

Model application

The model main discusses shooting angle and distance problems, therefore athletes in shooting moment should estimate release point height as well as release point and rim center horizontal distance, finally affect shooting effects. Therewith, according to different shooting cases, it provides conclusions. Take release point height $h_0=3.05$ m as an example, according to formula (26), it solves different dropping points’ release angles range as Table 1 shows.

Table 1: Different dropping points’ release angles range

<table>
<thead>
<tr>
<th>$x$</th>
<th>8.25</th>
<th>6.25</th>
<th>5.25</th>
<th>4.25</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>45.50°--45.54°</td>
<td>45.67°--45.71°</td>
<td>45.80°--45.85°</td>
<td>45.97°--45.60°</td>
<td>46.34°--46.50°</td>
<td>46.34°--46.40°</td>
</tr>
</tbody>
</table>

From Table 1, it is clear when shooting release point height $h_0=3.05$ m, best angle in free throw line projection is 46.5°, best angle in 3-point shooting is 45.7°, it has very small differences with actual shooting results.

CONCLUSION

Through shooting angle and shooting speed analyses, it carried out researching on actual basketball shooting problems, only targeted to basketball players’ air ball shooting status, in case that air resistance and wind force effected on shooting were ignored, assume it had reasonable and effective release angle the hypothesis is helpful for simplifying problems analysis, not affect shooting actual results, according to different athletes’ different heights, different shooting positions and different release point, established the model and made reasonable analysis, mainly considering release speed and release angle the two maximum influences factors on shooting effects, gradually deduced shooting release point and its location’s best release angle.

REFERENCES