Badminton 21-point system athletes performance correlation research based on classical probabilistic model

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ABSTRACT

After implementation the new competition system of badminton 21 points every service is scored, it has drawn a mixed response from every country athletes, coaches and umpires. This paper utilizes classical probabilistic model establishing relationships between one rally winning probability and one game winning probability, through observation it adopts hyperbolic tangent curve fitting images method to simplify the express formulas, from which it raises two evaluation indicators model that are occasionality indicator and intensity indicator. To occasionality indicator, it defines through hyperbolic tangent functions inclination, and so gets conclusions that occasionality by adopting 15 points three out of five sets is 20% above that of 21 points three out of five sets; While for competition intensity indicator, we use single match winning probability's approximate degree to 0.5 to represent. For comprehensive evaluation model indicator, it adopts variable weight functions optimal indicator evaluation model to determine, defining one variable weight function to distinguish occasionality elements and intensity elements weights when values of are different. TOPSIS method evaluation model, by calculating every evaluation objects distance from ideal solution and negative ideal solution, then makes comparison and so gets 4 kinds of competition systems comprehensive evaluation. Every competition system doesn’t have absolute advantages or disadvantages. This paper adopts relativity analysis and comprehensive evaluation analysis four schemes that mentioned in subject, it is known those 21 points two out of three sets and 15 points three out of five sets are reasonable competition systems.

Key words: Classical probability, TOPSIS algorithm, badminton circuit, competitive capacity

INTRODUCTION

Presently, Asia have monopolized superpower status in world badminton events, while Europe and American have less opportunity to win the prize, in this way it gradually shakes badminton roles in Olympic Games. By referencing Ping-pong reform successful experience, badminton 21 points rally-points new competition system is debut in the eyes of public. Through previous competitions, it draw a mixed response from every country athlete, coach and umpire, audiences to reporters that participating interviewing [1-3].

In Dec.11th of 2005, international badminton joint association council decided that 21 points rally-points new competition system should be trial implemented comprehensively since Feb.1st, 2006, adopted men and women singles, doubles, mixed doubles total 5 events [4, 5]. The “Thoma-Uber” that held in Japan in April, 2006, which adopted 21 points rally-points new competition system, and voted in the international badminton association conference Tokyo, Japan. Voting through international badminton association whole member representatives, they decided to abolish 15 points alternate service system and formally started to implement 21 points rally-points system, including Beijing Olympic Games in 2008. New competition system makes the circuit shorter, more interesting and also of great antagonism [6, 7].
In the classical probabilistic model, this paper through establishing 4 sub models those are player competitive capacity model, single rally probability model, single game probability model as well as single match probability model, so that determines function relationships between single rally winning probability and single game winning probability as well as single rally winning probability and single match winning probability, and uses curve fitting method simplifies functions express formula. Every competition doesn’t have absolute advantages or disadvantages, therefore in evaluation process, this paper adopts relativity analysis and comprehensive analysis method to analyze four schemes mentioned in subjects, gets which score way should be adopted in special competition system and normal competition system.

EVALUATION MODEL ESTABLISHMENTS
Classical probabilistic model
Classical probabilistic model is mainly including 4 sub models as player competitive capacity model, single rally probability model, single game probability model as well as single match probability model. We define player on spot competitive capacity model in the first sub model; output on spot competitive capacity and single rally winning probability function relations. In the third, fourth sub models, we establish function relationships between single rally winning probability and single game winning probability as well as single rally winning probability and single match winning probability, and use curve fitting method simplifies functions expressions [8-11].

(1) Sub model one: Player on spot competitive capacity model
A badminton player technical level is affected by variety of factors, including strength, techniques, speed and reaction capacity and psychological quality etc. Meanwhile, different players have their specialty in service, return of the service and first three strokes, central and farther station stalemate capacity. It is difficult to make comprehensive evaluation of players’ competitive levels so as to weigh each factor. To discuss the simplified model, on the pre-conditions of no affect validity, it can be thought that players’ fixed technical level can use standardized indicator $\mu$ to weight, and we define $\mu$ as players fixed technical level; only when $\mu_a > \mu_b$, it can think that player $a$ technical level is higher than that of player $b$. Meanwhile, in competitions also should take variety of non-technical factors into consideration. Considering above causes, players levels in competition not only have correlations with variety of factors, but also these factors all belong to random variables, so it is difficult to establish precise models. According to Independent identically distributed central limit theorem give player single rally competition on spot competitive capacity as random variable $X$, it gets:

$$X \sim N(\mu, \sigma^2)$$  \hspace{1cm} (1)

From which $\mu$ represents players’ fixed technical level (technical levels display in average state), $\sigma$ represents player steady state playing (deviation degree of game level and fixed level).

(2) Sub model two: Single rally probability model
Assume player $a$ and $b$ during one rally competition, $a$ spot performance is $X_1$, $b$ spot performance is $X_2$. $a$ winning probability in the rally is $P\{X_1 > X_2\}$. Due to basic assumption without considering mutual interference among players, then it is thought that random variables $X_1$, $X_2$ are independently identically distributed. Since $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$, so that X1 probability density function is:

$$f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$$  \hspace{1cm} (2)

X2 probability density function is:

$$f_2(x) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right)$$  \hspace{1cm} (3)

X1, X2 joint probability density function is:


\[ f(x, y) = \frac{1}{2\pi \sigma_1 \sigma_2} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2} - \frac{(y-\mu_2)^2}{2\sigma_2^2}\right) \]  

(4)

\( a \) Winning probability in single rally competition is:

\[ p = \int_{x, y} \frac{1}{2\pi \sigma_1 \sigma_2} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2} - \frac{(y-\mu_2)^2}{2\sigma_2^2}\right) dx dy \]  

(5)

Then b winning probability in single rally competition is: \( q = 1 - p \).

(3) Sub model three: single game probability model

From null hypothesis (2), we can work out player \( a \) and \( b \) single rally competition winning probability with classical probability theory’s method. With the background information that put forward in problems analysis, we can respectively calculate no follow probability and follow probability. We take player \( a \) winning status as examples to make analysis (The analysis would not affect question’s essence, because \( a \), \( b \) have symmetry here, so we only analyze \( a \) winning status).

(i) The status that draws (i-1) not held between two parties is that \( a \) won by achieving \( i \) balls, and the rally ended. Through analysis we can know:

\[ P(A_i) = \sum_{n=1}^{\infty} A_n \]  

(6)

Every known rally can be regarded as Bernoulli experiment, then:

\[ P(An) = C_{n-1}^{i-1} p^{i-1} q^{n-i} p = C_{n-1}^{i-1} p^1 q^{n-i} \quad (n = i + k, k = 0, 1, 2, \cdots , i - 2) \]  

(7)

(ii) In case that two parties held draw(i-1), then the two continue to play until \( a \) got 2 score more than \( b \), and then \( a \) won the game.

From above analysis, it is known that total \( n \) balls in competition at this time, \( n = 2(i - 1) + 2m \), from which \( m = 1, 2, 3, \cdots \). For the question, we can analyze in 3 stages, the first stage is previous \( 2(i - 1) \) balls, \( a \), \( b \) each win \( i - 1 \); in the second stage in order to play \( 2m \) follows, from which every two rallies can be regarded as one round; In each round of previous \( m - 1 \) rounds, \( a \), \( b \) each won 1 ball, while in \( m \) round, \( a \) continuously won two balls and won the game that is:

\[ P'(An) = C_{2(i-1)}^{i-1} p^{i-1} q^{i-1}(2pq)^{m-1} p^2 = C_{2(i-1)}^{i-1} p^{i-1} q^{i-1}(2pq)^{m-1} \]  

(8)

So under i points system, every competition \( a \) winning probability is:

\[
\begin{align*}
    f_i(p) & = \sum_{k=0}^{i-2} C_{i+k-1}^{i-1} p^k q^{i-1-k} + \sum_{m=1}^{\infty} C_{2(i-1)}^{i-1} p^{i+1} q^{i-1}(2pq)^{m-1} \\
    & = p^i \sum_{k=0}^{i-2} C_{i+k-1}^{i-1} q^k + C_{2(i-1)}^{i-1} p^{i+1} q^{i-1} \frac{1}{1 - 2pq}
\end{align*}
\]  

(9)

Use MATLAB software draw relation figure between single rally winning probability \( p \) and single game winning probability \( f_i(p) \) (as Figure 1), in which we extend the definition \( i \), define \( i = 7, 11, 17, 21, 27 \) (In hypothesis,
only according to subjects define $i = 11, 21$, and make extension of the value $i$ determining here, i still keeps definition by original hypothesis in the following). Figure 1 is relation figure between single rally winning probability $p$ and single game probability $f_i(p)$ when $i = 7$.

![Figure 1: 7 points system single rally winning probability and single game probability functional relations](image)

In Figure 1, function line successively represents 7 points system single rally winning probability and singe game probability functional relations. Similarly, it draws single rally winning probability and single game probability functional relations figure under $i = 15, 17, 21, 27$. From Figure 1, we can think single rally winning probability shows player comprehensive competitive capacity. According to questions defining competition occasionality, we can think when single game winning probability completely unrelated to single rally winning probability that as no matter what two parties players competitive capacities would be, the winning probability is always 0.5, the competition result is completely accidental; when single game winning probability entirely depends on single rally winning probability that as higher competitive capacity players would surely win, then competition is completely non-accidental(as Figure 1); The higher points system that adopted in competition laws, the closer function curve would get to non-accidental function line, the lower points system that changed in competition laws, the curve function would get to completely accidental function line. In figure, inclination of curve reflects contingency under different points system, from 7 points system to 27 points system, $\alpha$ in each curve is gradually increasing, while occasionality successively deceasing. It can be found that negative correlation existing between $\alpha$ and occasionality. Figure 1 defining $x = 0.5$ and $y = 0.5$ are two asymptotic lines. When $\alpha$ changes from 0 to $\infty$, curve sweeps $[0.0.5, 0.5], [0.5]$, and $[0.5, 1], [0.5, 1]$ two areas. In subsequently models, we just use $\alpha$ to reflect different competition systems' occasionality indicators that bring great convenience to calculation.

(4) Sub model four: Single match probability model

Badminton circuit normally adopts $2h - 1$ games $h$ winning competition law, we assume that in player $a$, $b$ $2h - 1$ games competition, each game competition is independent, which is also giving player $a$ the same winning probability as $f_i(p)$. Given event $B(s)$ to be "b totally won $s$ games, and finally $a$ won", then it has:

$$P(B(s)) = C_{h+1}^s (1 - f_i(p))^s f_i(p)$$  \hspace{1cm} (10)

Given event $B$ to be "a won the competition", then it has:

$$P(B) = \sum_{s=0}^{h-1} P(B(s)) = f_i(p) \sum_{s=0}^{h-1} C_{h+1}^s (1 - f_i(p))^s$$  \hspace{1cm} (11)
Therefore we have expressions as following:

\[
\varphi_i(2h-1, h, p) = f^h_i(p) \sum_{s=0}^{h-1} C^s_{h+s-1}(1 - f^h_i(p))^s
\]  \hfill (12)

According to above computational methods, single match winning probability is the function of single game winning probability, therefore it is also the function of single rally winning probability. We utilize MATLAB presenting functional figure about subjects' required 4 schemes single match winning probability and single rally winning probability correlations. Through observation of function graphics, we found that graphics all go through fixed point \((0.5, 0.5)\), the graphics is very similar to hyperbolic tangent function figure, so we adopt hyperbolic tangent function to fit curve here.

Through formula (9) and (12), we can know that if directly output single match winning probability and single rally winning probability functional expressions by compound functional relations, the expressions would be very complicated, therefore we approximate complicated polynomial into hyperbolic tangent function through curve fitting, actually is to implement power series expansion inverse calculation. We bring into fitting function \(g(x)\), here, that is:

\[
g(x) = \frac{2}{1 + \exp(-2\alpha(x - 0.5))} - 0.5
\]  \hfill (13)

Through fitting functions, it can make approximate calculation of \(\varphi_i(2h-1, h, p)\) as following:

\[
\varphi_i(2h-1, h, p) = g(p, \alpha^{(i)}_h) = \frac{2}{1 + \exp(-2\alpha^{(i)}_h(p - 0.5))} - 0.5
\]  \hfill (14)

Among them, \(\alpha^{(i)}_h\) is corresponding fitting functions inclination achieved when competition system is \((i, h)\).

We get 4 cases inclination \(\alpha^{(i)}_h\) by MATLAB curve fitting functions; the exact values refer Table 1.

Table 1: 4 kinds of competition system plans inclination

<table>
<thead>
<tr>
<th>Competition system</th>
<th>Inclination (\alpha^{(i)}_h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 points three out of five sets</td>
<td>4.1481</td>
</tr>
<tr>
<td>21 points two out of three sets</td>
<td>4.2938</td>
</tr>
<tr>
<td>15 points four out of seven sets</td>
<td>4.2970</td>
</tr>
<tr>
<td>21 points three out of five sets</td>
<td>4.4600</td>
</tr>
</tbody>
</table>

In fitting process, in order to improve curve fitting precise, used value \(P\) range is \((0.3, 0.7)\), such breaking both two ends fitting method increases fitting curve consistency to great extent; According to actual, we get fitting curve variable \(P\) range as \((0.3, 0.7)\), when \(p \in (0,0.3)\), function value is always 0, when \(p \in (0.7,1)\), function value is always 1.

By above analysis, we can make below approximate to \(\varphi_i(2h-1, h, p)\) (the approximate adopting in all subsequent calculations); the exact explanatory notes are as following:

\[
\varphi_i(2h-1, h, p) = \begin{cases} 
0 & , \quad (0 < p < 0.3) \\
\frac{2}{1 + \exp(-2\alpha^{(i)}_h(p - 0.5))} - 0.5 & , \quad (0.3 < p < 0.7) \\
1 & , \quad (0.7 < p < 1)
\end{cases}
\]  \hfill (15)
Moving average indicator model
Moving average indicator model mainly including 3 sub models, which are occasionality indicator model, intensity indicator model, time type indicator model. Sub model 1 through single match winning probability and single game winning probability functional curve inclination reflect occasionality indicator. In sub model 2, through comparison of two competition systems single match winning probability approximate degree to 0.5 define competition intensity, simplify calculation process by previous stated model fitting curve functions. In sub model 3, it just targets for side-out scoring system influences on total competition time, assuming other conditions are same. According to $A, B$ two people whole competition each rally data, makes statistics and gets probability of continuously achieving, obtains that time before reform is 1.9 times to that after reform. It can fully see that competition time is largely shortened.

(1) Occasionality indicator model
Through analyzing figure 1, we can know that curve inclination angle can be used to reflect occasionality indicator under the competition system, due to $\alpha$ be negative correlated to occasionality $OC$, and we define below functions:

$$OC(i, h) = \frac{10}{1 + (\alpha_h^{(i)})^2} - 0.1$$

From formula (16) we can get each scheme occasionality indicators (refer to Table 2).

Table 2: 4 kinds of competition system plans occasionality indicators

<table>
<thead>
<tr>
<th>Competition system</th>
<th>Occasionality indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 points three out of five sets</td>
<td>0.4492</td>
</tr>
<tr>
<td>21 points two out of three sets</td>
<td>0.4145</td>
</tr>
<tr>
<td>15 points four out of seven sets</td>
<td>0.4138</td>
</tr>
<tr>
<td>21 points three out of five sets</td>
<td>0.3787</td>
</tr>
</tbody>
</table>

From Table 2 we can get 3 conclusions as below, 15 points system three out of five sets occasionality is higher than that of 21 points system two out of three sets; 15 points system four out of seven sets occasionality is higher than that of 12 points system three out of five sets; 15 points system four out of seven sets occasionality is roughly the same as that of 21 points system two out of three sets.

Solved occasionality indicator in the model actually is a kind of competition system approximate probability that is players’ individual difference effects on occasionality indicators would not be considered in the competition system, though certain errors existing, its approximate effects are good.

(2) Intensity indicator model
Before modeling, we firstly should make definition of intensity indicators here. If previous stated occasionality indicators lay particular importance on analysis of competition results, then intensity indicators even lay particular importance on competition process. On the pre-conditions that feasibility not losing in model, it sees from the angle of each game scores status, but we don’t use single game scores approximate degree to express competition inspiration here while simplify intensity indicator functional expressions through different competition system one player single match winning probability approximate degree to 0.5 to express competition intensity. In the following we provide intensity indicator functional expressions:

Intensity indicator function $\Omega_h^{(i)}(p) = 1 - 4(\varphi(2h - 1, h, p) - 0.5)^2$, then it has:

$$\Omega_h^{(i)}(p) = \begin{cases} 0 & \text{if} \quad \Omega_h^{(i)}(p) \leq 0.5 \\ \left[1 - 4\left(\frac{2}{1 + \exp(-2\alpha_h^{(i)}(p - 0.5))} - 1\right)^2 \right] & \text{if} \quad \Omega_h^{(i)}(p) > 0.5 \end{cases}$$

In formula we apply squared differences algorithm still can extract value of $P$ in the interval $(0, 1)$. From the expression, we can know $\Omega_h^{(i)}(p)$ value range is also in the interval $(0, 1)$. We can think that when $\Omega_h^{(i)}(p)$ is 0, competition almost has no intensity; while $\Omega_h^{(i)}(p)$ is 1, competition has already entering into the much more intensive stage. By MATLAB, we present 4 kinds of plans intensity indicator curve as Figure 2.
From functions image, we can provide corresponding analysis, 15 points system competition intensity is above that of 21 points system competition. For subjects mentioned 4 kinds of plans, we mark them according to intensity, then it has got 15 points system four out of seven sets>15 points system three out of five sets>21 points system three out of five sets>21 points system two out of three sets. Due to every competition system intensity degree affected by $P$ that is different from occasionality factors; it is not a fixed value.

(3) Time type indicator model
The model is just targeted on side-out scoring system influences on total competition time, assuming other conditions are the same. According to $A, B$ two people whole competition each rally data, make statistics and get probability of continuously achieving $i$ scores. Make statistical summary on player $A$ whole competition data; get the proportion that player $A$ continuously achieving $i$ balls of total scores as $P_i(A)$ $(i=1,2,3,4,\ldots)$. Determine above formula sum according to below formula, get before reform $N_A$ and after reform $N_A'$ total score probability:

$$N_A = \sum_{i=1}^{n} i \cdot P_i(A) \quad N_A' = \sum_{i=1}^{n} (i - 1) \cdot P_i(A)$$

Proportional relations between established total scores probability and competition time consumption before and after reform is

$$h_A = \frac{N_A}{N_A'}$$

after reform is $h_A' = \frac{N_A'}{N_A}$. Make total competition time consumption before reform under same circumstance

$$h_A = \frac{N_A}{N_A'}$$

and after reform $h_A' = \frac{N_A'}{N_A}$. Make statistical summary on player $B$ whole competition data; get the proportion that player $B$ continuously achieving $i$ balls of total scores as $P_i(B)$ $(i=1,2,3,4,\ldots)$. Determine above formula sum according to below formula, get before reform $N_B$ and after reform $N_B'$ total score probability:

$$N_B = \sum_{i=1}^{n} i \cdot P_i(B) \quad N_B' = \sum_{i=1}^{n} (i - 1) \cdot P_i(B)$$

Proportional relations between established total scores probability and competition time consumption before and after reform is

$$h_B = \frac{N_B}{N_B'}$$

after reform is $h_B' = \frac{N_B'}{N_B}$. Due to make respective data statistics among opponents, their data have a certain correlation, therefore

In order to better verify before and after side-out scoring system canceling effects on total competition time consumption, using badminton men singles final Li Zhong-Wei verse Lin Dan data in London Olympic Games, 2012 as an example like following Table 3.

Table 3: London Olympic Games men badminton single final scores statistics (2012.08.05)

<table>
<thead>
<tr>
<th>The first game</th>
<th>The second game</th>
<th>The third game</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 0</td>
<td>0: 1</td>
<td>1: 1</td>
</tr>
<tr>
<td>2: 1</td>
<td>1: 2</td>
<td>2: 2</td>
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<tr>
<td>3: 2</td>
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<td>15: 16</td>
<td>15: 16</td>
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<td>16: 16</td>
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<td>16: 17</td>
</tr>
</tbody>
</table>

Utilize model 3 to make statistics and calculate each rally scores as Table 4.

Table 4: Each rally scores statistics and calculation

<table>
<thead>
<tr>
<th>Scores</th>
<th>Li Zhong-Wei</th>
<th>Lin Dan</th>
<th>Scores</th>
<th>Li Zhong-Wei</th>
<th>Lin Dan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.34</td>
<td>0.4</td>
<td>0</td>
<td>0.34</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.32</td>
<td>1</td>
<td>0.28</td>
<td>0.32</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>0.18</td>
<td>2</td>
<td>0.3</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>0.24</td>
<td>3</td>
<td>0.08</td>
<td>0.24</td>
</tr>
<tr>
<td>2.12</td>
<td>2.54</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Through finals total time consumption comparison, get time before reform is 1.9 times to time after reform. It can fully see that competition time is greatly shortened. In this way, international badminton federation can more leisurely arrange competitions, and audience can also watch more competitions.
Competition schemes evaluation model

For competition system schemes evaluation, we can explain from relativity evaluation and comprehensive evaluation such two aspects.

(1) Relativity evaluation: The subjects provide us four competition system schemes, for each scheme, no absolute good or bad; it only has relative adequacy and inadequacy. By above comparison, it can find that generally speaking, 15 points competition occasionality is far larger than that of 21 points system. From Table 2 it can see that adopts 15 points three out of five sets occasionality is nearly 20% higher than that of 21 points three out of five sets. Occasionality improving increase badminton circuit ornamental that make the competition of more suspense, but excessive occasionality would let competition lose its competitive significance. According to results in Table 2, this paper thinks that adopts 15 points system three out of five sets leads to too large competition occasionality, it need to avoid to use the scheme in the important international competitions; Adopt 15 points system four out of seven sets and 21 points system three out of five sets are generally equal that are relative reasonable competition systems.

On the condition that presently badminton players and audiences are widely believed that 15 points system occasionality too large that affects competition equality, it can think about to adopt competition system between 15 points system and 21points system, such as 17 points system, 13 points system and so on, so as to better achieve badminton promotion and increase competitions ornamental, meanwhile maintain equal competition objects.

(2) Comprehensive evaluation model

(i) Evaluation model based on TOPSIS grey correlation degree

We will not repeat the method concrete steps here but direct provide evaluation table, determine each scheme comprehensive evaluation values. We will not consider weight changes here, value intensity indicator weight as 0.6, and occasionality indicator weight as 0.4. In previous model, we have already mentioned that intensity indicator is a function with regard to $p$, but we use $p = 0.4$ to approximate one competition system average intensity here (now temporarily ignore players differences), we can get following evaluation table (refer to Table 5).

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Occasional indicator</th>
<th>Intensity degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 points three out of five sets</td>
<td>0.4492</td>
<td>0.3548</td>
</tr>
<tr>
<td>21 points two out of three sets</td>
<td>0.4145</td>
<td>0.2991</td>
</tr>
<tr>
<td>15 points four out of seven sets</td>
<td>0.4138</td>
<td>0.3975</td>
</tr>
<tr>
<td>21 points three out of five sets</td>
<td>0.3787</td>
<td>0.3447</td>
</tr>
</tbody>
</table>

Input weight value into table get weighting decision standard matrix $Z$

$$Z = \begin{pmatrix} 0.1797 & 0.2129 \\ 0.1658 & 0.1795 \\ 0.1655 & 0.2385 \\ 0.1515 & 0.2068 \end{pmatrix}$$

We select visual sequence $c = (0.1515, 0.2385)$ and respectively determine 4 schemes visual sequence grey correlation values that are $r_1 = 0.0174, r_2 = 0.0198, r_3 = 0.0217, r_4 = 0.0217, r_5 = 0.0208$. Through above analysis, 21 points three out of five sets and 15 points four out of seven sets are reasonable competition systems.

(ii) Optimal indicator evaluation model based on variable weight function.

Before modeling, we firstly provide below definition, record subjects mentioned 4 schemes respectively as $j = 1, 2, 3, 4$, make corresponding relations existing as $(i, h) \rightarrow j$ we record as:

$$(15,3) \rightarrow 1, \ (21,2) \rightarrow 2, \ (15,4) \rightarrow 3, \ (21,3) \rightarrow 4 \quad (18)$$

From formula (16), (17) it is known that for competition system $j$, its occasionality is a constant value without influence from participating players levels differences, while competition intensity degree obviously affected by players’ levels( that is influence of value $P$), therefore we need to define a variable weighting function to distinguish occasionality and intensity degree weight when $P$ values are different. In the following we present

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evaluation function, variable function as well as optimal evaluation value. Here based on symmetry analysis, we assume that $0 < p < 0.5$, we all adopt such symmetry assumption in the subsequent models’ analysis.

Define $M^{(j)}(p)$ as evaluation function in $j$ scheme, $\lambda(p, \alpha)$ is corresponding variable weight function, $W(j)$ is optimal evaluation value in $j$ scheme. Then it has:

$$M^{(j)}(p) = (1 - \lambda(p, 8)) \Omega_h^{(j)}(p) + \lambda(p, 8) \varphi(2h - 1, h, p)$$

$$W(j) = \text{Max}\{M^{(j)}(p)\}$$

$$\lambda(p, \alpha) = \begin{cases} 
1 & (0 < p < 0.3) \\
\frac{2}{1 + e^{2\alpha(p - 0.3)}} & (0.3 < p \leq 0.5) 
\end{cases}$$

Here we make new definition of occasionality indicator that is the probability that when one party player is weaker (indicates that $P$ values intervals is $(0,0.5)$), it still won the game, therefore occasionality indicator that got at this time is function of $P$ not the constant value in previous stated model. We carry out comprehensive evaluation through different schemes $W(j)$ comparing.

**Optimal competition system exploring model**

Though this paper mainly judge the merits of 15 points competition system, we cannot deny that there is another competition system whose function value according to formula (20) evaluation plan is better than 15 points system. We deduce exploring optimal competition system scheme based on variable weight function optimal indicator evaluation model here, according to above model definition, it is known that $P$ range is $(0, 0.5)$ here.

Here, we need to extend $i$ and $h$ intervals, we make following definition.

$$(i_1, h_1) \rightarrow j_1 = \begin{cases} 
i_1 \\
h_1 
\end{cases}$$

From which, $i_1$ is all odd number values in the interval $[5, 27]$, $h_1$ is all integral values in the interval $[2, 9]$, $j_1$ is corresponding scheme of $(i_1, h_1)$. In the following, we provide the exact model:

$$G(i_1, h_1) = \text{Max}F(i_1, h_1, p)$$

$$F(i_1, h_1, p) = (1 - \lambda(p, 12)) \Omega_h^{(i_1)}(p) + \lambda(p, 12) \varphi(2h - 1, h, p)$$

$$\Omega_h^{(i_1)}(p) = \begin{cases} 
0 & 0 < p < 0.3, 0.7 < p < 1 \\
1 - \frac{2}{1 + \exp\left(-2\alpha_{h_1}(p - 0.5)\right)} & 0.7 < p < 1
\end{cases}$$

In above defined ranges, through exchanging different combines $(i_1, h_1)$ can get corresponding value of $G(i_1, h_1)$. We get the optimal value of $G(i_1, h_1)$ through searching $i_1, h_1$ in given ranges; its corresponding $(\hat{i}_1, \hat{h}_1)$ is the optimal competition system scheme that we explored.
CONCLUSION

This paper simply adopted normal distribution model on players’ competition performance, actually players’ performance influence factors except for technical levels, it had steady performance and other factors. Such as, 15 points system due to its short schedule causes great psychological pressures to players, players psychological quality would be one of the important factors that affect competition results. Besides, players performance is not fixed, actually players performance in each rally is a random variable. Meanwhile, variety of factors affect one badminton players technical levels, which including strength, techniques, speed and reaction ability and so on. Different players in service return of service and smash other segments have their characteristics. It is difficult to make comprehensive evaluation on players’ competitive levels and weight each factors. This paper through one standardized indicator $\mu$ measured players’ competitive levels, and it also considered other non technical factors, from which players’ psychological quality and psychological state have great effects on technical levels playing.

REFERENCES