A weighted compensation of coordinate error localization algorithm based on RSSI

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ABSTRACT

Node self-localization is one of the key technologies in wireless sensor network (WSN) applications. Aiming at the problem of inaccurate RSSI ranging process due to environmental interference, this paper presents an algorithm of weighted compensation of coordinate error after taking the environment similar feature into account between reference node and blind node. The algorithm first selects the three suitable reference nodes to calculate the preliminary coordinate of blind node by using the signal propagation loss model and the centroid of intersection model, then getting three groups coordinate errors by reference nodes location each other. Finally, as coordinate error weighted to the blind node coordinate, getting the ultimate estimate coordinates of blind node. This paper also makes some analysis on the choice of reference distance. Simulation results in MATLAB show that this algorithm can improve the positioning accuracy effectively, and choose a different reference distance which has different positioning accuracy.

Keywords: Wireless Sensor Network, Localization Algorithm, RSSI ranging, Error Compensation, Weighting

INTRODUCTION

In order to achieve the detection and tracking of target information in wireless sensor network [1-2], we need to know the specific location of monitored events, so node self-localization is a foundation of sensor network application.

At present, several algorithms and techniques have been proposed for sensor network location estimation. All these techniques can be divided into two broad categories based on the type of information used in localization: range-based and range-free. Range-based techniques need point-to-point distance or angle information to calculate the location which is done by triangulation, trilateration or maximum likelihood estimation, it mainly includes Received Signal Strength Indicator (RSSI), Time of Arrival (TOA) [3], Time Difference of Arrival (TDOA), and Angle of Arrival (AOA) techniques. Range-free techniques do not need distance information; while they estimate the location depending on the information of proximity of the reference nodes. Compared with the range-based techniques, it reduces the demand for hardware, but increases the corresponding position error. Centroid, DV-Hop, Amorphous [4], and APIT [5] are four important range-free techniques.

RSSI ranging technology is the most commonly used location technology, because of general RF circuit has the ability to capture the value of RSSI, and it requires no additional hardware support but easy implement, low cost, the positioning accuracy of the technique is easily affected by environmental factors. How to improve the positioning accuracy effectively has become one of the hot spots of current research. Hence, the author puts forward an improved localization algorithm based on RSSI ranging, the algorithm uses the reference node location to obtain the position error. Then, take weighted to the blind node coordinate into consideration. It can enhance the node adaptability to the environment, improve its positioning accuracy.
2 Description and analysis of algorithms model
2.1 RSSI ranging model
RSSI is the size of the received electromagnetic energy, it decreases accompanies with the increase of distance, and easy to be affected by various environmental factors. In the free space, the relationship between RSSI and distance from transmitter to receiver can be given by Friis transfer equation [6] in antenna theory, the equation is:

\[ P_t(d) = \frac{P_t G_s l^2}{(4\pi)^2 d^2} \quad (1) \]

Where \( P_t \) is the transmission power in Watts (W), \( P_r \) is the received power in Watts (W), \( G_s \) and \( G_r \) respectively is the sender and receiver’s antenna gain, \( l \) is the wavelength. Generally, \( G_r \) and \( G_s \) are selected to be 1.

The path loss is expressed as the relative size between the effective radiated power and the receiving power, as the following formula:

\[ PL(d) = 10 \log \left( \frac{P_t}{P_r} \right) = -10 \log \left[ \frac{G_s G_r l^2}{(4\pi)^2 d^2} \right] \quad (2) \]

Further the following formula can be obtained:

\[ PL(d) = PL(d_0) + 20 \log \left( \frac{d}{d_0} \right) \quad (3) \]

Where \( PL(d) \) is the total path-loss measured in Decibels(dB), \( d \) is the length of the path in meter, \( d_0 \) is the reference distance in meter, \( PL(d_0) \) is the pass-loss at the reference distance \( d_0 \) in dB.

But in the practical environment, the signal transmission is often anisotropic due to the factors of multipath reflections, diffraction, obstructions, etc. It will cause the energy received within a certain distance which has some volatility, the larger errors path-loss measured. So the Shadow model is obtained through the combination of analysis and experience, as shown in equation (4):

\[ PL(d) = PL(d_0) + 10n \log \left( \frac{d}{d_0} \right) + X_s \quad (4) \]

Where \( n \) is the path-loss distance exponent, usually in the range of 2–6; \( X_s \) meet the Gaussian distribution which mean is 0 and variance is \( s^2 \) (usually between 4-10). The \( RSSI \) value which is received by the unknown node from the reference node is:

\[ RSSI [dBm] = P_t [dBm] - PL(d) \quad (5) \]

Where \( P_t [dBm] \) is the transmission power in dBm. Assumption that A is the RSSI at reference distance \( d_0 \), as the following formula:

\[ A = P_t - PL(d_0) \quad (6) \]

Simultaneous solution formula (4) to (6), we obtain the following equation:

\[ RSSI = A - 10n \log \left( \frac{d}{d_0} \right) - X_s \quad (7) \]

\( X_s \) is a random variable with zero mean:
\[
\overline{\text{RSSI}} = A - 10n \log\left(\frac{d}{d_0}\right) \tag{8}
\]

Where \(\overline{\text{RSSI}}\) is the average value of \(\text{RSSI}\), we can obtain the following formula:
\[
d = d_0 \cdot 10^{\frac{A - \overline{\text{RSSI}}}{10n}} \tag{9}
\]

We can calculate the distance \(d\) between transmitter and receiver by formula (9).

\(A\) and \(n\) are the two parameters of formula (9) that determine the relationship between the RSSI and the distance \(d\). Now, in order to analyze the impact of \(A\) and \(n\) ranging, suppose that the reference distance was 1 meter. Firstly, when \(n\) is fixed to take 3, \(A\) ranges from -60dBm to -35dBm, then, \(A\) is fixed to take -45dBm, \(n\) ranges from 2 to 4.5. The simulation results are as follows (Fig.1 to Fig.2).

Fig.1 \(A\) constant but \(n\) changed, the relationship between RSSI and \(d\)

Fig.2 \(n\) constant but \(A\) changed, the relationship between RSSI and \(d\)

Fig.1 and Fig.2 show that when short distance wireless signal propagation attenuation is very fast, the long distance signal attenuation is very slow, we can draw a conclusion that the closer the distance measurement is, the higher the localization accuracy will be by RSSI ranging.

In the application, for the convenience of reference distance is to select 1 meter, but in fact, according to the exponential relationship between RSSI and distance, if the reference distance \(d_0\) taken a value close to the distance \(d\), it would make the distance measurement more accurate. For example, if the distance \(d\) from blind node to beacon nodes all were in the vicinity of 5 meters, then reference distance \(d_0\) may select 5 meters, \(A\) was the RSSI value at 5 meters; in a similar way, if the distance \(d\) was in the vicinity of 10 meters, \(d_0\) may select 10 meters, \(A\) was the RSSI value at 10 meters.
After obtaining the multiple distances between nodes by RSSI value, the coordinates of the blind node is calculated.

### 2.2 Centroid of the intersection algorithm model

After obtaining the multiple distances between nodes by RSSI value, the coordinates of the blind node is calculated.

![Fig.3 Schematic diagram of Trilateration localization](image)

The traditional trilateration localization is shown in Fig.3. Suppose that A, B, C are reference nodes, their coordinates are $A(x_a, y_a)$, $B(x_b, y_b)$ and $C(x_c, y_c)$, the coordinate of undetermined node is $M(x, y)$, we can know the distance $d_a$, $d_b$ and $d_c$ respectively by formula (9). A non-linear group of equations as formula (10) is obtained according to the computing formula of two-dimensional distance [7]:

$$
\begin{align*}
\sqrt{(x - x_a)^2 + (y - y_a)^2} &= d_a \\
\sqrt{(x - x_b)^2 + (y - y_b)^2} &= d_b \\
\sqrt{(x - x_c)^2 + (y - y_c)^2} &= d_c
\end{align*}
$$

(10)

Coordinates of the undetermined location node $M$ can be obtained as shown in formula (11) using linearization methods.

$$
\begin{align*}
\hat{e}_x &= \frac{1}{d_a} (\hat{e}_{x_a} - x), \quad \hat{e}_{y_a} = \frac{1}{d_a} (\hat{e}_{y_a} - y), \\
\hat{e}_y &= \frac{2}{d_a} (\hat{e}_{y_a} - y), \quad \hat{e}_{x_a} = \frac{1}{d_a} (\hat{e}_{x_a} - x) + d_a \hat{e}_{y_a} \\
\hat{e}_x &= \frac{1}{d_b} (\hat{e}_{x_b} - x), \quad \hat{e}_{y_b} = \frac{1}{d_b} (\hat{e}_{y_b} - y), \\
\hat{e}_y &= \frac{2}{d_b} (\hat{e}_{y_b} - y), \quad \hat{e}_{x_b} = \frac{1}{d_b} (\hat{e}_{x_b} - x) + d_b \hat{e}_{y_b} \\
\hat{e}_x &= \frac{1}{d_c} (\hat{e}_{x_c} - x), \quad \hat{e}_{y_c} = \frac{1}{d_c} (\hat{e}_{y_c} - y), \\
\hat{e}_y &= \frac{2}{d_c} (\hat{e}_{y_c} - y), \quad \hat{e}_{x_c} = \frac{1}{d_c} (\hat{e}_{x_c} - x) + d_c \hat{e}_{y_c}
\end{align*}
$$

(11)

In the real environment, owing to the influential factors of multipath reflections, diffraction, obstructions, etc. The distance $d$ measured by RSSI value will always larger than the actual distance. Thus leads to three rounds will not meet in a point, but there is a cross region as Fig.4. Six coordinates are obtained through the trilateration localization, and then select three points inside form a triangle, finally, consider centroid of the triangle as the final estimate of the blind node coordinate. But if any two of the three circles do not intersect, it may have a plural solution. Now, we take the centroid of intersection algorithm which is the improved method for traditional trilateration localization. The square of equation (10) can be written equation (12) after the transformation.

$$
\begin{align*}
\frac{2}{d_a} (x - x_a)^2 - \frac{2}{d_a} (y - y_a)^2 &= \frac{1}{d_a} \hat{e}_{x_a}^2 + \frac{1}{d_a} \hat{e}_{y_a}^2 + \frac{2}{d_a} d_a \hat{e}_{y_a} \\
\frac{2}{d_b} (x - x_b)^2 - \frac{2}{d_b} (y - y_b)^2 &= \frac{1}{d_b} \hat{e}_{x_b}^2 + \frac{1}{d_b} \hat{e}_{y_b}^2 + \frac{2}{d_b} d_b \hat{e}_{y_b} \\
\frac{2}{d_c} (x - x_c)^2 - \frac{2}{d_c} (y - y_c)^2 &= \frac{1}{d_c} \hat{e}_{x_c}^2 + \frac{1}{d_c} \hat{e}_{y_c}^2 + \frac{2}{d_c} d_c \hat{e}_{y_c}
\end{align*}
$$

(12)

Solving matrix can be obtained:
The \((x, y)\) is the coordinate of blind node M. The centroid of intersection algorithm all can obtain the real solution of coordinate of blind node regardless whether any two of the three circles do intersect.

### 2.3 Optimization of beacon nodes compensation model

After receiving multiple RSSI values from the beacon nodes, Blind node M ranks the RSSI from large to small. Now for analysis, select the nearest four beacon nodes which are near to far order of A, B, C, D. As shown in Flg.5.

**Fig.5 Analysis diagram of optimization of beacon nodes model**

Assuming that the actual coordinates of the four beacon nodes are \(A (x_a, y_a)\), \(B (x_b, y_b)\), \(C (x_c, y_c)\), and \(D (x_d, y_d)\). In order to obtain the coordinate error of reference nodes, we let reference nodes locate each other. Select the front of the three nodes A, B, C to locate mutually. Firstly, put node A as the unknown node, assuming coordinates is \(A' (x'_a, y'_a)\), put B and C as reference nodes, and the distance from A to B, C are \(d_{ab}\), \(d_{ac}\) respectively, then:

\[
\begin{align*}
\frac{1}{2} \sqrt{(x'_a - x_b)^2 + (y'_a - y_b)^2} &= d_{ab} \\
\frac{1}{2} \sqrt{(x'_a - x_c)^2 + (y'_a - y_c)^2} &= d_{ac}
\end{align*}
\]  

Solution formula (14) can get two sets of coordinates, select a set of solutions which is close to actual node A as the estimates of node A, we can get the coordinate error values \(\text{DA} (\text{DA}_a, \text{DA}_a)\), where \(\text{DA}_a = x_a - x'_a\), \(\text{DA}_a = y_a - y'_a\); Similarly, we can obtain the coordinate error values of node B through putting node B as the unknown node, put A and C as reference nodes, the error is \(\text{DB} (\text{DB}_b, \text{DB}_b)\); In such a push to get the coordinate error of node C, the error is \(\text{DC} (\text{DC}_c, \text{DC}_c)\). When positioning for blind node M, we select nodes A, B, C as the reference node which are the closest, this covered region of the mutual positioning of the three nodes is the \(S_{ABC}\), the area covered when location for blind node M is the \(S_{MABC}\), area \(S_{MABC}\) and \(S_{MABC}\) are almost overlapped as shown in Flg.5, it can be considered that they have the same environment of position. In other words, they have the same impact factors for environment, so using the coordinate errors \(\text{DA} \), \(\text{DB} \), \(\text{DC}\) to compensate for the blind node will be effective.

Now it is the analysis of the node D. When positioned on the node D, it is close to B and C, so put B and C as reference nodes, similarly we can obtain the coordinate error of node D, the error is \(\text{DD} (\text{DD}_d, \text{DD}_d)\), the area covered when location for node D is the \(S_{DBC}\), area \(S_{DBC}\) and \(S_{MABC}\) are almost no overlapped, even if there were only a little, they are not the same environment of positioning, so using the coordinate error \(\text{DD}\) to
compensate for the blind node is meaningless. Assuming again that the position of the node D is in $D'$ as Fig. 5, then select recent node A and C as the reference nodes, compared the area $S_{DAC}$ of location covered with $S_{MABC}$ also has no overlapped.

Another analysis is when node A is a unknown node, if you select B, C, D reference nodes to locate node A, compared the area $S_{ABCD}$ of location covered with $S_{MABC}$ will have few overlapped. So this option is not desirable.

We can make a conclusion from comprehensive analysis: for location, the blind node M only selects the nearest three nodes A, B and C as reference nodes and uses the coordinate error to compensate is effective.

Choosing what is the weighting factors of influence when using compensation of coordinate error. According to the path loss propagation model we can know that the further the reference node to blind node, the greater the error of distance by RSSI ranging. Then the smaller decision weight of reference node for the blind node.

For example, assume that the node M is close to coincide with node A, in other words the distance is close to 0, then the coordinate error of node A can be approximately regarded as the coordinate error of blind node M, this also shows that the closer distance is, the greater weight will be. So choosing the weight $W = 1/d_i$ can reflect the influence of reference node to the blind node. The coordinate error of blind node $(M, (x, y))$ can be expressed as,

$$
\Delta x_M = \frac{\Delta x_a / d_a + \Delta x_b / d_b + \Delta x_c / d_c}{1/d_a + 1/d_b + 1/d_c}
$$

$$
\Delta y_M = \frac{\Delta y_a / d_a + \Delta y_b / d_b + \Delta y_c / d_c}{1/d_a + 1/d_b + 1/d_c}
$$

(15)

The final estimated coordinate of blind node M can be expressed as,

$$
\begin{align*}
\hat{x}' &= x + \Delta x_M \\
\hat{y}' &= y + \Delta y_M
\end{align*}
$$

(16)

### 3 Process of the algorithm

1) Blind node sends location request. After receiving the information, beacon nodes send its own ID and location information periodically to other nodes within its communication radius.

2) Blind node records RSSI value only from the same beacon node, and then calculates the average of RSSI value when receiving the information more than threshold $K$.

3) After receiving multiple RSSI from the different beacon nodes, rank the RSSI from large to small order, choose the nearest three nodes as a reference node.

4) Calculate the distance from blind to beacon node by formula (9), then calculate the preliminary estimate coordinate of blind node by formula (13).

5) According to the mentioned in section (C), the position error of each of reference node is using three reference nodes to locate each other.

6) Obtaining the final estimated coordinate of blind node by considering positioning error weighted compensation to the preliminary estimate of blind node.

### 4 Simulation results

Experiments are carried out in MATLAB7.10.0. In the 50 m $^2$ 50 m square area, there are four beacon nodes, respectively setting their coordinates are $(0, 0)$, $(50, 0)$, $(0, 50)$ and $(50, 50)$; the total of 100 blind nodes are distributed randomly. Transmit power is selected to be 1mW, and the path-loss distance exponent $n$ is assumed 3.875. In order to reflect the influencing factors of multi-path reflections, diffraction, obstructions, etc., Gauss noise $X_s$ with the mean of 0, the standard deviation $s$ is selected to be 8. The coordinates of blind nodes are obtained by using the weighted compensation of coordinate error localization algorithm, then compared with real coordinates, we get the localization error as formula (17).
\[
e_i^2 = (x - x_i)^2 + (y - y_i)^2
\]  
(17)

Where \(i\) is changed from 1 to 100, \((x_i, y_i)\) is the estimated coordinate, \((x, y)\) is the actual coordinate.

The experimental results are shown as Fig.6, which abscissa represents the number of the positioning, and the ordinate represents the positioning error. When using the traditional algorithm to locate, the reference distance is selected to be 1 m, the mean of positioning error is: 0.196829; and when using the algorithm in this paper to locate, the reference distance is selected to be 1 m, the mean of positioning error is: 0.050727; the reference distance is selected to be 5 m, the mean of positioning error is: 0.042713; the reference distance is selected to be 10 m, the mean of positioning error is: 0.039294; We can come to the conclusion that the weighted compensation of coordinate error localization algorithm has higher positioning accuracy than the traditional algorithm. Choosing 10 meters as the reference distance is better than 5 meters or 1 meter, due to the distance from blind node to the reference node is more greater than 10 meters in the 50 m × 50 m square area, this also indicates that select the appropriate reference distance also has some good effects on improving the positioning accuracy.

CONCLUSION

After analyze the principle of RSSI ranging, consider the environmental factors of positioning, put forward a weighted compensation of coordinate error localization algorithm, and do the simulation analysis under the condition of different reference distance. The results show that the algorithm has been greatly improved in accuracy and stability, and with a strong resistance to environmental interference can be used in an actual indoor environment.

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