



## A novel tire-road friction estimation for electric vehicle with in-wheel motors based on process model

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### ABSTRACT

For four-wheel independently driven in-wheel motors electric vehicle, vehicle dynamic control systems such as direct yaw moment control (DYC) can be easily achieved. Accurate estimation of vehicle state variables and uncertain parameters can improve the robustness of vehicle dynamic control system. In order to estimate tire-road friction better, the process mathematical model ( $\mu = K \cdot s$ ) was built on the basis of the proportional relations in the linear region of  $\mu$ -s curve and then identified parameter  $K$  by using Recursive Least Square algorithm (RLS). The rotational dynamics of the wheel model has been taken here to characterize the tire force. The paper designed parameter  $K$  identification algorithm and computed the input and output value of the proposed process mathematical model by the application of Matlab/Simulink software and acquired real-time data by means of AMESim. The results of simulation proved the feasibility of the proposed tire-road friction identification theory better.

**Key words:** Tire-road friction estimation; Process mathematical model; RLS algorithm; Parameter identification

### INTRODUCTION

In the field of automotive chassis electronic control technology, real-time identification of road adhesion conditions is the key technology of active safety control system (ABS, TCS, VSC, ACC, etc.), such as the tire-road friction coefficient in the VSC system determines the ideal yawing angular velocity of the vehicle. Therefore, some domestic and foreign scholars put forward the real-time identification of tire-road friction coefficient of a variety of different ways in recent years, however, using the curve relationship ( $\mu$ -s curve) between adhesion coefficient and slip rate is the most influential theory. The  $\mu$ -s curve is divided into linear and nonlinear parts in this method, to identify the pavement through the different  $K$  values of linear slope in  $\mu$ -s curve on different pavement.

Real vehicle tests have been carried out using this method in reference[3], the experimental results verify the reliability of the theory, however, the  $\mu$ -s curve can be divided into linear area part, nearly linear part, nonlinear part according to  $\mu$ -s curve actually. Scholars like Wang classified nearly linear area as linear area when identifying the  $K$  value, this will cause inaccurate in identification of  $K$  value inevitably, resulting in the unreliability of the tire-road friction coefficient during real-time identification. Take these factors into consideration, this paper divides  $\mu$ -s curve into linear area, nearly linear area and nonlinear area, then identifies, only, the  $K$  value of linear zone in order to eliminate misalignment in  $K$  values, and then estimate the reliable tire-road friction coefficient.

### THE TIRE -ROAD FRICTION COEFFICIENT IDENTIFICATION THEORY BASED ON M-S CURVE

The tire adhesion coefficient and slip rate curve ( $\mu$ -s curve) can be divided into linear area (OA), nearly linear area (AB) and nonlinear area (BC) as shown in figure 1. During (OA), the used adhesion coefficient and slip rate is in the proportion relationship absolutely, while area (AB) is not in proportion relationship. During the process from the

static to the movement, normally, the vehicle wheel slip ratio is gradual change from zero, and the relationship between the adhesion coefficient and the wheel slip ratio is also changing. In other words, the relationship between the adhesion coefficient and the wheel slip ratio changes from (OA) to (AB) area eventually into the (BC). Thus, we just can use their relation curve in (OA) section of the process to establish mathematical model, according the proportional relationship between them.

As shown in Figure 2, the relationship curves between adhesion coefficient and slip rate under different road conditions, which show that of the  $K$  values of the curve family are not the same in the linear area, so we can identify the corresponding road adhesion according to the  $K$  value in the linear zone. During the vehicles running process, the used adhesion coefficient and the slip ratio can be calculated in real time with the vehicle dynamics model and wheel speed sensor, that is to say, the input and output signal in the mathematical model of can be acquired in real time, then the next step is to identify the model parameter  $K$  and to achieve identification of road adhesion conditions.

To identify parameter  $K$ , this paper choose the widely used recursive least squares method (RLS), which has good convergence and tracking properties, and can revise the predicted value using the observed value, at the same time can also to estimate and correct the unknown or uncertain model parameters and noise statistical parameters.

To make it clear, thinks to the road recognition theory in this paper strictly distinguish the linear zone and the nearly linear zoneto identify the parameter  $K$  in a linear area, however, the length of the linear zone in the  $\mu$ -s curve is different under different road conditions. If the used length of linear zone overstep the boundary in the process of identification which results in larger slip rate in recognition, then the linear area will extend to the nearly linear area, where the process mathematical model  $\mu = K \cdot s$  is not true, resulting in great deviation of the parameter identification. This is the reason why this paper argues that the parameter  $K$  identification in ref. [3] is not accurate, so in view of the slip rate this paper take the principle of "make the linear zone narrow not wide".

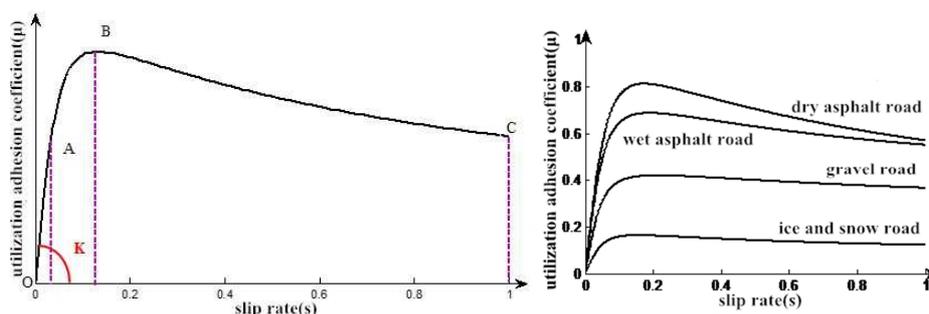


Fig.1 characteristics of  $\mu$  and s Fig.2  $\mu$ -s curve of different road conditions

**REAL-TIME ESTIMATION OF TIRE-ROAD FRICTION**

**1. Vehicle Model**

The Figure 3 shows the vehicle dynamics model. The model ignores the role of the suspension and tire characteristics change of the left and right wheels due to the change of the load. Only the vertical movement of the car body and the tire force isconsidered.

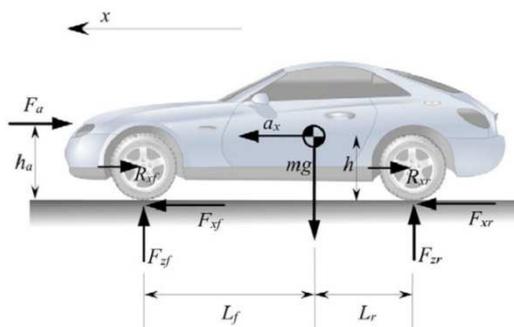


Fig.3 vehicle dynamic model

Based on vehicle dynamics balance equation, the tire longitudinal force( $F_x$ ) and tire vertical force( $F_z$ ) can be calculated:

$$F_x = F_{xf} + F_{xr} = m |a_x| \pm fmg \pm C_D A v_x^2 / 21.15 \quad (1)$$

$$F_z = \frac{mgL_r - ma_x h - C_D A v_x^2 / 21.15}{L} \quad (2)$$

$$F_z = \frac{mgL_f + ma_x h + C_D A v_x^2 / 21.15}{L} \quad (3)$$

Where,  $m$  refers to the mass of the vehicle;  $f$  refers to the coefficient of rolling resistance;  $C_D$  refers to the coefficient of air resistance;  $a_x$  refers to the longitudinal acceleration;  $v_x$  refers to the vehicle velocity;  $L_f$  refers to the distance between CG and front axle;  $L_r$  refers to the distance between CG and rear axle;  $L$  refers to the wheelbase.

## 2. Wheel Dynamics

The rotational dynamics of the 4 wheels are given by the following torque balance equation:

$$(F_{xi} + F_{zi} f_i) r_i = T_i - J_i \dot{\omega}_i \quad (4)$$

Here  $T_i$  refer to the drive/brake torque transmitted to the corresponding wheel.  $J_i$  refers to the rotational inertia and  $\omega_i$  refers to the wheel velocity of the corresponding wheel.  $r_i$  refers to the effective rolling radius of the corresponding wheel.  $i = f, r$

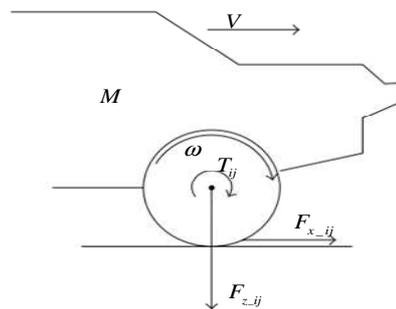


Fig.4 Quarter model of vehicle.

Hence, the lateral force can be expressed as

$$F_{xi} = \frac{1}{r} (T_i - J_i \dot{\omega}_i) - F_{zi} f_i \quad (5)$$

## 3. Computation of Slip Rate

The vehicle model in this article is front-wheel drive, so the vehicle speed can be calculated using the driven pulley wheel speed. It can also be estimated by the extended Kalman filtering state observer algorithm. In this article, computation formula of slip rate shows as follows:

$$s = \left| w_f \cdot r_w - v_x \right| / \{ \max(w_f \cdot r_w, v_x) \} \quad (6)$$

Where,  $w_f$  is the rotate speed of the front wheel;  $r_w$  is the rolling radius of the tire.

## 4. Process Mathematical Model

In order to illustrate the usefulness of this road recognition theory, an example of how to build the mathematical model based on the front-wheel drive vehicle in detail, is given as follows:

Assumes that the vehicle runs on the road with a single attachment coefficient, and the relationship between  $\mu$  and  $s$  is linear, namely  $\mu = Ks$ , then the tire longitudinal force( $F_x$ ) will conform to the equation (7).  $K_f$ 、 $K_r$  values relate to the quantity and structure characteristics of the front and back wheels, and have nothing to do with the road adhesion conditions. Therefore, equation(7) can be converted into (8),  $\alpha$  is the scaling factor, which depends on the quantity and structure characteristics of the front and back wheels.

If the vehicle is front-wheel drive or rear-wheel drive, then  $\alpha$  (or  $1/\alpha$ ) equals 0. The equation(8) can be converted into (9).

$$F_x = F_{xf} + F_{xr} = \mu_f F_{zf} + \mu_r F_{zr} = K_f \cdot s_f \cdot F_{zf} + K_r \cdot s_r \cdot F_{zr} \quad (7)$$

$$\begin{aligned} F_x &= F_{xf} + F_{xr} = K_f \cdot s_f \cdot F_{zf} + K_r \cdot s_r \cdot F_{zr} = K(\alpha F_{zf} s_f + F_{zr} s_r) \\ &= K(F_{zf} s_f + \frac{1}{\alpha} F_{zr} s_r) \end{aligned} \quad (8)$$

$$F_x = K \cdot F_{zr} \cdot s_r \quad (F_x = K \cdot F_{zf} \cdot s_f) \quad (9)$$

According to the foregoing parts, the tire force  $F_x$ 、 $F_{zr}$ 、 $F_{zf}$  and slip rate  $s_r$ 、 $s_f$  all can be obtained in real-time, then the mathematical model  $\mu = Ks$  can be build based on equations (7),(8) and (9).

### 5. Identification of Parameter K by RLS

The stand form of parameter K identification based on  $\mu = Ks$  and the basic principle of RLS algorithm is:

$$y(t) = u^T(t)\theta(t) + e(t) \quad (10)$$

In this article,  $y(t)$  means real-time measured or calculated tire longitudinal force  $F_x$ , the  $\theta(t)$  means parameters  $K(t)$ ,  $u(t)$  stands for slip rate multiply front wheel vertical load  $F_{zf}$ ,  $e(t)$  means the error between the output value  $y(t)$  and the estimated value.

Figure 5 shows the process of identification of  $K$  values. The parameters  $K$  recognition program was written in the Matlab/Simulink according to the flow chart, at the same time, the signals of operation state of the vehicle in the AMESim is acquired in real-time before  $K$  value was program output. It is important to note that the above parameters  $K$  identification process combined with the "make the linear zone narrow not wide" principle, namely setting the parameter identification boundary in the RLS algorithm. When the slip rate beyond boundaries, the program will automatically stop identification parameter K, thus ensuring the accuracy of road recognition.

### ANALYSIS AND EVALUATION OF SIMULATION RESULTS

A combination of Simulink and AMESim simulation model is set up based on the content above, using the visualization of road surface model in the AMESim software to verify the feasibility of this road recognition theory. During the process of simulation, at first, the vehicle velocity is 10 km/h, and began to accelerate after maintain 6 s, to the target speed(80 km/h). Vehicles run in the dry asphalt pavement, wet asphalt pavement, gravel road pavement, ice and snow pavement respectively. The real time sampling frequency of the vehicle motion is 1Hz in all the conditions.

As shown in figure 6 to figure 9:the actual speed and the parameter  $K$  curve under different pavement. It can be seen that parameters  $K$  can be well identified in the following curve, regardless that the vehicle is normal acceleration to the target speed or appear skid phenomenon lead to abnormal. In other words, the identification of parameter  $K$  has nothing to do with the traffic conditions.

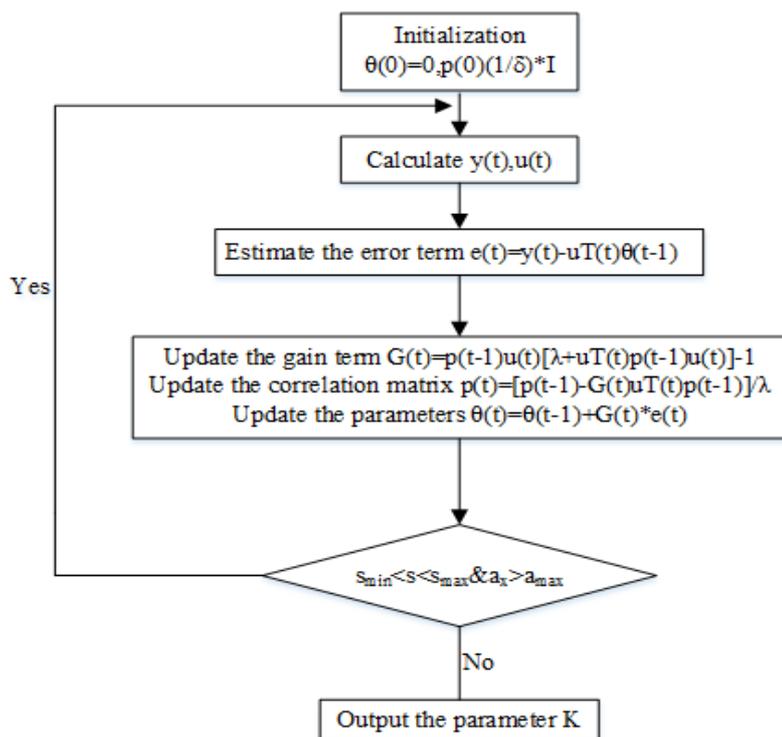


Fig.5 Flow chart of parameter K identification

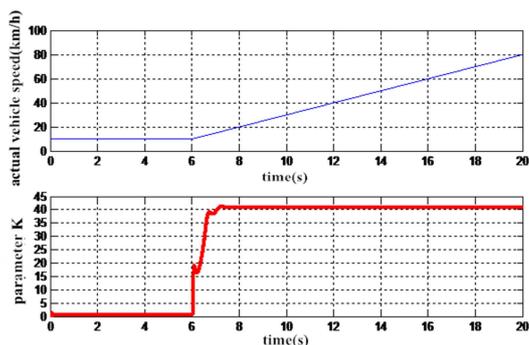


Fig.6 The curve of actual vehicle speed and parameter K on the dry asphalt road surface

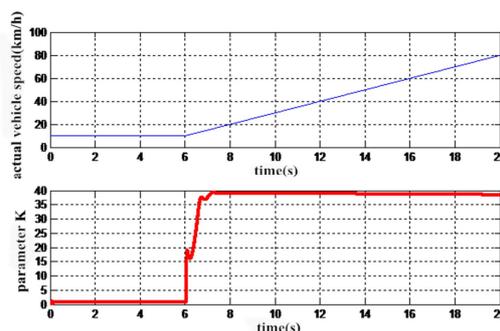


Fig.7 The curve of actual vehicle speed and parameter K on the wet asphalt road surface

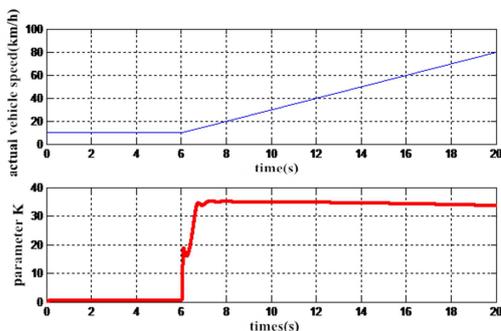


Fig.8 The curve of actual vehicle speed and parameter K on the gravel road surface

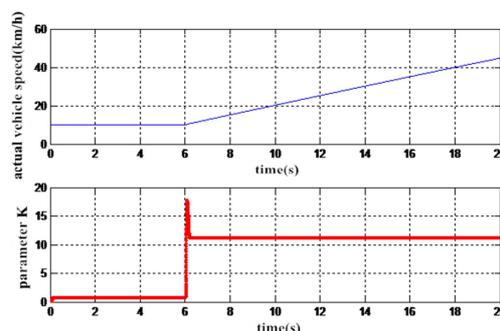


Fig.9 The curve of actual vehicle speed and parameter K on the ice and snow road surface

As shown in the curves above, the greater the parameter  $K$  value, the better the tire-road friction condition, such as on dry asphalt pavement, the parameter  $K = 41$ , while on icy pavement, the parameter  $K = 13$ . This conclusion is consistent with actual pavement properties in Figure 2, and verifies the road recognition theory proposed in this article.

## CONCLUSION

The theory analysis and simulation results show that parameter identification method based on the process mathematical model is very good to achieve the tire-road friction condition recognition, and explain why Wang and other scholars may not be accurate on the road recognition at the same time. In order to further verify the feasibility and practicability of road recognition theory, the team will use this road recognition theory to carry on the real vehicle test.

## REFERENCES

- [1] Rajamani, R. *Vehicle Dynamics and Control* (Second Edition) [M]. Springer, **2011**.
- [2] Gustaffson, F. *Automatical* Vol.33 (6), pp1087-1099, **1997**.
- [3] Wang, J. Alexander, L. and Rajamani, R., *ASME Journal of Dynamic Systems, Measurement and Control, Special Issue on Sensors*, Vol.126, No.2, pp.265-275, June **2004**.
- [4] Hahn, J.O., Rajamani, R. and Alexander, L., *IEEE Transactions on Control Systems Technology*, Vol.10, No. 3, May **2002**.
- [5] Hwang, W. and Song, B.S., "Road Condition Monitoring System Using Tire-road Friction Estimation," Proceedings of AVEC 2000, Ann Arbor, Michigan, pp437-442, Aug. **2000**.
- [6] Müller, S., Uchanski, M. and Hedrick, J.K., "Slip-Based Tire-Road Friction Estimation During Braking," Proceedings of 2001 ASME International Mechanical Engineering Congress and Exposition, New York, **2001**, pp. 213-220.
- [7] Yi, K., Hedrick, J.K. and Lee, S.C., *Vehicle System Dynamics*, Vol. 31, p. 233-261, **1999**.
- [8] Yu Zhisheng. *Vehicle Theory*(Fifth Edition)[M].China Machine Press, **2009**.
- [9] Fang Congzhi, Xiao Deyun. *Process Identification* [M]. Tsinghua University Press, **1988**.
- [10] Simon Haykin. *Adaptive Filter Theory*(Forth Edition) [M]. Electronic Industry Press, **2010**.