A non-cooperative game model between the call centre and the customers

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ABSTRACT

A decision behavior of a call center and the customers are addressed. For a M/M/N+M call center model, a non-cooperative Stackelberg game model is formulated for maximizing the utility value based on the impact of delay time on the customers’ impatient psychology. The performance value of the queuing system is calculated by the combination of the bisection algorithm and the fixed point algorithm based on queue theory. The impacts of the delay time reliability and the patient change rate on the system performance are analyzed with numerical results. Further, Comparison under different game strategies illustrates the reasonability and effectiveness of the method.

Key words—call center, delay information, customer patience, game model, balking and reneging

INTRODUCTION

With the maturity and progress in technology, call centers have been widely used in service industry such as communication, finance, electronic commerce, etc. Call centers have become the essential department of service businesses [1-2]. Studies show that the uncertain wait is much longer than the foreseeable wait so that this kind of invisible queuing system has become the significant defect of call centers [3-4]. Thus, how to overcome this defect and improve the level of customer service is the important issue that call center operation management needs to face.

Guo et al. studied to improve the customers’ service rate by delay information [5]. Armony et al. used an analytic method to estimate the queue waiting time [6-7]. Jouini et al. addressed a further study on how to achieve the optimal behavior probability distribution of balking and reneging by controlling the reliability strategy of delay information [8-9].

Actually, delay information can be regarded as an active method of customer behavior induction to guide and control the mentality and behavior of customers. The non-obligatory of information makes the decision and the reflection of information cue become a game relationship between managers and customers. The study of a call center’s customer behavior mainly adopts the strategy with the combination of queue theory and behavioral science together [10-11], the theoretical work which adopts a master-slave game model for determining the prompt time and the customer patience is very few so far. In this paper, the main goal is to formulate the master-slave game model of a call center’s delay information and the rate of customer patience for the optimal utility of the call center and its customers.

II. PROBLEM DESCRIPTION

In this paper, we address a call center with multiple agents who just can operate a skill, which is described by a model as M/M/N+M. Assuming that there are S service agents in the call center, and the Poisson distribution with rate \( \lambda \) represents the customers’ coming call arrival rate, and the service time is represented by the exponential distribution with parameter \( \mu \), and \( n \) is the number of customers in the queue waiting for service( \( n \geq 0 \) ). The call center queuing service process is shown in figure 1. The initial value of the customer patience is \( T \), i.e. the
maximum waiting time represented by the exponential distribution with parameter $\theta$. The actual waiting time of each customer is $W_n$ and the waiting time prompted to the customers by the call center is $\omega_n$.

When the number of the customers in the queuing system is less than $s$, the first customer in the queue will get the service at once; otherwise, some of them will choose to balk directly with probability $\alpha$, and the others will wait for the delay information, if the delay time promoted by the call center is longer than the patience value, the customers will choose to balk with probability $p_n^b$, as shown in formula (1):

$$p_n^b = P(T < w_n) = 1 - e^{-\omega_n}$$

(1)

Then the remaining customers will enter into the system and wait with probability $\alpha'$ shown in formula (2):

$$\alpha' = (1 - \alpha)(1 - p_n^b)$$

(2)

The call center adjusts the relationship between the customer actual waiting time $W_n$ and the waiting time $w_n$ prompted to the customers by setting up the reliability of the delay information, and the reliability can be defined as follow:

Definition 1: Reliability probability $\beta$: the delay information reliability given by the call center is the probability that the actual waiting time $W_n$ is shorter than the prompted waiting time $w_n$, as shown in formula (3):

$$\beta = P(W_n < w_n)$$

(3)

In generally, the original value of customer patience $T$ will change to a new patience value after they get the delay information, which can be defined as follow:

Definition 2: The new value of customer patience $T'$: When customers get the delay information, the maximum willing waiting time $T$ will change to $T'$ by the patience changing coefficient $\varphi$, $T'$ obeys the exponential distribution with parameter $\theta'$ approximately. $\theta'$ is the customer abandon rate in the queue, which is shown in formula (4):

$$T' = \varphi w_n$$

(4)

In general, $\varphi$ is a constant bigger than 1 and its practical significance is: when customers get the delay information, their psychological activity will be like this: since they have already been waiting for $W_n$, they are willing to wait for a longer time. Certainly the increase of patience changing coefficient will make the customers satisfaction degree decrease.

Definition 3: The customer actual waiting time $W'_n$: refers to that the new customers of the queuing system need to wait for a period of time to get service until the customers ahead leave the queue, and this process is a pure death
stochastic process. Here $W_n$ is the distribution when concerned the circumstance that the value of customer patience has changed.

The variable $W_n$ is the waiting time from state $s+n+1$ to absorbing state $s$. The process from $s+n+1$ to $s$ represents the system outflow state. Therefore the distribution of $W_n$ is the convolution of $n$ independent exponential distributions that the parameters are $s\mu + \theta', s\mu + 2\theta', \ldots, s\mu + n\theta'$, which obey the hyperexponential distribution.

It is assumed that $g_n(t)$ is the probability density function of $W_n$, and $G_n(t)$ is the cumulative distribution function of $W_n$, $E(W_n)$ is the expectation value of $W_n$:

$$E(W_n) = \sum_{i=1}^{n} \frac{1}{s\mu + i\theta'}$$

III. STACKELBERG GAME MODEL FOR INFORMATION RELIABILITY AND DEGREE OF CUSTOMER PATIENCE

Through the analysis of the call center service process, we can find that the information reliability, customer patience and the variation of system performance are relevant, and the relationship between the two sides from their respective interests is a game.

A. The call center utility function

The call center's strategy is induced to control customer abandonment by information reliability. The utility function of the call center:

$$U_{callcenter}(\beta, \phi) = r_{c}p^{s} - c_{1}E(W_{s})$$

$p^{s}$ is the probability customers service, $E(W_{s})$ is the average waiting time for customers, $r_{c}, c_{1}$ is the coefficients for the call center service revenues and customer waiting for the call center loss, respectively.

B. The utility function of customers

Customer strategy is chosen according to the call center prompt information to patient value adjustment and waiting behavior. The utility function:

$$U_{customer}(\beta, \phi) = r_{c} - c_{2}E(W_{s})$$

$U_{customer}$ represents utility customers value type, direct exit and halfway up the customer loss to limit the constraints in the model. $E(W_{s})$ is the average waiting time for customers, $r_{c}, c_{2}$ represent customers get services revenue and customer waiting for its loss, respectively.

C. The two level programming model

The call center and its customers choice action is in accordance with the sequence of call center, to provide a wait time information, then the customer according to wait for information to choose the right of their behavior. According to the above process description, as well as the decision making process of customer call center, can adopt two layers planning to modeling, constructing two level programming models are as follows:

$$\begin{align*}
\max_{\beta_i} U(\beta_i, \phi) &= p^{s} \cdot r_{c} - c_{1}E(W_{s}) \\
\max_{\phi_j} U(\beta_j, \phi) &= r_{c} - c_{2}E(W_{s}) \\
\text{s.t.} c_{p}p^{a} + c_{p}p^{r} &\leq \gamma \\
0 &< \beta_i < 1 \\
1 &< \phi_j < 2
\end{align*}$$

(8)
Among them, the constraints are probability weight and less than constraint value, penalty coefficient, and center and customer choice for programs call representative.

RESULTS AND DISCUSSION

This article investigates the delay information reliability and the customer patience under different operational models, and the effects of different game plan on the system performance through numerical experiments. Using Matlab programming implemented model solving algorithm running on the computer which CPU is Intel Core 2 (2.67 GHz), memory is 2GB. Experimental global parameters set: \( a = 0.01 \), \( \mu = 1 \), \( \theta = 0.5 \), \( S = 10 \), \( c_b = c_r = 1 \), \( \rho = \frac{\lambda}{(S\mu)} \) indicates the call center operations, \( \rho = 0.8 \), \( \rho = 1 \), \( \rho = 1.2 \) respectively represent the quality-driven call centers, quality and effectiveness and efficiency-driven balanced three kinds of operational strategies.

To analyze the influence of the prompts reliability to the average waiting time, it assumes that customer arrival rate \( \lambda = 10 \), customer patience coefficient of variation \( \phi = 1.2 \).

![Figure 2. The reliability impact on system performance comparison under different loads](image)

Fig.2 shows that the reliability is proportional to the average waiting time, and as load increases \( \beta \) influence on the average waiting time becomes greater.

<table>
<thead>
<tr>
<th>Strategy set ((\beta, \phi))</th>
<th>(U_{call})</th>
<th>(U_{customer})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0.5, 1.2))</td>
<td>2.4538</td>
<td>0.6072</td>
</tr>
<tr>
<td>((0.5, 1.3))</td>
<td>2.4593</td>
<td>0.5752</td>
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<tr>
<td>((0.5, 1.4))</td>
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<td>0.541</td>
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<tr>
<td>((0.5, 1.5))</td>
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<td>0.5058</td>
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<tr>
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<tr>
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<td>((0.7, 1.5))</td>
<td>2.4842</td>
<td>0.2413</td>
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</tbody>
</table>
It can be seen from table 1, when the gain coefficient and the opportunity cost ratio is greater than a certain threshold coefficient, the call center should make the higher information prompt reliability strategy, and otherwise it should take the lower information prompt reliability strategy. In the customer aspect, no matter how coefficient ratio, the lower patient value strategy should be taken.

CONCLUSION

This article concentrates on call center prompt information reliability decision and customer behavioral decision and establishes a non-cooperative game model of the master-slave. Through numerical experiments analysis found several conclusions: Prompt information reliability is positively related to the customer service rate, and is negatively correlated with the average waiting time. As call center load is larger, the prompt information reliability sensitivity is greater; Customers patient size is positively related to the average waiting time; On the basis of meeting customer service rate, when the gain coefficient and opportunity coefficient ratio are high, call centers should take high reliability prompt information, otherwise, should take low reliability prompt information; and the customer should take a low value of patience. The variation plays a reference role of prompting information with a call center queue management and customer coordination.

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REFERENCES